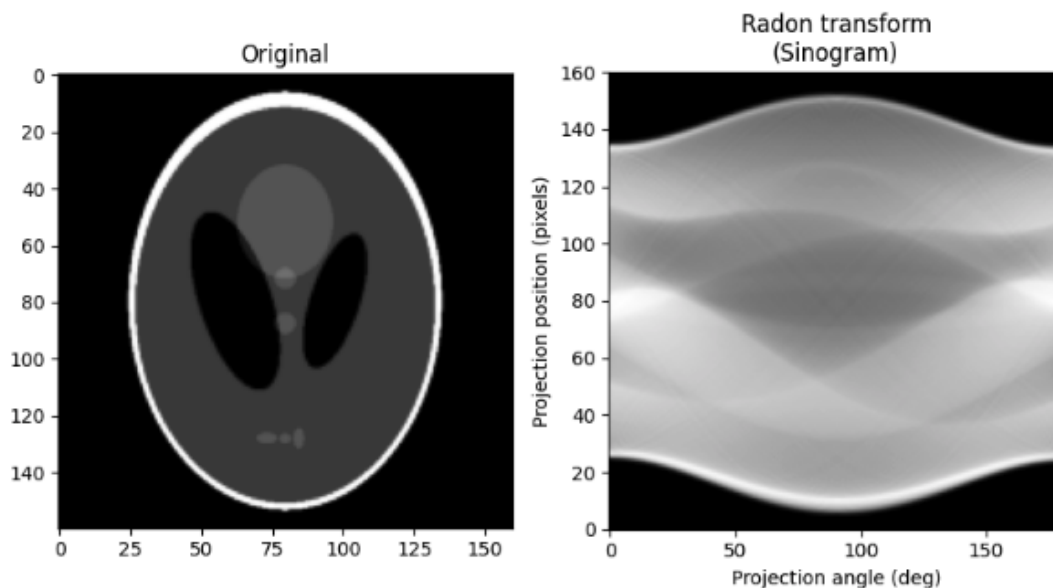


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CT Reconstruction

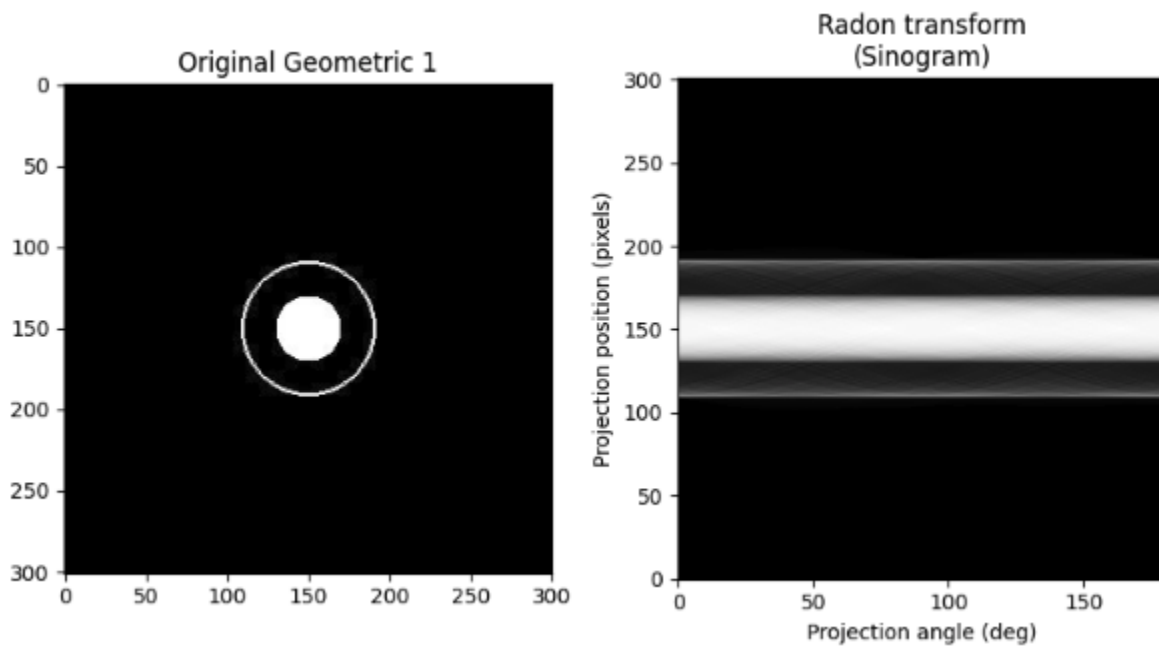
In this exercise, we were tasked with exploring the basic principles of image reconstruction in computed tomography (also known as CT), while particularly focusing on filtered back projection and iterative reconstruction. CT is a volumetric medical imaging technique based on X-Ray transmission but involves taking multiple X-Ray images of the body from different angles and then reconstructing these into a 3D image.

In step 1, we were tasked with creating a head phantom or using the Shepp-Logan head phantom to generate a bright ellipse that corresponds to the skull and internal structures that correspond to various brain tissues and lesions. The Shepp-Logan head phantom mimics the density of human tissue to represent various anatomical features, and it is a standardized test object in CT scans. We did this using the `shepp_logan_phantom` function from the `data` module of `skimage` library. We then calculated the synthetic projection using radon transform (parallel beam geometry) for the head phantom, which is used to simulate the 1D projections obtained by a CT scanner. The sinogram plots the projection data as a function of the projection angle and distance from the center of the rotation.

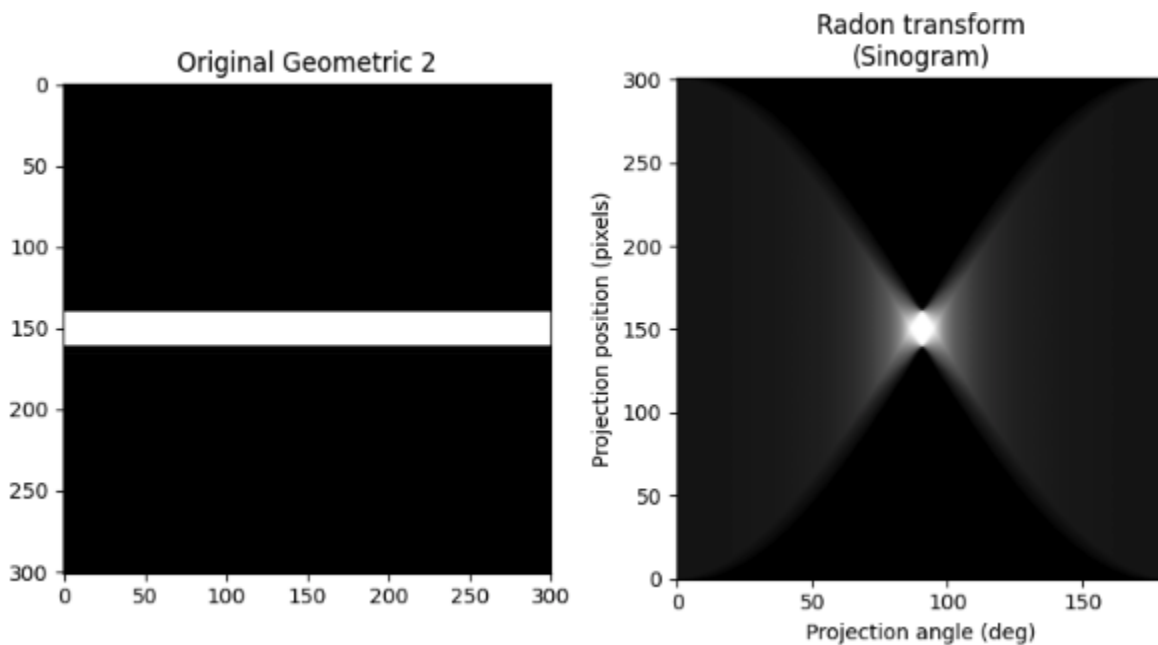


The sinogram shows two peaks that correspond to the main shapes within the phantom. Each shape is represented as a cylinder, and overlapping shapes produce a darker hue.

Additionally, here is the synthetic projection using radon transform for geometric shapes:

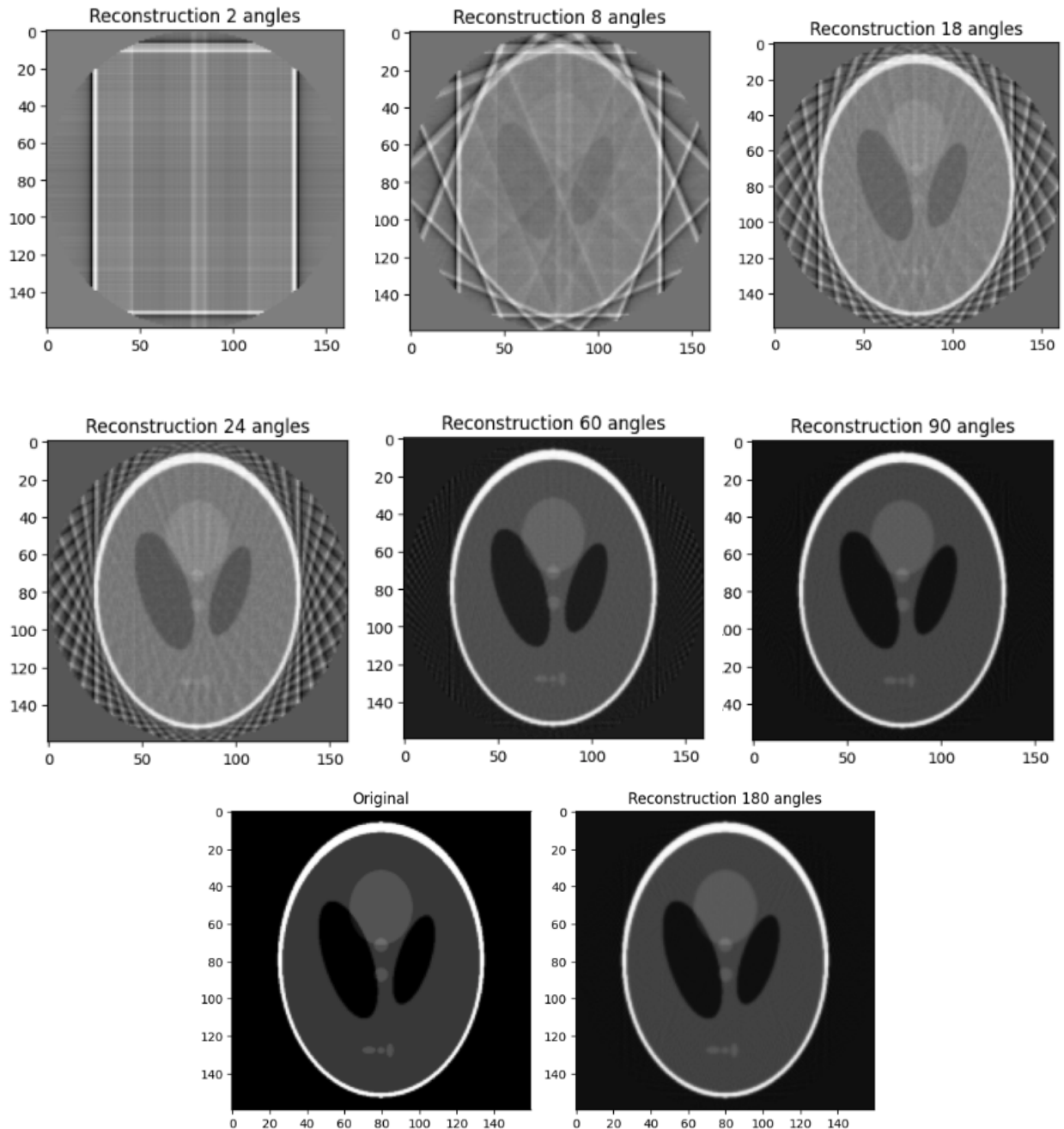


In this case, the sinogram represents the direct beam that passes through the object without being attenuated. It even accounts for the intensity of the X-ray beams through the color brightness.



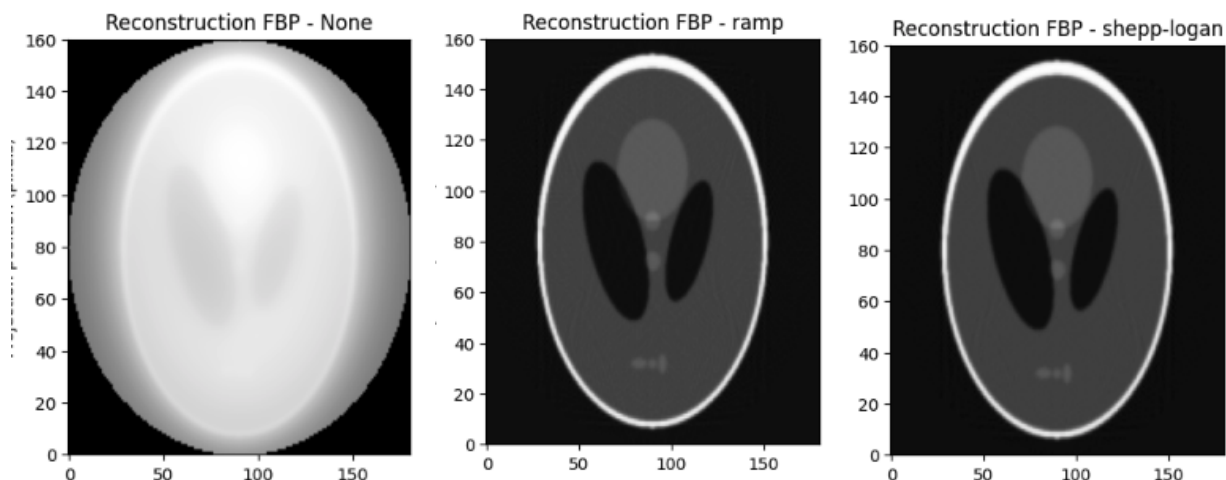
In this case, the original beam is represented as a point of bright light in the sinogram. The area of brightness corresponds to the area where the X-ray passed through without being altered.

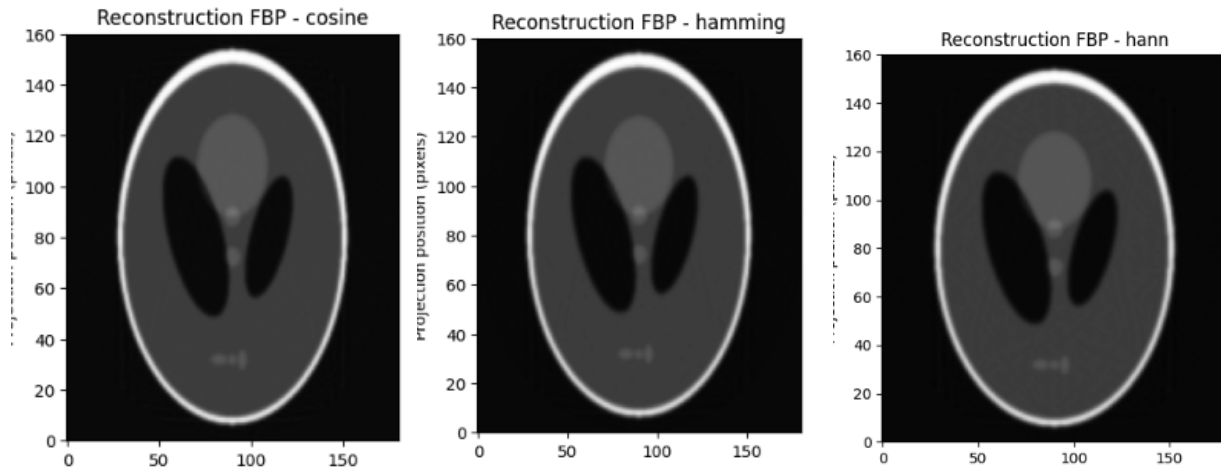
In the next step, we varied the number of projection angles and performed a synthetic reconstruction for each angle increment. In general, when the number of projection angles is low, there is less information available for the reconstruction so the sinogram has lower resolution and captures a limited amount of information about the internal structures. This results in more artifacts, blurring and loss of details. At 2 angles, the reconstruction only shows the rough shape. At 8 angles, the reconstruction captures the real shape and starts to detect the internal structure. At 18 angles, the shape and structures become less blurry. At 60 and 90 angles, the background color comes through and the color of the internal structures come through. The best reconstruction is at 180 angles, which has the best quality image and less artifacts, so the details are better differentiated.



In the third step, we demonstrated the differences of performing back projection and filtered backprojection. Both are used to reconstruct images from the X-Ray data, but filtered backprojection (FBP) incorporates a filtering step to enhance the quality of the reconstructed image by removing high frequency noise and other artifacts. We compared the original image to the FBP reconstruction using different filters, as well as to regular backprojection (unfiltered). The FBP is based on the Fourier slice theorem, which allows us to obtain the 2D Fourier Transform of an object by taking the 2D Fourier transform of each projection and then assembling the slices along different directions to get the complete Fourier Transform. In FBP, we transformed the original image to the Fourier domain using the Fourier Slice Theorem to obtain the frequency domain representation of each projection. From here, we can apply the filters in the frequency domain to remove the high frequency noise and artifacts. The different filters are slightly different, but they are all high pass filters that emphasize the low frequency regions of the sinogram and suppress the high frequency components. The full image is backprojected for each projection onto the 2D image plane using the inverse Fourier transform and then the full image is reconstructed by merging the back projected images from all the angles. The reconstruction image that doesn't use any filter (regular backprojection) has a lot of high frequency noise and artifacts. Additionally, it is more blurry because the high frequency signals were not removed. Ramp, Shepp-Logan, Cosine, Hamming and Hann are all different kinds of filters used in FBP, where each varies in their frequency response.

- Ramp filter has a frequency response that increases linearly with frequency
- Shepp-Logan filter has a frequency response similar to the ramp filter, but with added smoothing to the reconstructed image
- Cosine filter is a product of the ramp filter and cosine function to reduce edge artifacts
- Hamming filter is a product of the ramp filter with the hamming window function to improve contrast
- Hann filter is a product of the ramp filter with the Hann window function to improve sharpness in the image





In the final step, we used SIRT to the Shepp-Logan head phantom to view the differences. SIRT (Simultaneous Iterative Reconstruction Technique) iteratively updates the reconstructed image by estimating the contribution of each projection to the image and subtracting the difference between the measured and estimated projections. This method is computationally expensive and requires long running times, as it requires solving a set of linear equations in each iteration. In standard CT imaging, beam hardening (a type of artifact that occurs in CT imaging where X-Ray beams pass through an object and are more penetrating as they travel deeper into it) can lead to streaks and shading in the reconstructed images. The SIRT algorithm fixes these artifacts by estimating in parallel the beam hardening correction as it is reconstructing the image in an iterative optimization process. The SIRT reconstruction seems to result in improved image quality, specifically with dense objects.

