Laboratory of Evolutionary Computing

Ant Systems

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1 Introduction

Ant algorithms were first proposed in early 1990s as a multi-agent approach to solve difficult stochastic combinatorial optimization problems [1], [2]. The main characteristics of this approach are the use of a population based search, robustness and a positive feedback. The ant algorithms in some applications can be outperformed by more specialized methods but as a meaningful general approach they offer a great degree of flexibility to different problems.

2 Biological Inspiration of Ant Algorithms

Ant algorithms were inspired by the study of real ant populations. Ant are social animals known for their hard work and cooperation. They are almost blind and their behavior is directed more to survival of the ant colony as a social system then to that of a single individual. They present a very great degree of structuration in their colonies compared to the relative simplicity of an individual. One of the most interesting and important properties of ant populations is their ability to find the shortest paths between sources of food and their nest.

While searching for a food sources and during their way back to the nest, ants deposit on the ground a substance called *pheromone*, forming in this way a pheromone trail. Ants can smell the pheromone on the ground and choose their path with a probability proportional to the pheromone concentration. On the chosen path ants deposit a new dose of pheromone altering the previous concentration. This concentration is also changed by a pheromone evaporation process. The pheromone trail can be exploited by other individuals to discover the most interesting (shortest, less dangerous) path from their nest to the food resources and back.

In order to study the ants behavior the following experiment was performed. The nest and a food source have been separated by a bridge (see Figure 1). This bridge has two branches of equal length. The ant population has been left for one hour to move free between the nest and the food source. During this time the number of ants which choose the right (R) or left (L) branch was counted. Initially on the branches there was no pheromone and the ants selected both branches with equal probability. After this transitory stage, more and more ants tend to select only one bridge branch. These ants deposit during their route a new quantity of pheromone and in this way stimulate more ants to choose this path, forming a positive feedback. At the end of this experiment almost all ants were walking on the same branch.

The mathematical model that follows the experimental observation can be now described. The model assumes that the quantity of pheromone on a branch is proportional to the number of ants that used this branch before. This assumption implies that the pheromone evaporation process is not taken into account. Such simplification is valid as long as the experiment lasts not longer than several hours (in this time the amount of the evaporated pheromone is negligible). Let R_m and L_m denote the number of ants that have used the right and left bridge branch after m ants have crossed the bridge, $R_m + L_m = m$. The probability $P_R(m)$ with which the $(m + 1)$-th ant selects

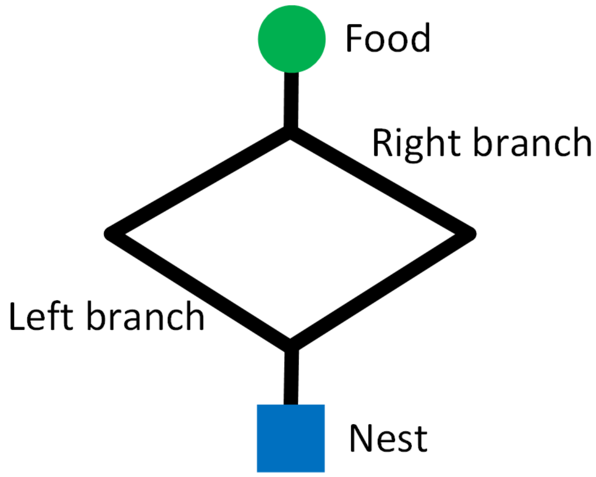

Figure 1: A single, symmetric bridge.

the right branch is

$$P_R(m) = \frac{(R_m + k)^d}{(R_m + k)^d + (L_m + k)^d}. \tag{1}$$

The probability $P_L(m)$ that this ant follows the left branch is given by

$$P_L(m) = 1 - P_R(m). \tag{2}$$

The $(m+1)$-th ant chooses its path according to the following conditions

$$\begin{cases} R_{m+1} = R_m + 1, L_{m+1} = L_m, & r \leq P_R(m), \\ R_{m+1} = R_m, L_{m+1} = L_m + 1, & \text{otherwise,} \end{cases} \tag{3}$$

where r is a random variable with uniform distribution in the interval $[0, 1]$. The parameters k and d let fit the model to the data and their values were found experimentally: $k \approx 20$ and $d \approx 2$.

The model above can be easily extended to the case in which the bridge branches are of different length. In this case the pheromone laying mechanism is the same as in the previous situation and the shortest branch is quickly chosen. The first ants that arrive to the food are the ones that took the shortest branch. When these ants start to return to their nest there is more pheromone present on the shorter branch than on the longer one.

The above described process is a kind of a distributed optimization behavior. Although a single ant is in principle capable to find a solution (i.e., to find a path between the nest and a food source), the optimal (shortest) path finding mechanism is only a property of the ensemble of ants – the ant colony. It is very interesting to note that this optimization behavior is developed by using only a very limited form of indirect communication. This form of communication can be interpreted as a stimulation of co-workers by the performances they have achieved and is characterized by the physical nature of the information and its local nature (the information is accessible only to these individuals that visit the neighborhood of the state where the information was released). Another important aspect of the ant behavior is the positive feedback and the implicit evaluation of solutions. The shorter paths will be completed more quickly than the longer ones and the pheromone reinforcement will be more important. The shorter the used path is, the sooner the pheromone is deposed and the more ants use the shorter path.

3 Ant Colony Optimization

The ant colony optimization method is an approach to find good solutions to difficult discrete optimization problems using a population of artificial, cooperating and simple individuals - artificial ants. The artificial ants, on the one hand, are an abstraction of the real ants. In particular, they:

- are a colony of cooperating but asynchronous individuals searching for a good solution.
- use a pheromone (artificial) trail for local communication. Only ants that visit the place where the pheromone trail was deposed can receive this information from the previous ants.
- search for a shortest (minimum cost) path from the origin (nest) to the destination (food). They perform only local moves (do not jump) moving step by step through neighbor states.
- build the solution using stochastic local decisions. They do not know the future or try to predict future states.

On the other hand, artificial ants are enriched with some properties that are not found in the real world. In fact, the goal of this approach is to create an effective optimization mechanism and not to simulate the behavior of real insects so it is justified to give ants some additional capabilities. In particular, the artificial ants:

- live in a discrete world and their moves are also discrete.
- have an internal state which contains the history of the past actions of the individual.
- deposit a quantity of pheromone that is a function of the quality of their performances (the quality of the solution found).
- often deposit the pheromone trail only after having generated a solution.
- can be given some additional capabilities like: local optimization, look ahead, backtracking, etc.

4 Application of Ant Colony Optimization

The Ant Colony Optimization is particularly suitable for difficult combinatorial optimization problems: static and dynamic. Static problems are the ones where their characteristics are given once and do not change during the search for a solution. An example of a static problem is the traveling salesman problem. Dynamic problems can be divided into two categories. First category includes problems where the environment (cost function) changes over the time and the optimization algorithm must take into account this changes. Second class of dynamic problems consists of problems where the topology of environment (available paths and nodes) changes during the optimization process. The structure of the ant colony optimization algorithm highly depends on the nature of the optimization problem. The following part of this manual presents a more detailed view on the application of the ant colony optimization to the traveling salesman problem.

5 Traveling Salesman Problem

One of the first applications of the ant colony optimization method was the optimization of the traveling salesman problem. The traveling salesman represents a very easy and straightforward adaptation of the idea of ant colony optimization due to its direct structure and goal correspondences.

The traveling salesman problem consists in finding a minimal length circuit on a graph composed of N nodes (cities) connected by arcs (routes) (see Figure 2). The connection is full (every node is connected with each other) and the length of arc (distance between cities) (i, j) is d_{ij}. The minimal length circuit represents a closed tour where all the cities were visited once and only once. The final (total) length of the tour is given as a sum of the lengths of all used arcs.

6 Ant System

Ant System is one of many variants of ant colony optimization algorithms. In this approach solutions are constructed by moving m artificial ants on the problem graph. The ants build a tour visiting $n = N$ cities using a stochastic decision rule presented later. After a complete tour is constructed the artificial ants deposit the pheromone trail (update a variable associated with each arc). The amount of pheromone $\tau_{ij}(t)$ deposed in iteration t on arc (i, j) represents the desirability to move from city i to city j. The quantity of pheromone deposed by every ant is proportional to the quality of the solution found.

The internal memory of each ant contains the list of already visited cities and is used to identify the details of the chosen path. By exploiting its memory an ant k can build a set of feasible solutions.

The ant decision in node i is based on a decision table $\mathbf{A}_i = [a_{ij}(t)]_{[N_i]}$ obtained

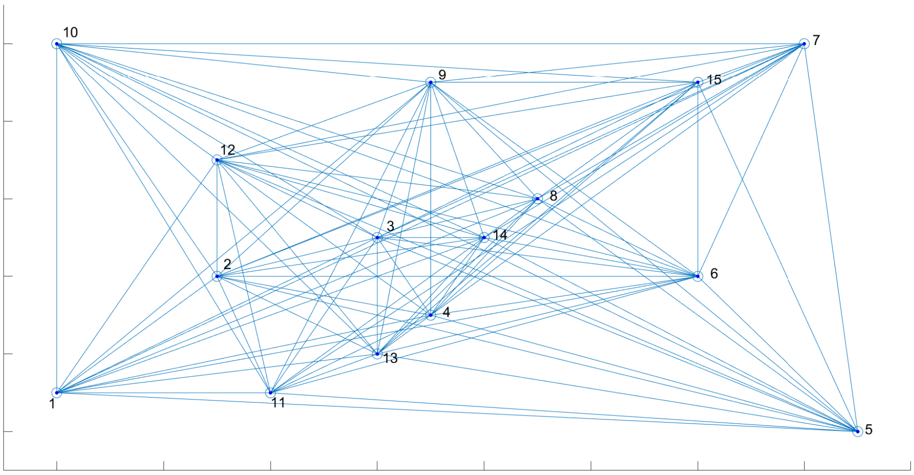

Figure 2: An example of a city network ($N = 15$).

using the following rule:

$$a_{ij}(t) = \frac{(\tau_{ij}(t))^{\alpha} (\eta_{ij})^{\beta}}{\sum\limits_{l \in N_i^k} (\tau_{ij}(t))^{\alpha} (\eta_{ij})^{\beta}}, \quad \forall j \in N_i, \tag{4}$$

where τ_{ij} is the amount of pheromone on arc (i, j) at time t, $\eta_{ij} = 1/d_{ij}$, N_i is the set of neighbors cities of city i, $N_i^k \subseteq N_i$ is a set of neighbor cities of city i that ant k has not visited yet, and α and β are control parameters.

The ant k chooses to go from city i to city $j \in N_i^k$ with probability

$$p_{ij}^k = \frac{a_{ij}(t)}{\sum\limits_{l \in N_i^k} a_{ij}(t)}. \tag{5}$$

After all m ants have completed their tour the deposition and evaporation of pheromone mechanism is started. This mechanism can be described by the formula

$$\tau_{ij}(t+1) = (1-\rho)\,\tau_{ij}(t) + \Delta\tau_{ij}(t), \tag{6}$$

where $\rho \in [0, 1]$ is the pheromone evaporation coefficient, and the total quantity of pheromone $\Delta\tau_{ij}$ deposed on arc (i, j) is specified by

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^k(t). \tag{7}$$

The quantity of pheromone $\Delta\tau_{ij}^k(t)$ deposed by ant k on arc (i, j) (that it has used) is given by

$$\Delta\tau_{ij}^k(t) = \begin{cases} 1/L^k(t), & \text{if } (i, j) \in T^k(t), \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

where $L^k(t)$ is the length of the tour $T^k(t)$ made by the k-th ant.

Before the start of the optimization algorithm a small amount of pheromone $\tau_0 > 0$ is deposed on all arcs $\tau_{ij}^k(0) = \tau_0$. A good choice for parameter values is $m = n$, $\alpha = 1$, $\beta = 5$ and $\rho = 0.5$ but the optimal set depends on the investigated problem.

7 Tasks

1. Write a computer program simulating the ants colony behavior in an environment where the nest and the food are separated by a double, asymmetric bridge (see Figure 3).

2. Implement the Ant System to solve the traveling salesman problem. Use a map of cities provided by a tutor ($m = N = 10$, the maximum number of tours $T_{\max} = 200$. What was the minimal total distance traveled? What is the sequence of cities to be visited ensuring the minimal total distance traveled? Show the results in a graphic form (as a network of connections).

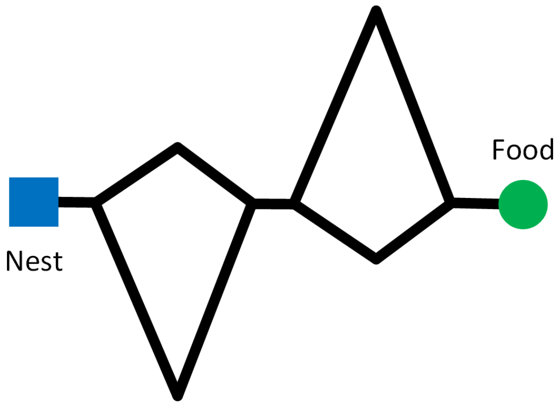

Figure 3: A double, asymmetric bridge.

References

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[2] M. Dorigio, L.M. Gambardella, *Ant Colony Systems: A Cooperative Learning Approach to the Traveling Salesman Problem*, IEEE Trans. on Evolut. Comp., vol. 1(1), pp. 53–66, 1997.