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KK BIRLA GOA CAMPUS

Length Analysis in Transition zones of a sinusoidally forced Hodgkin-Huxley Neuron

Project Report
by

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Abstract

When electrodes are inserted in Neuron Membrane and different types of external currents(stimuli) are given to them, the membrane potential shows rich dynamics. In this study we are going to give periodic stimuli and do analysis of the response which is in the form of membrane potential. Mode-locking is seen in the membrane potential. The transition zones between these mode locked states have been studied in this work. This study provides a deeper insight in the phase transition patterns(frequency and lengths) when the Hudson-Huxley Neuron is excited by a sinusoidal external current.

1.Introduction

Study of Neurons is necessary .With the help of neurons ,different biological processes are coordinated. Nowadays the study of neurons is necessary for the field of cognitive science also. Neurons have very rich dynamical properties. In this study, the behaviour of membrane potential is studied .Membrane potential is the potential difference between the outside and inside of a cell .Action potential is a speedy rise and subsequent fall in membrane potential with a characteristic pattern. Sufficient current is required to initiate a voltage response in a cell membrane; only if the potential is above a threshold value then the action potential is triggered

This membrane current (external) acts as a stimulus and the response is in the form of membrane potential .This external current has two control parameters : (i) amplitude of current and (ii) flow of the current injected(steady, pulse, periodic, etc,). The study of the dynamics of neurons is necessary as it gives us information about neural mechanisms.This data can further be used in cognitive science,medical research,etc.

A steady external current with constant amplitude(greater than threshold value) gives rise to a periodic train of spikes , this is due to Hopf Bifurcation. Under periodically varying stimulus with a fixed frequency and amplitude we get a rich variety of responses. In this kind of stimulus, we can see two different frequencies namely the natural frequency of the neuron and the forcing frequency of the external current. The interaction between these frequencies give rise to complex dynamical responses which have already been studied in previous publications[2-4]. One special case of such periodic stimulus is sinusoidal current, from earlier work [2-3] it can be seen that for a large range of amplitude and frequency of the external current, mode-locked oscillations are observed between the external current and action potential. While in some smaller regions other periodic oscillations and chaos are observed. With help of nonlinear dynamics,bifurcation patterns near these states have already been studied [3].

The terminologies used in our work are the same as that of previous reports and publications. The study of underlying patterns of “Freq of spike groups vs External Current” for transition regions has already been studied in [4]. This report is in continuation to [4] where we have done analysis of length of the signals in these transition regions.

In Sec 2 we have discussed the Hodgkin-Huxley model of the Neuron .In Sec 3 we have discussed the precious work on the topic done by publications and reports before. Sec 4 covers the discussion on the results obtained in our work. Sec 5 covers the conclusion and scope for future work.

2.Hodgkin-Huxley Model

One such model that describes the dynamics of real neuron membrane is the Hodgkin-Huxley model which is a conductance based model developed by Alan Hodgkin and Andrew Huxley .This model was developed from electrophysiological experiments on squid giant axons . The model is based on nonlinear conductance of potassium and sodium channels.

The model is basically a set of four coupled nonlinear ordinary differential equations. External current (I_{ext}) is the control variable. Fundamental neurodynamic variables are V, m, n, and h. V is the membrane potential, and m, n, and h are the gating variables for the ion channels through which ions cross and produce neural current. the model is described by the equations given below:

$$C\frac{dV}{dt} = -\bar{G}_{Na}m^3h\left(V - V_{Na}\right) - \bar{G}_K n^4\left(V - V_K\right) - \bar{G}_L\left(V - V_L\right) + I_{ext} \quad (2.1)$$

$$\frac{dm}{dt} = \alpha_m(1-m) - \beta_m m \quad (2.2)$$

$$\frac{dh}{dt} = \alpha_h(1-h) - \beta_h h \quad (2.3)$$

$$\frac{dn}{dt} = \alpha_n(1-n) - \beta_n n \quad (2.4)$$

where,

$$\alpha_m = \frac{0.1(25-V)}{\exp[(25-V)/10]-1}, \quad \beta_m = 4\exp[-V/18])$$

$$\alpha_h = 0.07\exp[-V/20], \quad \beta_h = \frac{1}{\exp[(30-V)/10]+1}$$

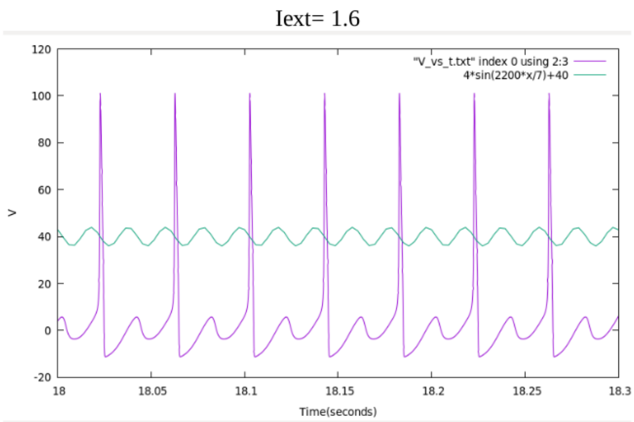
$$\alpha_n = \frac{0.01(10-V)}{\exp[(10-V)/10)]-1}, \quad \beta_n = 0.125\exp[-V/80]$$

here,
Capacitance of the axonal membrane C = $1\mu F/cm^2$.
The reversal potentials of sodium, potassium, and leakage channels are V_{Na} = 115mV, V_K = 12mV and V_L = 10.5995mV respectively.
The maximal conductances of the membrane for sodium, potassium, and leakage currents are $\overline{G}_{Na}$ = 120mS/cm^2,$\overline{G}_K$=36mS/cm^2, $\overline{G}_L$ = 0.3mS/cm^2 respectively.

In simulation for computing solutions of the above differential equations, the fourth order Runge Kutta method is used. As mentioned before, the type of external current(forcing) used is sinusoidal .I_{ext} = $Asin(2\pi\omega t)$, with amplitude A and frequency ω as the control parameters. But in our case, we have kept the frequency to be constant at 50Hz. The amplitude of the current varies and the simulation runs over current steps .These ranges of amplitude are selected such that we are able to observe the transition zones between two mode locked states .For each current value,fourth order Runge Kutta method is used to find solution and time step used is dt=T/1000 where T is time period of oscillation of the external(forcing) current.

3.Previous Work

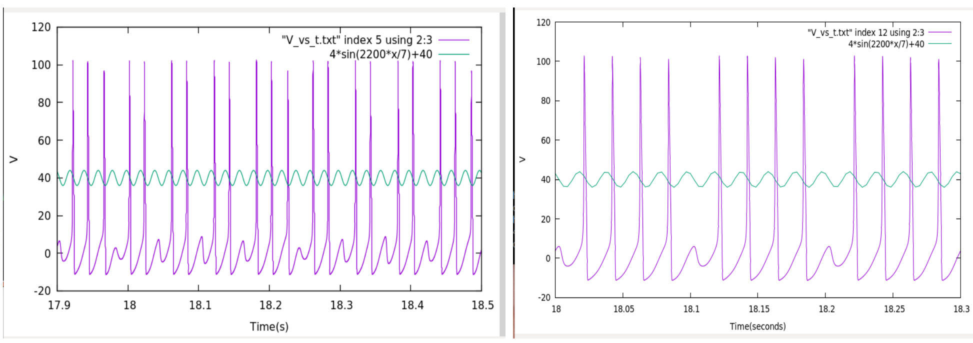
Various mode-locked states have been observed, studied using nonlinear dynamics and analysed based on bifurcations[3] . The mode-locking of the form 1/n (where n=1,2,3…) are known as fundamental locked states. 1/n over here is the firing rate. Firing rate is the number of spikes that rise in one time period of the forcing(driving) signal , which in this case is the external current. for eg. If m spikes appear in a time interval nT(where T is the time period of the forcing current) then firing rate is m/n. As mentioned before ,for fundamental mode-locked states , m=1.



1/2 locked state

FIGURE 3.1: ½ Fundamental mode locking of Hodgkin-Huxley Neuron under sinusoidal current

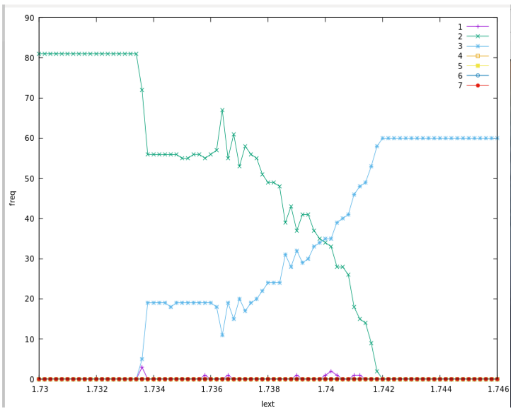
Other than fundamental mode-locking, there are other general mode-lockings that appear.



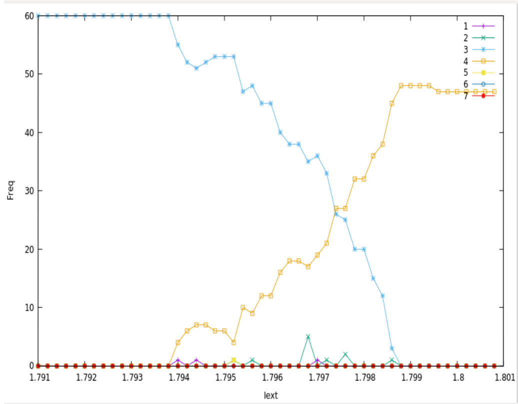
(a)Iext=1.7345 (b)Iext=1.82

FIGURE 3.2: general mode locking .
In figure(a) we can see a 9/13 mode locked state repeating sequence of {3.2.2.2} in the spike group,In figure (b) we can see 4/5 mode locked state with {1...1} repeating sequence

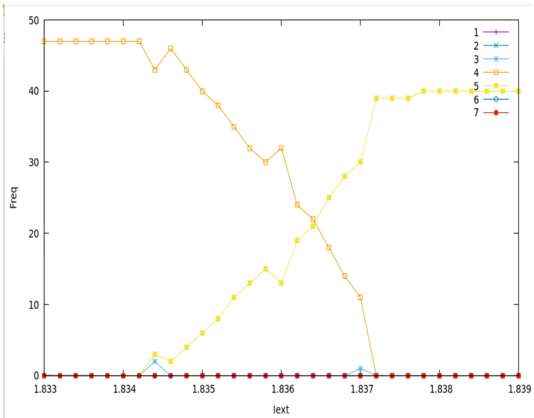
In earlier work, the transition zones between any general (m-1)/m and m/(m+1) mode-locked states were studied for underlying patterns . In [3] it was shown that tangent bifurcation and period doubling bifurcation is responsible for the transitions. The patterns showed a particular behaviour which is shown in [4]



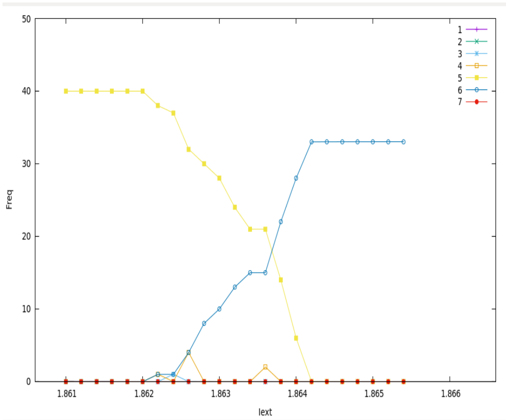
(a)m=3



(b)m=4



(c)m=5



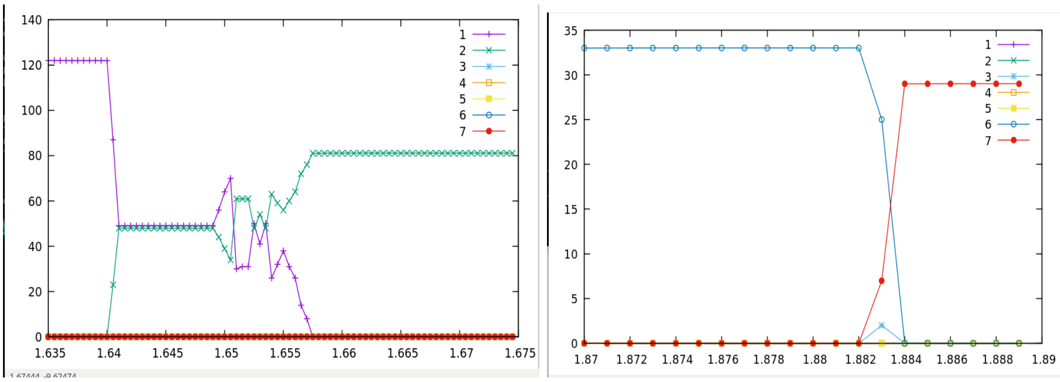
(d)m=6

FIGURE 3.3: Frequency of Spike groups vs External Current Amplitude at a constant forcing frequency of 50Hz.

As we can see in Figure 2.3 (a),(b),(c),(d) two frequencies of spike groups crossover with respect to current in these transition zones.

4.Results

In [4] the patterns in the transition zones were studied as mentioned in Sec 3, similar patterns were observed in the vicinity of the transition zones.



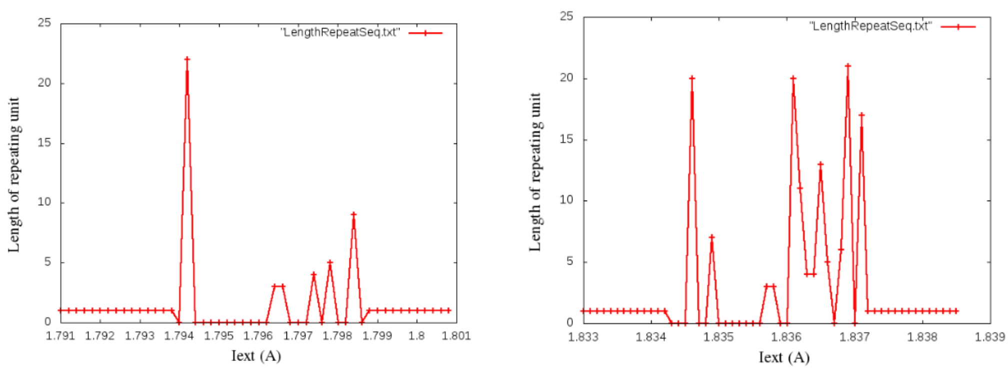
(a)m=2 (b)m=7

FIGURE 4.1: Frequency of Spike groups vs External Current Amplitude at a constant forcing frequency of 50Hz.The transition occurs from mode locking state of (m-1)/m to m/(m+1).

As we can see in figure 4.1 , the previous conclusion on the frequency of the spike groups holds for these cases also. An important observation over here is that as m increases the range of external current over which the transition occurs decreases .

We define **Length (L)** of a sequence as the number of spike groups in a repeating unit.For eg in figure(3.2a) length is 4. For mode-locked state (m-1)/m, the repeating unit is {(m-1).}, hence the length of the repeating unit for any general (m-1)/m locked state is 1 and the lengths of chaotic regions is 0. Thus the regions on both of the sides of transition regions, the length is 1.

We define **Resolution** as the external current step that we take for simulation. For eg at resolution of 10^{-4} there are 20 steps in between external current values of 1.872 and 1.874.

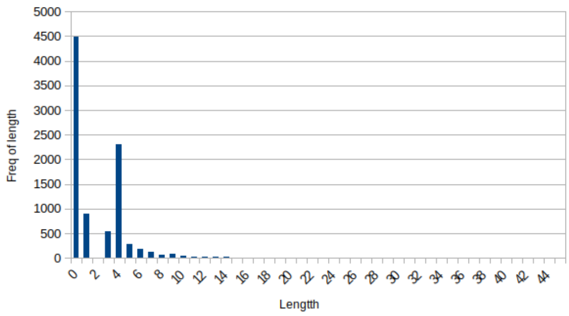


(a)m=4 (b)m=5

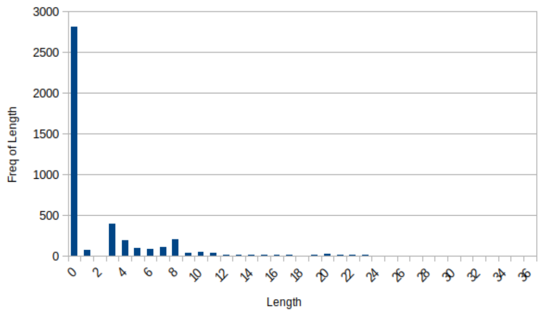
FIGURE 4.2: Length vs External current at a resolution of $2x10^{-4}$

In earlier study, resolution of $2x10^{-4}$ which was not enough and as a result many points were skipped and we were not able to observe enough lengths.which can be seen in the figure 4.2

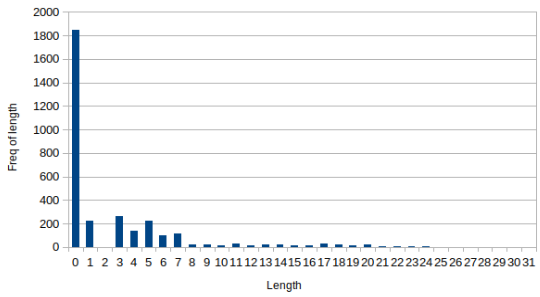
But when we increase the resolution to 10^{-6} , the number of steps for the same region increases, instead of 20 steps we will have 2000 steps. i.e for m=5 transition we will have 3200 steps and practically it's not possible to observe so many points on graphs so instead we will plot frequency of length vs length for that particular transition.



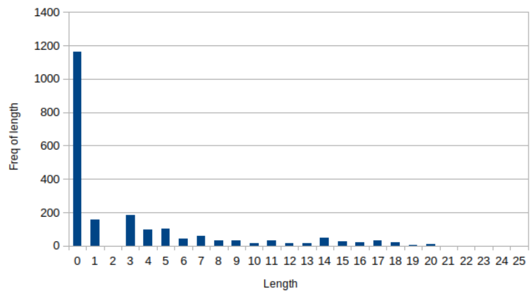
(a)m=3(Iext=1.7332 to 1.7425)



(b)m=4 (Iext=1.7939 to 1.7987)



(c)m=5(Iext=1.834 to 1.8372)



(d)m=6(Iext=1.862 to 1.8641)

FIGURE 4.3: Length vs Frequency of length (resolution of 10^{-6})

Note: the x axis scale is shifted to the left note that 1st bar is 0, 2nd is 1 and so on .As some of the data points which have very low frequency cannot be seen clearly in these plots,the data has been attached as an appendix.

As we can see from Figure 4.3 (b),(c), we can see that all lengths from minimum to maximum i.e from 1 to 30 and from 1 to 24 occur for transition m=4 and m=5 respectively which was expected after seeing the pattern of the transition.

But from figure 4.3(a),(d) we can see that some lengths are missing from m=3 and m=6 transitions . Which means that this resolution is not enough to analyse our assumption for these transitions so we need to increase the resolution further .For resolution of 10^{-7} data points for the transition m=6 increase from 2100 to 21000.

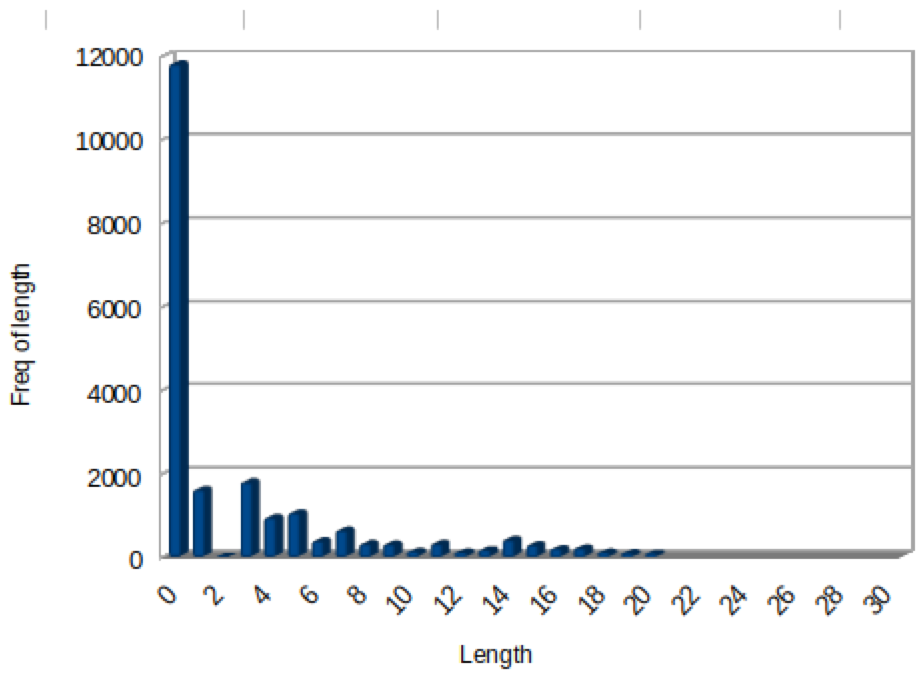


FIGURE 4.4: Length vs Frequency of length (resolution of 10^{-7})

Note: he x axis scale is shifted to the left note that 1st bar is 0, 2nd is 1 and so on .As some of the data points which have very low frequency cannot be seen clearly in these plots,the data has been attached as appendix

As we can see in Figure 4.4 , all the lengths from 0 to 20 appear which is expected for the transition of m=6

5.Conclusion

In our work we have studied the transition zones between the mode locked states . Few more transitions in the vicinity of the previous zones have been discovered .From this analysis we can see that region in vicinity of a transition of (m-1)/m to m/(m+1) is dominated by “m-1” and “m” spike groups.Length analysis at different resolutions has given expected results , thus verifying the assumptions. The patterns of all these results have been plotted and more descriptive data of each of these analysis is given in the excel sheets where results of each sub simulation has been stored.The length analysis as well as the transition zone analysis is consistent with the previous findings [2,3].

The analysis of different properties of spike groups is necessary because by analysis of different patterns of properties we can predict the properties and with the help of data that we have collected along with the previous findings, we can get an idea of the system .All this data is very useful in the streams of medical sciences, neuroinformation processing, cognitive science ,etc. .The predictability of the system will save us many resources such as simulation time in future . This is necessary as the resolution of the simulation increases, the data points increase by logarithmic scale.

In future work the same findings can be proved for other transition regions and different properties of these Spike groups can be studied to reveal their underlying patterns and relations.

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Bibliography

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3. https://neuronaldynamics.epfl.ch/online/Ch2.S2.html

Simulation

1. System used for simulation: Hp laptop 8gb RAM, 4gb Nvidia, intel i5 8th gen
2. Operating System: Ubuntu 18.04
3. Simulation time for each current value t=0 secs to t=20 secs.
4. Run time for one current value(including both codes): 5 secs (approx)
5. Total Run time for the results obtained in this paper: 60+ hours

Appendix

Resolution 10^{-6}

m=3

Length	Freq of L
0	4497
1	890
2	0
3	546
4	2303
5	276
6	194
7	134
8	73
9	77
10	39
11	34
12	22
13	19
14	18
15	13
16	13
17	12
18	6
19	9
20	11
21	8
22	7
23	8
24	11
25	12
26	12
27	10
28	9
29	9
30	11
31	5
32	3
33	2
34	5
35	2
36	1
37	0
38	1
39	0
40	0
41	2
42	0
43	0
44	0
45	0

m=4

Length	Freq of L
0	2808
1	77
2	1
3	392
4	199
5	97
6	91
7	112
8	203
9	38
10	45
11	44
12	17
13	18
14	14
15	14
16	10
17	14
18	9
19	12
20	32
21	14
22	18
23	19
24	5
25	7
26	8
27	6
28	1
29	1
30	3
31	0
32	0
33	0
34	0
35	0

m=5

Length	Freq of L
0	1848
1	220
2	1
3	263
4	138
5	224
6	103
7	114
8	20
9	25
10	13
11	33
12	17
13	18
14	20
15	14
16	13
17	32
18	24
19	16
20	22
21	7
22	8
23	6
24	4
25	0
26	0
27	0
28	0
29	0
30	0
31	0

m=6

Length	Freq of L
0	1162
1	159
2	0
3	181
4	94
5	104
6	44
7	57
8	33
9	32
10	17
11	33
12	14
13	17
14	47
15	26
16	21
17	29
18	18
19	6
20	7
21	0
22	0
23	0
24	0
25	0

Resolution 10^{-7}

m=6

Length	Freq of L
0	11815
1	1615
2	9
3	1802
4	947
5	1059
6	384
7	638
8	314
9	302
10	133
11	315
12	112
13	170
14	416
15	289
16	195
17	211
18	111
19	93
20	86
21	0
22	0
23	0
24	0
25	0