How Competition Shapes Information in Second-Price Auctions*

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Abstract

We consider second-price auctions where buyers can acquire costly information about their valuations and those of others, and investigate how competition between buyers shapes their learning incentives. In equilibrium, buyers find it cost-efficient to acquire some information about their competitors so as to only learn their valuations when they have a chance of winning. We show that such learning incentives make competition between buyers less effective: losing buyers often fail to learn their valuations precisely and, as a result, compete less aggressively for the good. This depresses revenue, which remains bounded away from what the standard model with exogenous information predicts, even when information costs are negligible. Finally, we highlight some implications for auction design.

Keywords: Auction, Information Acquisition, Competition, Market Design

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1 Introduction

In many auctions, participants spend significant time and resources learning about the goods for sale before bidding. Relevant examples are the sales of complex, high-value assets such as companies, broadband licenses, or procurement contracts, during which interested buyers conduct thorough due diligence. For instance, bidders in takeover auctions get access to extensive information about the target's operations and finances, allowing them to assess synergies and estimate how much they value its acquisition.

Accessing and processing such information is often costly, and buyers only want to undertake this investment if they have a decent chance of winning the auction.[1] Effective auction design then needs to account for buyers' learning incentives, and existing literature has mostly focused on the *intensive* margin—that is, *how much* information buyers acquire about their values under various auction formats (see, e.g., Persico (2000); Bergemann and Välimäki (2002); Shi (2012)). Much less attention has been paid to how auction design affects *what types* of information they seek.

In practice, auction participants have access to a wide range of information and have ample flexibility as to what to learn about. Some information helps them assess their own willingness to pay for the good while other pertains to the rival bids they will face. Anecdotal evidence suggests that participants in high-stake auctions seek both types of information. For instance, GTE's bidding team prepared for the 1994 spectrum auction run by the Federal Communications Commission by assessing both the value of each license to GTE and the level of competition in each market (Salant (1997)). To assess competition, they looked at which bidders were eligible to bid on which licences, estimated their budgets, and identified synergies between each license and their rivals' existing networks.[2] This gave GTE a better sense of which licences they could reasonably win, which were then the focus of bid preparation efforts.

Naturally, what buyers choose to learn is likely to affect how they bid, and thus the outcome of the auction. In the above-mentioned FCC auction, the Los Angeles license raised "little" revenue,[3] perhaps because the participation of Pacific Telesis,

[1] Due diligence for the acquisition of a company can take months and involves legal and accounting fees that average to 47 basis points of the deal value (Cole et al. (2016)), in addition to consulting fees.

[2] Buyers have access to some information about their competitors in other settings too. For instance, bidders in highway procurement auctions can learn about the location of their rivals' machines and their backlogs, and bidders in takeover auctions about their rivals' financial constraints.

[3] It was sold at the price of \$26 per capita, which was considered low as less profitable licenses were sold at higher prices, e.g., the Chicago license was sold at \$31 per capita. See

the main company in California, scared away the competitors. GTE's bidding team indeed thought they had little chance of winning that license, and so focused their efforts elsewhere (Salant (1997)). Beyond such anecdotal evidence, we do not however have a good theoretical understanding of what information bidders seek when given the flexibility, nor of how it affects the performance of the auction.

This paper proposes a tractable model of multidimensional learning in second-price auctions. We focus on the canonical auction setting with independent private values, in which buyers' valuations are drawn i.i.d. from some common knowledge distribution. Importantly, buyers do not know their valuations ex ante and can acquire costly information before bidding. The main innovation of our model is that buyers have flexibility in *what* information they can seek. Specifically, buyers can acquire two signals—one about their own valuations for the good and one about those of their competitors—and can choose in which order to acquire them. They also have some flexibility in choosing each signal's informativeness and, in particular, can choose how each signal partitions the set of possible valuations. Information is costly, and we require that the cost satisfies appropriate notions of monotonicity and convexity. We allow the cost of a signal about the competitors to scale with the number of competitors $N-1$.

Our first main set of results is that buyers cannot converge to becoming fully informed of their valuations in equilibrium, even as information costs vanish (Proposition 3). Instead, they find it cost-efficient to first assess the valuation of their toughest competitor, and only then learn about their own, which they do only when they have a chance of winning. We find necessary conditions that must be satisfied by equilibrium information structures in high-stake second-price auctions (i.e., when information costs are small relative to the value of the good). We show that buyers fully learn their valuations only if it falls in a similar range as that of their toughest competitor (Theorem 1). These results rely on the prior distribution of values being sufficiently uncertain. Theorem 1 implies that buyers' private information when entering the auction (i.e., their types) are interdependent: not only do buyers have information that is relevant to others, but their own expected values may depend on what they learned about the competitors. The information buyers acquire is then deeply shaped by the competitive pressure they impose on each other.

We then examine how buyers' learning incentives, in turn, affect the performance

https://www.fcc.gov/auctions-summary for the auction outcome and Klemperer (2002) for a more detailed account.

of the second-price auction. We show that expected revenue remains bounded away from what the standard model predicts, even as information costs become arbitrarily small (Theorem 2). Indeed, losing buyers often fail to learn their valuations precisely, and since they bid their expected valuations for the good in equilibrium, this leads to a regression to the mean of bids. Losing bids are then less dispersed than in the standard model, which depresses the expected second-highest bid and hence expected revenue whenever the number of buyers $N \geq 3$.[4]

Our main results highlight a new adverse effect of competition on revenue, and we investigate some implications for auction design. We show that attracting an additional bidder can be less valuable than setting an optimal reserve price, unlike the seminal result of Bulow and Klemperer (1996). There are several forces at play. First, an additional bidder does not raise revenue as much as in the standard model, as that bidder does not fully learn his valuation. Second, a carefully chosen reserve price is more valuable as losing bidders oftentimes fail to learn their valuations for the good, leaving a larger expected gap between the highest and second-highest bids.

We then show how the seller can mitigate the revenue loss by maintaining uncertainty over the extent of competition. If the seller can directly control how much information buyers have about their competitors, then revenue is monotonically decreasing in the precision of that information. This result might explain why bidders in takeover auctions are often required to sign non-disclosure agreements preventing them from revealing, among other things, their participation in the sale. Even if the seller cannot directly control such information, she can still induce uncertainty over *effective* competition by randomizing access to the auction. We show that doing so unambiguously improves expected revenue in high-stake auctions, as buyers can no longer predict whose bids they will be facing in the auction, which reduces their incentives to learn about their competitors.

Finally, we close the paper with a discussion of the model's key assumptions, which are the ones imposed on the process of information acquisition. We see Theorems 1 and 2 as the main results of the paper, and show that they are robust to a weakening of these assumptions and alternative model specifications. The market design implications rely more heavily on our assumptions, but capture practically relevant insights.

[4] This effect persists as information costs vanish, highlighting a discontinuity between the standard model where buyers know their valuations ex-ante (information is then effectively free), and ours.

1.1 Related Literature

First, we build on a previous paper (Gleyze and Pernoud (2023)), which investigates whether the seller can design a mechanism under which participants only have an incentive to learn about their own preferences. We show that it is impossible: most selling mechanisms incentive participants to learn about others' preferences as well, leading their types to be endogenously interdependent. Gleyze and Pernoud (2023) however does not characterize what an equilibrium information structure looks like—it just shows that it has to feature some interdependencies, but not what form they take—nor how it affects the performance of the selling mechanism. The present paper addresses these questions in the context of the second-price auction.[5]

Second, our paper contributes to the literature on entry and learning costs in auctions. Levin and Smith (1994) characterize the symmetric equilibrium when buyers pay a fixed cost to learn their values before bidding in a second-price auction. They show that equilibrium entry decisions are revenue-maximizing in the IPV setting.[6] Several papers highlight the benefits of dynamic auction formats (Compte and Jehiel (2007)), and in particular of the Dutch auction (Miettinen (2013); Kleinberg et al. (2016)), in coordinating learning across buyers, as dynamic formats endogenously reveal information on the toughness of competition. Another strand of the literature allows buyers to flexibly choose *how much* information to acquire (Hausch and Li (1993); Persico (2000); Bergemann and Välimäki (2002); Shi (2012); Kim and Koh (2022)). Importantly, these papers consider information acquisition about a one-dimensional random variable—usually buyers' own valuations or a component that is common to all buyers.

Our key contribution is to propose a model of *multidimensional* learning in which buyers can separately choose how much to learn about self and others. To our knowledge, the only papers that study multidimensional learning in auctions are Larson and Sandholm (2001b,a) and Bobkova (2024). Larson and Sandholm (2001b,a) also consider learning about both self and others. They show that computationally limited agents have an incentive to learn about others in Vickrey and ascending auctions and

[5] We focus on the second-price auction, not only because it is a widely-used auction format but also because it is *strategy-proof*: if buyers knew their valuations, they would have a dominant strategy and would have no incentive to inquire about the competition. We can thus isolate the detrimental effect of competition on learning incentives.

[6] A related literature assumes that buyers have some private information when making entry decisions (Ye (2007); Quint and Hendricks (2018); Lu et al. (2021), etc.). Entry then serves as a screening mechanism that the seller can leverage. Such considerations are absent in our paper.

compare different models of costly deliberation. In Bobkova (2024), buyers' valuations are composed of a private and a common component, and buyers choose how much to learn about each.[7] She shows that buyers only seek information about their private components in second-price auctions, which contrasts with our results.

A relatively small literature studies buyers' incentives to learn about the competition in first-price auctions (Tian and Xiao (2007)) and auctions with interdependent values (Kim and Koh (2020)). Information about opponents' types is valuable as it allows buyers to either shade their bids more aggressively (in the former) or alleviate the winner's curse (in the latter). Such incentives are absent in our setting as buyers compete in a second-price auction and their valuations are independent and private.

Our paper also speaks to the literature highlighting the value of competition in selling mechanisms. Bulow and Klemperer (1996) show that attracting just one more buyer has more value than using an optimal reserve price. Relatedly, Bulow and Klemperer (2009) show that with costly entry, actual competition in an auction dominates potential competition from a sequential entry mechanism.[8] These results, however, take buyers' information as fixed. Instead, our paper asks how competition affects the information buyers acquire and reaches different conclusions. Gershkov et al. (2021) also qualify the value of competition and show that it can even hurt revenue if buyers can invest to increase their values before bidding.

Finally, several papers study the performance of auctions when buyers are ex-ante asymmetric (Maskin and Riley (2000); Compte and Jehiel (2002); Kim and Che (2004); Cantillon (2008); Jehiel and Lamy (2015); Marquez and Singh (2024)). In particular, Marquez and Singh (2024) shows that the presence of an asymmetrically strong buyer can hurt revenue. In our paper, buyers are ex-ante identical, but we show that asymmetries in private information arise endogenously, even in symmetric equilibria.

We end by noting that an extensive literature examines the different, though related, question of optimal information *disclosure* in auctions (see, e.g., Bergemann and Pesendorfer (2007); Prummer and Nava (2023) and references therein). In our paper, the seller has no information and does not intervene in buyers' learning process.[9]

[7] They cannot however learn about their competitors' values directly.

[8] Roberts and Sweeting (2013) extend their model with ex-ante noisy signals and asymmetries and find that the sequential entry mechanism can dominate the auction.

[9] Note that the equilibrium distribution of types in our setting is highly subopimal in the eyes of the seller since it leads to a revenue loss compared to full information.

1.2 A Motivating Example

Two buyers compete in a second-price auction to acquire a good. Their values are drawn i.i.d. from a finite set of possible valuations V. For the sake of this example, assume that buyer j is exogenously informed of his valuation v_j, and plays his dominant strategy, such that he always truthfully bids his valuation. The example focuses on the incentives of buyer i.

Buyer i does not know his valuation ex-ante but can acquire costly information before bidding. Importantly, buyer i has flexibility as to what to learn about: he can acquire a signal about his own valuation v_i and one about his opponent's v_j. He has full flexibility in how to design each of these two signals and can furthermore choose in which order to acquire these signals. Information is costly and, for this example only, we consider the entropic cost function. Informally, the cost of each signal is proportional to the expected reduction in uncertainty as measured by the entropy of beliefs:

$$\text{cost of a signal} = \lambda\Big(\text{prior entropy} - \mathbb{E}[\text{posterior entropy}]\Big)$$

where λ is a scaling parameter.

We examine what information buyer i acquires as the size of information costs λ goes to zero. From buyer i's perspective, this is simply a decision problem: the behavior of buyer j is fixed in this example, so it is as if i faced a random price v_j and could learn both about the realized price and his realized valuation for the good. For small enough information costs, buyer i must learn enough to avoid (non-trivial) mistakes. That is, i must learn enough so that he (almost always) submits a winning bid $b_i > v_j$ whenever $v_i > v_j$ and a losing bid $b_i < v_j$ whenever $v_i < v_j$.[10]

Interestingly, many learning strategies guarantee that buyer i makes no mistakes when bidding. For instance, buyer i can learn his valuation fully and learn nothing about his competitor's. That is, i can partition the state space $V \times V$ as depicted in Figure 1 (left panel) and bid his value v_i in any state (v_i, v_j). Another such learning strategy is depicted in the right panel of Figure 1: i first learns whether the opponent's value is above or below some threshold v^*; if $v_j > v^*$ (resp. $v_j \leq v^*$), he learns his value fully if it is above v^* (resp. below v^*) but bundles together all values below v^* (resp. above v^*). This is enough for i to bid optimally in the auction as well, as he learns enough to know whether or not it is optimal for him to win.

[10]Let us ignore ties for the sake of this example.

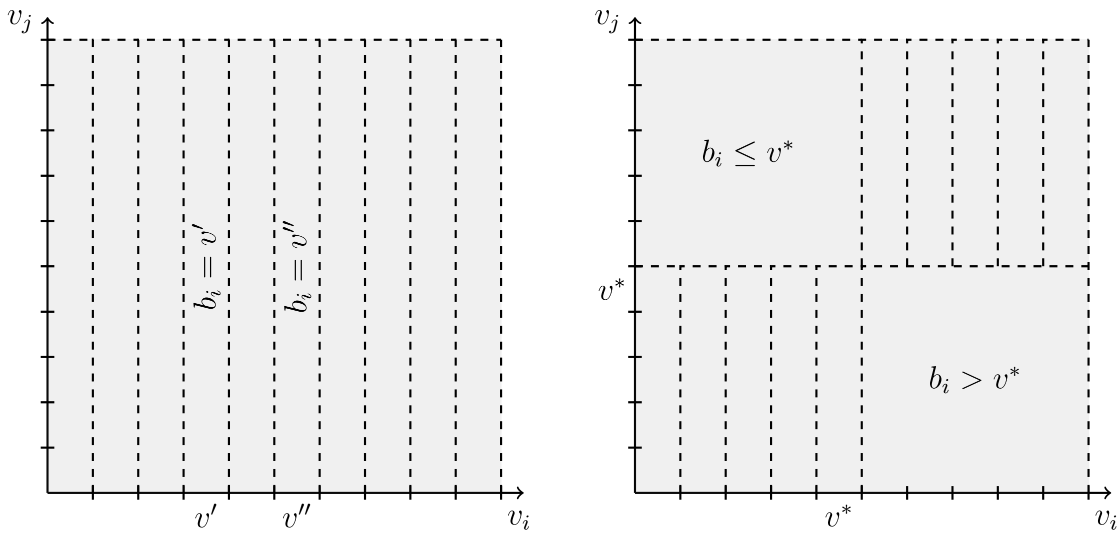


Figure 1: Two learning strategies that guarantee buyer i bids optimally, leading to two different partitions of the state space $V \times V$.

As information costs vanish, buyer i optimally chooses the *cheapest* information structure that ensures he makes no mistakes in the auction. In this paper, we show that when information costs are sufficiently increasing in a signal's informativeness, fully learning one's value is not cost-efficient. For instance, when there are $|V| = 10$ possible valuations and they are all ex-ante equally likely, learning v_i fully costs $\lambda \log 10$ under the entropic cost. The information structure depicted in the right panel of Figure 1 however only costs $\lambda[\log 2 + \log 10 - 0.5 \log 5]$, which is strictly cheaper.

This example assumes that buyer j knows and bids his value to simplify the argument, but the same reasoning works in equilibrium when all buyers simultaneously choose how to learn and bid. The added difficulty is that if i fails to learn his valuation *fully* and j learns something about v_i, then j has information that is relevant to i. Buyers' information—or *types*—is then interdependent and characterizing equilibrium bidding is much less straightforward. In our model, we thus add structure to the learning process and require signals to be monotone partitions of V.[11] This allows us to characterize the equilibrium information structure as information costs vanish, which looks like the one in Figure 1 (right).

[11]Note that, in this motivating example, a monotone partition is the optimal type of signal that i would acquire about his value as information costs vanish.

2 The Model

We introduce a tractable model of multi-dimensional learning in auctions.

A seller puts a unique, indivisible good for sale through a second-price auction.[12] There are N buyers, and buyers' valuations for the good $(v_i)_i$ are drawn i.i.d. from a finite set $V \subset \mathbb{R}_+$ according to a probability distribution $\mathbb{P} \in \Delta V$. Buyers have quasilinear utility functions. Buyer i's gross payoff from the auction in state $(v_j)_j$ at bid profile $(b_j)_j$ equals

$$U(v_i, b_i, b_{-i}) \equiv \begin{cases} \dfrac{v_i - \max_{j \neq i} b_j}{|\{j = 1, \dots N \text{ s.t. } b_j = b_i\}|} & \text{if } b_i = \max\limits_j b_j \\ 0 & \text{otherwise} \end{cases}.$$

Note that we are considering a setting in which buyers' valuations are independent and private. Hence, if buyers knew their own valuations v_i, it would be a dominant strategy for them to bid truthfully in the auction, and the seller's expected revenue would be the expected second highest value.

Information Structures. Buyers start with no private information, but they can learn about the realization of $(\tilde{v}_i)_i$ at some cost before competing in the auction.

We assume that buyers can acquire two signals, one about their own valuations $\tilde{v}_i$ and one about others' $\tilde{v}_{-i}$. Without loss of optimality, buyers first acquire information about others' valuations and, conditional on the realization of this signal, acquire information on their own. This is without loss in the sense that all our results go through if we allow buyers to choose in which order to acquire these two signals (see Section 6.1 for a formal argument). To reduce the dimension of the problem, we furthermore assume that buyers can only learn about $\max_{j \neq i} \tilde{v}_j$, and not the full vector $\tilde{v}_{-i}$.[13]

We model information acquisition about any random variable as the choice of a partition $\Pi = \{\pi_1, \dots \pi_L\}$ of the set of possible realizations V. That is, if a buyer chooses information partition Π, then the buyer learns to which element of the partition the realization of the random variable belongs. If the chosen partition is $\Pi = \{V\}$, then

[12] We discuss the focus on second-price auctions in Section 6.3.

[13] For small learning costs, this is a sufficient statistic for the highest bid faced by i in any symmetric equilibrium, which is what matters for i's gross payoff. This assumption however prevents i from learning more about buyer j than about buyer k. One can then think of this assumption as selecting equilibria in which buyers learn about their competitors symmetrically, as we show in Section 6.1.

no information is acquired. If $\Pi = \{\{v\}_{v \in V}\}$, then the partition is fully revealing. We furthermore require that buyers choose *monotone* partitions, meaning that if $v', v'' \in \pi_l$ with $v' < v''$, then all $v \in (v', v'')$ also belong to the element π_l of the partition.

Information is costly. Letting $\mathcal{P}$ denote the set of all possible monotone partitions of V, the cost of a signal $c : \mathcal{P} \times \Delta(V) \longrightarrow \mathbb{R}_+$ is a function of both the chosen partition and the prior belief p. Furthermore, since a signal about $\max_{j \neq i} \tilde{v}_j$ aggregates information about $N-1$ random variables, we scale its cost by some increasing function $\gamma : \mathbb{N} \longrightarrow \mathbb{R}_+$. Thus a partition Π costs $c(\Pi, \mathbb{P})$ if it is acquired about a buyer's own value v_i and costs $\gamma(N-1)c(\Pi, \mathbb{P})$ if it is acquired about $\max_{j \neq i} v_j$.

Strategies and Solution Concept. Buyers have two decisions to make. First, they decide what information to acquire. Then, conditional on their information set, they submit a bid to the seller.

As described above, information acquisition is sequential, and an information strategy consists of two parts. Each buyer i first chooses an information partition about others $\Pi_i^{other} \in \mathcal{P}$. Then, conditional on his information set about others $\pi_i^{other} \in \Pi_i^{other}$, he chooses an information partition about his own valuation $\Pi_i^{self} : 2^V \longrightarrow \mathcal{P}$. Finally, each buyer chooses a bidding strategy $\beta_i : 2^V \times 2^V \longrightarrow \mathbb{R}_+$, which outputs a bid given the buyer's overall information $\pi_i = (\pi_i^{other}, \pi_i^{self})$.

Let $\Sigma \equiv \left\{ (\Pi_i^{other}, \Pi_i^{self}, \beta_i) \in \mathcal{P} \times \mathcal{P}^{2^V} \times \mathbb{R}_+^{2^V + 2^V} \right\}$ be the overall set of pure strategies,[14] and $\sigma_i \in \Delta\Sigma$ a strategy for buyer i.

Buyer i's ex-ante expected utility under strategy profile $(\sigma_i)_i$ writes

$$\mathbb{E}_{\sigma_i, \sigma_{-i}} \left[U(v_i, \beta_i(\pi_i), \beta_{-i}(\pi_{-i})) - \lambda \left(\gamma(N-1) c \left(\Pi_i^{other}, \mathbb{P} \right) + c \left(\Pi_i^{self} \left(\pi_i^{other} \right), \mathbb{P} \right) \right) \right],$$

where $\lambda > 0$ is a parameter that scales the cost of information.

A Nash equilibrium is a strategy profile such that each buyer's equilibrium strategy σ_i maximizes his ex-ante expected utility given that all others follow theirs. As usual in a second-price auction, there exist many unappealing Nash equilibria, and even more so now that the information structure is endogenous. In particular, the concept of Nash equilibrium imposes no discipline on which (losing) bid a buyer submits if, at some information set, he can predict his toughest opponent's bid and knows that

[14] Technically, a buyer could condition his bid not only on what he *learned* π_i but also on the partitions he *chose* $(\Pi_i^{other}, \Pi_i^{self})$—i.e., not only on the signal *realizations* but on the signals themselves. However, that would never be strictly optimal as π_i is a sufficient statistic for a buyer's posterior belief.

he does not want to outbid it. To rule out unrealistic equilibria, we use the following trembling-hand-like refinement.

Definition 1. *A tremble-robust symmetric equilibrium σ^* is a Nash equilibrium in which,*

(i) all buyers choose the same strategy $\sigma_i = \sigma^*$,

(ii) there exists a sequence of strictly positive numbers $\{\varepsilon^{(k)}\}_{k=1}^{\infty}$ converging to zero such that $\sigma^* = \lim_{k \longrightarrow \infty} \sigma^{(k)}$, where $\sigma^{(k)}$ is a symmetric Nash equilibrium of a perturbed game in which, with probability $\varepsilon^{(k)}$, each buyer's bid is drawn from some continuous distribution F with support $[\min_{v_i \in V} v_i, \max_{v_i \in V} v_i]$ independently of his chosen strategy.

Intuitively, we require that the equilibrium remains virtually unchanged if, with vanishing probability, buyers tremble and make a bid drawn from a full-support distribution. This refinement allows us to discipline the bids of losing buyers: if buyer i knows that one of his opponents will bid at least 10 and that he does not want to win at that bid, it ensures that i will bid his expected valuation instead of, e.g., 9.9. This refinement serves no other purpose.

Assumptions on the Cost of Information. First, we impose a monotonicity condition that disciplines how the cost varies with the fineness of a partition. Given some partition Π and prior $\mathbb{P} \in \Delta V$, let $\mathbb{P}_\Pi$ be the distribution over elements in Π.[15] Formally, we require that there exists a real-valued concave function H such that, if Π is finer than Π', then

$$c(\Pi, \mathbb{P}) \geq c(\Pi', \mathbb{P}) + \kappa \left(H\left(\mathbb{P}_\Pi\right) - H\left(\mathbb{P}_{\Pi'}\right) \right),$$

for some $\kappa > 0$. The function H takes as input any finite-support distribution and captures the amount of uncertainty in the distribution. Indeed, the concavity of H ensures that if Π is finer than Π', then $H\left(\mathbb{P}_\Pi\right) \geq H\left(\mathbb{P}_{\Pi'}\right)$. If a distribution $\mathbb{P}_\Pi$ is more uncertain, then learning about its realization provides more information, and the above condition requires that the associated cost be strictly higher. The cost must furthermore scale at least linearly with the uncertainty in the distribution, though the constant of linearity κ need not be large. To make H a valid measure of uncertainty, as defined in Frankel and Kamenica (2019), we set the uncertainty of a degenerate distribution to zero: $H(\mathbb{P}_\Pi) = 0$ if $|\operatorname{supp} \mathbb{P}_\Pi| = 1$. We also require H to be invariant under permutation,

[15] That is, $\operatorname{supp} \mathbb{P}_\Pi = \Pi$ and $\mathbb{P}_\Pi(\pi_l) = \mathbb{P}(\tilde{v} \in \pi^l)$ for all $\pi_l \in \Pi$.

such that H only depends on the probability mass of each realization and not on its label.[16]

Absent more structure on H, the above condition does not imply anything stronger than (weak) Blackwell monotonicity. Indeed, if H is constant at zero, then the above condition only means that finer partitions are (weakly) costlier. Our results, however, rely on the cost of a partition scaling non-trivially with how fine the partition is. To that end, we require that H be unbounded: for any $\bar{H} \in \mathbb{R}_+$, there exists a probability mass function q such that $H(q) \geq \bar{H}$. In words, the measure of uncertainty is not exogenously capped and can be arbitrarily large for sufficiently diffuse distributions.

Second, we impose a notion of convexity on the cost. Given some partition Π, let Π_J denote the restriction of Π to values in $J \subseteq V$. If $\Pi_J = \Pi'_J$ for some $J \subset V$ but $\Pi_{V \setminus J}$ is finer than $\Pi'_{V \setminus J}$, then

$$c(\Pi, \mathbb{P}) - c(\Pi', \mathbb{P}) \geq \mathbb{P}(v \notin J) \left[c(\Pi, \mathbb{P}(\cdot \mid v \notin J)) - c(\Pi', \mathbb{P}(\cdot \mid v \notin J)) \right].$$

In words, if two partitions coincide over some subset of values J but one is finer than the other over the complement set, then the difference between their cost is weakly larger than if all values in J had zero probability. This captures some weak notion of convexity as it means that the cost of the information about values in $V \setminus J$ is weakly higher when some information has been acquired about values in J than when all those values have zero probability (such that the set of values is effectively $V \setminus J$). Note that the above condition always holds with equality when the cost is uniformly posterior separable (Caplin et al. (2022)).

Third, we discipline how the cost of a partition depends on the prior. Specifically, we assume that the prior only impacts the cost of a partition through its effect on the distribution over signals, that is through its effect on the distribution over elements in the partition $\mathbb{P}_\Pi$. Formally, if $\mathbb{P}_\Pi = \widehat{\mathbb{P}}_\Pi$, then $c(\Pi, \mathbb{P}) = c(\Pi, \widehat{\mathbb{P}})$. We furthermore require c to be continuous in $\mathbb{P}$ over the interior of the simplex.

Finally, we normalize the cost of a completely uninformative partition to zero: $c(\{V\}, \mathbb{P}) = 0$ for any $\mathbb{P} \in \Delta V$.

[16]That is, if there exists a bijection $\rho : \Pi \longrightarrow \Pi'$ such that $\mathbb{P}_\Pi(\pi_l) = \mathbb{P}_{\Pi'}(\rho(\pi_l))$ for all $\pi_l \in \Pi$, then $H(\mathbb{P}_\Pi) = H(\mathbb{P}_{\Pi'})$. For instance, let $V = \{v^1, v^2, v^3\}$ and $\mathbb{P}(v^k) = 1/3$ for all k. Partition $\Pi = \{\{v^1\}, \{v^2, v^3\}\}$ leads to distribution over signals $\mathbb{P}_\Pi$ that puts probability $1/3$ on signal $\{v^1\}$ and probability $2/3$ on signal $\{v^2, v^3\}$. Similarly, partition $\Pi' = \{\{v^1, v^2\}, \{v^3\}\}$ leads to distribution over signals $\mathbb{P}_{\Pi'}$ that puts probability $2/3$ on signal $\{v^1, v^2\}$ and probability $1/3$ on signal $\{v^3\}$. We say these two distributions have the same uncertainty.

We give two examples of cost functions that satisfy our assumptions.

Example 1. *The cost of a partition is an increasing and weakly convex function of the number of elements in the partition:*

$$c(\Pi, \mathbb{P}) = C\left(|\Pi_{\operatorname{supp}\mathbb{P}}|\right),$$

where $C(\cdot)$ is strictly increasing and weakly convex, with $C(1) = 0$. The associated measure of uncertainty is $H(\mathbb{P}_\Pi) = |\operatorname{supp}\mathbb{P}_\Pi| - 1$.

Example 2. *The cost of an information partition equals the expected reduction in the entropy of the buyer's belief:*[17]

$$\begin{aligned} c(\Pi, \mathbb{P}) = & \\ & -\sum_v \mathbb{P}(\tilde{v} = v)\log[\mathbb{P}(\tilde{v} = v)] + \sum_{\pi^l \in \Pi} \mathbb{P}(\tilde{v} \in \pi^l) \sum_v \frac{\mathbb{P}(\tilde{v} = v)}{\mathbb{P}(\tilde{v} \in \pi^l)} \log\left(\frac{\mathbb{P}(\tilde{v} = v)}{\mathbb{P}(\tilde{v} \in \pi^l)}\right) \\ & = -\sum_{\pi^l \in \Pi} \mathbb{P}(\tilde{v} \in \pi^l)\log\left(\mathbb{P}(\tilde{v} \in \pi^l)\right). \end{aligned}$$

The entropic cost has been widely used in the applied literature, in particular in models of rational inattention. It has the advantage of being tractable and having solid information-theoretic foundations.

Note that in Example 1, the cost depends on the prior $\mathbb{P}$ only through its support. Thus, the cost need not depend in rich ways on the prior for our results to go through.

Together, the above assumptions imply that it is cost-efficient for buyers to acquire *some* information about the competitors (i.e., choose an informative, though fairly coarse, partition Π^{other}) to avoid the cost of becoming *fully* informed about their own valuations, whenever the prior $\mathbb{P}$ is sufficiently uncertain.

To formalize this, let $\Pi^{other}_{v^*} \equiv \{\{v : v \leq v^*\}, \{v : v > v^*\}\}$ be the partition that divides the set of valuations V into two subsets: valuations that are below some threshold v^* and those that are above. Arguably, this is a fairly coarse partition whenever the set of valuations V is rich. If the equilibrium is efficient, a buyer i who learns $\max_{j \neq i} v_j > v^*$ has little to no incentive to learn to distinguish all valuations $v_i \leq v^*$

[17] With the convention $0 \log 0 = 0$, since the entropy is only defined for full-support beliefs. When a buyer learns $v \in \pi^l \neq V$, he however puts zero probability on all $v \notin \pi^l$: his posterior does not have full support. We extend the domain of the entropy function as follows: for any belief p on the boundary of the simplex, we define $H(p)$ to be the limit of $-\sum_v \hat{p}(v)\log[\hat{p}(v)]$ as $\hat{p} \longrightarrow p$ for some full-support $\hat{p}$.

since he loses the auction at all of these. Let $\Pi^{self}_{>v^*} \equiv \{\{v : v \leq v^*\}, \{v\}_{v>v^*}\}$ be the partition that bundles all these lower valuations together. This partition is less costly than becoming fully informed of v_i, and potentially significantly so.[18] Hence even a coarse signal about others can significantly reduce how finely a buyer should learn about his own valuation.

Lemma 1. *There exists $\overline{T}$ such that if $\sum_v [\mathbb{P}(v)]^2 \leq \overline{T}$, then it is less costly for buyers to acquire* some *information about others instead of becoming fully informed about themselves. Formally, for some $v^* \in V$,*

$$\gamma(N-1)c\left(\Pi^{other}_{v^*}, \mathbb{P}\right) + \Pr\left(\max_{j\neq i} v_j \leq v^*\right) c\left(\{\{v\}_{v\in V}\}, \mathbb{P}\right) + \Pr\left(\max_{j\neq i} v_j > v^*\right) c\left(\Pi^{self}_{>v^*}, \mathbb{P}\right)$$
$$< c\left(\{\{v_i\}_{v_i\in V}\}, \mathbb{P}\right),$$

Proofs of all the results are in Appendix B. In Lemma 1, $\sum_v [\mathbb{P}(v)]^2$ captures the precision of the prior belief. It is equal to one when the prior is deterministic (i.e., when $\mathbb{P}(v) = 1$ for some v), and decreases as the prior gets more diffuse. To see why the condition on the prior is necessary, take the extreme case in which only two valuations have strictly positive prior probability: $V = \{\underline{v}, \overline{v}\}$. Then any information acquired about others must fully reveal $\max_{j\neq i} v_j$: the only non-trivial information partition is the fully revealing one $\{\{\underline{v}\}, \{\overline{v}\}\}$. There is then no scope for buyers to save on information costs about their own values by learning a bit about others.

Example 1 (continued)**.** *Let $c(\Pi, \mathbb{P}) = |\Pi_{\text{supp}\,\mathbb{P}}| - 1$ and $\gamma(N-1) = N-1$. Let $V = \{\frac{1}{K}, \frac{2}{K}, \ldots, \frac{K}{K}\}$ with $\mathbb{P}(v_i) = \frac{1}{K}$ for all v_i. The fully revealing partition $\Pi^{self} = \{v_i\}_{v_i\in V}$ costs*

$$c\left(\{v_i\}_{v_i\in V}, \mathbb{P}\right) = K - 1.$$

Suppose K is even. Learning whether at least one competitor has value v_j above $v^* = \frac{1}{2}$ costs

$$\gamma(N-1)c\left(\Pi^{other}_{v^*}, \mathbb{P}\right) = N - 1.$$

With probability $1 - 0.5^{N-1}$, buyer i learns that some v_j is greater than $v^* = 0.5$. He then only needs to learn his valuation precisely when it is also greater than $v^* = 0.5$, which costs

$$c\left(\Pi^{self}_{>v^*}, \mathbb{P}\right) = 0.5K.$$

[18] Such partitions are sometimes called lower censorship in the literature.

Hence learning about the competition allows him to reduce learning costs about his value by $[1 - 0.5^{N-1}][0.5K - 1]$*, and so it is cost-efficient if* $K > 2\frac{N-0.5^{N-1}}{1-0.5^{N-1}}$.

Example 2 (continued). *Consider the entropy-based cost and* $\gamma(N-1) = N-1$*. Let* $V = \{\frac{1}{K}, \frac{2}{K}, \ldots, \frac{K}{K}\}$ *with* $\mathbb{P}(v_i) = \frac{1}{K}$ *for all* v_i*. Similar calculations as above yield that learning about the competition is cost-efficient if* $\log K > \frac{2N-1-0.5^{N-1}}{1-0.5^{N-1}} \log 2$.

The above examples highlight that the uncertainty threshold at which it becomes cost-effective to learn about the competition depends on the number of competitors $N-1$. In particular, the higher the number of competitors, the more uncertain the prior needs to be for this incentive to arise. This is intuitive as it is costlier to learn about the competition if there are many competitors, and so it would need to lead to a significant reduction in information costs on self for it to be worth it. Thus, this model is most relevant for auctions of small or intermediate size.

For the remainder of the paper, we assume that the prior $\mathbb{P}$ is sufficiently uncertain that Lemma 1 holds.

3 How Competition Shapes Buyers' Information

This section investigates how the competitive pressure between buyers affects what information they seek and the resulting equilibrium information structure. We first consider two benchmark cases in which buyers are either exogenously informed of their valuations or can only acquire costly information about their own valuations. We show that these two benchmarks yield the same predictions when information costs are small relative to the value of the good. This is, however, not the case when buyers can also learn about their competitors.

3.1 Two Benchmark Cases

In the first benchmark we consider, buyers are exogenously informed of their valuations. This case is well understood, and bidding truthfully is a dominant strategy for buyers. Whether or not they know others' valuations, or can acquire information about them, is then irrelevant. We report the properties of the equilibrium for completeness.

Proposition 0. *Suppose buyers know their valuations ex ante. Then there exists a symmetric equilibrium in which expected revenue equals the expected second-highest valuation* $\mathbb{E}[v_{(2)}]$.

Now suppose buyers have no private information ex ante and can only acquire information on their own valuations. Most papers on information acquisition in auctions focus on this case.

Proposition 1. *Suppose buyers can only learn about themselves. Then, for λ small enough, there exists a symmetric equilibrium in which they all become fully informed about their own valuations, and expected revenue equals the expected second-highest valuation $\mathbb{E}[v_{(2)}]$.*

Hence, the two benchmarks yield similar predictions for small information costs. The intuition is direct: the gains associated with distinguishing two realizations of $\tilde{v}_i$ are always strictly positive, as a buyer might face a price (i.e., a highest bid) that falls precisely between these two realizations. If information costs are small enough, buyers must choose the fully revealing partition $\Pi^{self} = \{\{v\}_{v \in V}\}$.

Finally, it is worth noting what happens when buyers can only learn about the competition, but not about themselves. In any equilibrium, buyers acquire no information whatsoever and simply bid their expected value given the prior. Indeed, information about others is only valuable if it helps buyers assess how much they should learn about themselves, which is here precluded. Thus restricting (in potentially ad hoc ways) what buyers can learn about significantly affects equilibrium behavior.

3.2 Multi-Dimensional Learning

We now consider our main model specification, in which buyers can acquire information on their valuations as well as others'. We show that buyers have an incentive to learn a bit about their competitors, so as not to waste resources learning about their own valuations when such information makes no difference.

We start by establishing equilibrium existence.

Proposition 2. *There exists a symmetric equilibrium that is robust to trembles for any cost parameter λ.*

In what follows, we use the term "equilibrium" to refer to a tremble-robust symmetric equilibrium.

We now show that, contrary to our benchmarks, buyers cannot all become fully informed of their valuations in equilibrium. This is true even as the cost parameter λ becomes arbitrarily small.

Proposition 3. *Suppose buyers can learn both about themselves and others, and the prior is sufficiently uncertain. Then there does not exist a sequence of equilibria in which buyers become fully informed of their valuations as the cost parameter λ goes to zero. That is, there exists $\varepsilon > 0$ such that, for any sequence of equilibria $\{\sigma_\lambda\}_\lambda$,*

$$\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{self} = \{\{v_i\}_{v_i \in V}\} \mid \sigma_\lambda\right) \leq 1 - \varepsilon.$$

The intuition is the following. If buyers learn their valuations fully, they simply bid truthfully in equilibrium, and the good goes to the highest-valuation buyer. It is then cost-efficient for buyers to first assess how much competitive pressure they will face in the auction (i.e., what is the highest valuation among their competitors) and then only learn their own valuations when it is worth it, as this leads to strictly lower overall information costs (Lemma 1). In the proof, we show that doing so does not harm their gross payoff from the auction and must hence be a profitable deviation.

More generally, this highlights buyers' incentive to learn about their competitors. If they do so, then their private information (i.e., their types) when entering the auction will be interdependent. Indeed, not only will buyers have information relevant to others, but their beliefs about their own valuations will depend on what they learned about others. In other words, the equilibrium information structure will fail to satisfy the standard assumption of independent private types, despite buyers' valuations being statistically independent. This significantly complicates the analysis of equilibrium behavior, as buyers no longer have a dominant strategy when deciding how to bid.

To illustrate this, consider a buyer i who acquired no information whatsoever and let $N = 2$. If buyers were not able to acquire information about others, then bidding his expected valuation would be a dominant strategy for i. This is no longer the case in our model. For instance, suppose buyer j becomes fully informed about his competitor's valuation (i.e., $\Pi_j^{other} = \{\{v\}_{v \in V}\}$) and always bids just below it (i.e., $\beta_j(\pi_j) = v_i - \varepsilon$ for some small ε). Given j's strategy, buyer i *always* wants to win the auction since the price he pays always lies strictly below his valuation: a best response is for i to bid sufficiently high so as to be guaranteed a win. Bidding his expected valuation given *his own information set* results in a strictly lower payoff for i, and so it is no longer a dominant strategy.[19]

[19] Thus, the (more standard) refinement of weakly undominated strategy has little bite in our setting, which is why we require robustness to trembles.

Overall, buyers' ability to learn about each other can create rich interdependencies between their information and bids, potentially expanding widely the types of behavior sustainable in equilibrium. Yet, focusing on high-stake auctions (i.e., auctions with small cost parameter λ) enables us to characterize equilibrium information structures.[20]

Theorem 1. *Let the prior be sufficiently uncertain. Take any sequence of equilibria* $\{\sigma_\lambda\}_\lambda$ *and any information structure such that* $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$*. Then:*

1. *Buyers learn something about the competitors:* $|\Pi^{other}| > 1$*;*

2. *Buyers learn their valuation fully if it is "close" to that of their toughest opponent but bundle together all values that are below that of their toughest opponent:* $\forall \pi^{other} \in \Pi^{other}$,

$$(\star) \qquad \{v_i\} \in \Pi^{self}(\pi^{other}) \quad \forall \min_{v \in \pi^{other}} v < v_i < \max_{v \in \pi^{other}} v$$
$$\exists \pi^{self}_{<} \in \Pi^{self}(\pi^{other}) \text{ s.t. } v_i \in \pi^{self}_{<} \quad \forall v_i < \min_{v \in \pi^{other}} v$$

In words, condition ($\star$) requires that if buyer i knows that his toughest competitor's valuation belongs to some interval π^{other}, then buyer i learns his own valuation fully if it falls (strictly) in the same interval. Indeed, in equilibrium, i knows that the highest bid he might face is likely to be in that interval, and so there are strict benefits from learning whether or not he wants to buy the good at each of these prices. However, learning to distinguish values that are lower than that of his toughest opponent brings no benefit, such that an optimal signal bundles these values together. Overall, the competitive pressure that buyers impose on each other shapes the information that they acquire in significant ways. The rest of the paper examines how that, in turn, affects the value of competition.

4 Revenue Distortions

In this section, we analyze the impact of learning incentives on revenue.

Since buyers do not fully learn their valuations in equilibrium, expected revenue is likely to be different than in our benchmark cases. We show that revenue remains strictly lower and bounded away from the expected second highest valuation, even for

[20]Focusing on high-stake auctions ensures that any information that has non-trivial value must be acquired in equilibrium, thus reducing the noise in buyers' behaviors.

small information costs λ. This contrasts with our above benchmarks, where revenue converges to the expected second highest valuation as λ goes to zero.

Theorem 2. *Let $N \geq 3$. There exist $L > 0$ and $\overline{T} > 0$ such that, if $\sum_v (\mathbb{P}(v))^2 \leq \overline{T}$, the revenue generated in any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ is bounded away by L from the expected second highest valuation as λ goes to zero:*

$$\lim_{\lambda \to 0} \mathbb{E}\left[\text{ equilibrium revenue } \middle| \sigma_\lambda\right] < \mathbb{E}\left[v_{(2)}\right] - L.$$

What is driving the loss in revenue? In equilibrium, buyers first assess the competition and only learn their own valuations if they fall in a similar range as that of their toughest competitor. As a result, losing bidders often only learn that their valuations are below some threshold (and, in particular, below that of their toughest competitor) but fail to learn it exactly. Our equilibrium refinement guarantees that if losing bidders fail to learn their valuations, they bid their expected valuations given their information sets. This reduces the variance in losing bids and distorts the expected second-highest bid downwards whenever $N \geq 3$. Indeed, since the $\max$ is a convex function, the expected highest bid among losing bids is greater when losing bids are more dispersed. (With only $N = 2$ buyers, there is only one losing bid, and dispersion plays no role.)

The order of quantifiers in Theorem 2 matters and is worth noting. First, the constant L is independent of the cost parameter λ, such that a revenue loss persists in the limit. Second, the result requires that the prior be sufficiently uncertain ($\sum_v (\mathbb{P}(v))^2 \leq \overline{T}$), which guarantees that it is cost-efficient for buyers to acquire some information about the competition. Theorem 2 holds for any $N \geq 3$, but the threshold $\overline{T}$ is allowed to depend on N, such that the condition on the prior becomes increasingly restrictive as the number of buyers N grows large. This aligns with real-world intuition: if the number of auction participants is small, it seems reasonable that they can learn about each other and that it affects their equilibrium bidding behavior. That seems less realistic if the number of auction participants is large, and the condition in Theorem 2 is indeed less likely to hold in such a case.

We emphasize that for sufficiently small information costs λ, the equilibrium allocation of the good remains efficient in our model, namely the highest valuation buyer wins the good. Indeed, a buyer only fails to learn his valuation in equilibrium if he is sure of losing given what he learned about others. Hence a direct corollary of Theorem 2 is that endogenous information acquisition does not affect total surplus for small information costs, but redistributes surplus from the seller to the buyers. It furthermore suggests that, if possible, high-valuation buyers have a strong incentive to signal that they have a high valuation, so as to discourage others from learning about their own and competing aggressively. This is often seen in practice. For instance, jump bidding and toeholds are sometimes seen as signaling devices that aim at deterring competition (Bulow et al. (1999); Betton and Eckbo (2000); Hörner and Sahuguet (2007)).

A Uniform Example. Let $V = \left\{\frac{1}{K}, \frac{2}{K}, \ldots, \frac{K-1}{K}, \frac{K}{K}\right\}$ and $\mathbb{P}(v_i) = \frac{1}{K}$ for each $v_i \in V$. For K large enough, this approximates a uniform distribution on $[0, 1]$. We set c to be the entropy-based cost function as in Example 2.

We know from Theorem 1 that, for small enough cost parameter λ, buyers only put non-trivial weight on information structures satisfying ($\star$). However, there are many such information structures, and we do not know which one(s) can arise in equilibrium.[21] If we perturb the model and add some noise to buyers' valuations, we can rule out ties on path and say more about the equilibrium information structure.[22] In the Online Appendix, we show that buyers must furthermore (i) bundle all valuations that lie above that of their toughest competitor, and (ii) choose the cheapest such information structure. We find this information structure numerically and depict the equilibrium information partition about others Π^{other} in Figure 6 of Appendix A.1. We then simulate equilibrium bids and compute expected revenue for vanishing λ. We also compute expected revenue when buyers are fully informed of their valuations, which equals the expected second-highest valuation.

We plot the results in Figure 2 for two specifications of the scaling factor γ. Recall that γ captures how the cost of learning about the competition scales with the number of competitors. The revenue loss due to endogenous information acquisition is captured by the difference between the top (blue) and the bottom (red) lines. To assess the magnitude of this loss, we compare it to the loss in revenue associated with using a posted-price mechanism instead of an auction in the standard model.[23] In this example, the revenue loss due to endogenous information acquisition is similar in magnitude to the loss associated with using a suboptimal posted-price mechanism when γ is

[21] We do know from Theorem 2 that they all lead to a revenue loss.

[22] The noise terms can be interpreted as perturbations à la Harsanyi (1973), which guarantee that buyers cannot predict others' preferences and equilibrium bids *perfectly*.

[23] Under a posted-price mechanism, the seller chooses a (unique) price and, if several buyers express interest in buying the good at that price, allocates the good randomly between them.

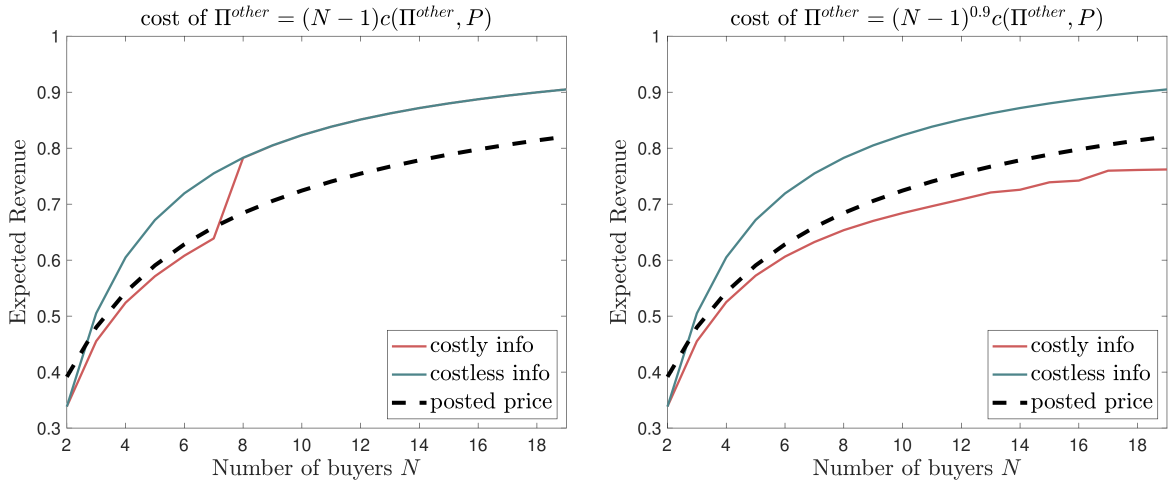


Figure 2: *The blue line plots expected revenue in the standard model (i.e., the expected second-highest value). The red line plots expected revenue in our model for small λ. For comparison, the dashed (black) line plots expected revenue in the standard model when the seller uses a posted-price mechanism. The difference between the blue and red lines is the revenue loss from Theorem 2. Parameter $K = 100$.*

slightly concave (right panel) and when γ is linear but N is not too large (left panel).

Why is there no revenue loss when γ is linear (left panel) and $N \geq 8$? Recall that, in this example, we fix the prior $\mathbb{P}$ and compute equilibrium revenue for different values of N. However, Theorem 2 only applies when the prior is sufficiently uncertain *given* N. So in this example and under linear γ, the prior is sufficiently uncertain for $N < 8$ but not for $N \geq 8$.

Remark 1. *Unlike in the standard auction model, realized revenue is not increasing in realized values $(v_i)_i$.*

When revenue equals the second-highest valuation among buyers, then a (component-wise) increase in $(v_i)_i$ leads to weakly higher revenue. This is no longer the case in our setting as a high realized value for a buyer can discourage others from acquiring information. For instance, suppose that $\Pi^{other} = \{\{v \mid v \leq v^*\}, \{v \mid v > v^*\}\}$ for some v^*, and consider what happens when the highest-value buyer has $v_{(1)} > v^*$ but all other buyers have values below v^*. As depicted in Figure 3, all these buyers learn that one of their competitors has a value greater than v^* and only learn that their own value is below v^*. They all bid $\mathbb{E}[v_i \mid v_i \leq v^*]$, which pins down revenue to the seller. Now consider what happens if we decrease the highest-value bidder's value to $v_{(1)} \leq v^*$. Then,

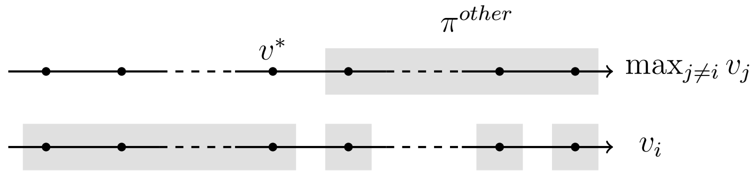


Figure 3: *Suppose* $\Pi^{other} = \{\{v \mid v \leq v^*\}, \{v \mid v > v^*\}\}$ *for some* v^**. If buyer* i *learns* $\max_{j\neq i} v_j > v^*$ *but* $v_i \leq v^*$*, he does not learn his valuation mfully and bid* $\mathbb{E}[v_i \mid v_i \leq v^*]$*.*

in equilibrium, all buyers learn that $\max_{j\neq i} v_j \leq v^*$, they all learn their valuations precisely since it is also below v^*, and they bid their valuations. Revenue then equals $v_{(2)}$, which is greater whenever $v_{(2)} \in (\mathbb{E}[v_i \mid v_i \leq v^*], v^*]$

5 Market Design Implications

So far, we have shown that losing buyers often fail to learn their valuations precisely, which leads them to bid less aggressively and depresses expected revenue. This has implications for market design. First, the value of an optimal reserve price can dominate the value of an additional bidder. This contrasts with common wisdom inherited from Bulow and Klemperer (1996) (thereafter, "BK"). Second, the seller gains by maintaining uncertainty over competition. Overall, this suggests that competition is most effective if it is carefully *designed* by the seller.

5.1 Additional Bidders vs Optimal Reserve Price

In a seminal paper, BK show that the value of an additional bidder in an auction always dominates the value of optimizing the reserve price under regular distributions. This is an important result as it gives market designers a very simple and actionable insight: attract as many bidders as possible.

We revisit this result in our setting. We show that, once we account for how competition shapes learning incentives, BK's theorem may no longer hold.

Proposition 4. *Revenue with* $N + 1$ *buyers but no reserve price is not always higher than revenue with* N *buyers but an optimal reserve price.*

The proof of Proposition 4 consists in finding a counterexample in which BK's result holds when buyers know and bid their valuations, but does not hold in our setting. The example we use in the uniform example from Section 3.3. Figure 4 plots the value of an additional bidder (dashed curve) and the value of a reserve price (solid curve) in our model for vanishing learning cost λ. We get the opposite of BK's result whenever the solid curve is above the dashed curve.

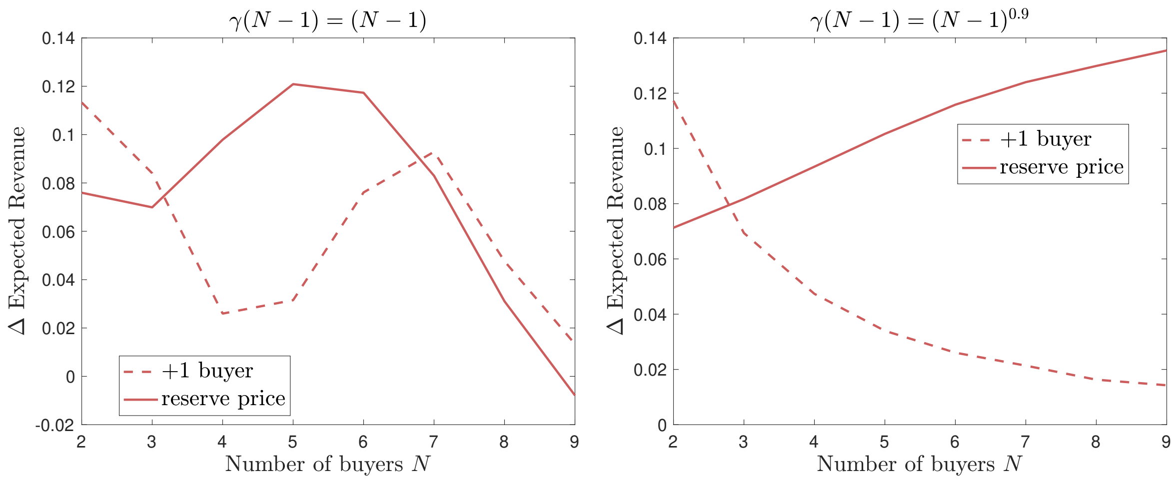


Figure 4: *The dashed lines plot the value of an additional bidder—i.e., the difference between expected revenue under $N+1$ bidders and expected revenue under N bidders—as a function of N. The solid lines plot the value of a reserve price—i.e., the difference between expected revenue under N bidders with an optimal reserve price and without a reserve price—as a function of N. All curves are fitted with a flexible (6^{th}-degree) polynomial to smooth small irregularities arising from the discreteness of the set of valuations V.*

That are several forces at play when comparing additional bidders to a reserve price. First, the standard BK force is still partly there: if buyers' valuations are close enough to each other, buyers learn their valuations precisely, and an additional draw in that range is better than a reserve price. Perhaps more importantly, there is a new force that favors additional bidders. Higher N means that learning about the competition is harder, such that moving to $N+1$ buyers might lead buyers to learn less about the competition and more about themselves. This is what happens at $N = 7$ in the left panel (linear γ): when going from $N = 7$ to $N = 8$, we go from an equilibrium in which Theorem 2 applies to one in which buyers learn nothing about the competition and learn their valuations fully. This leads to a steep increase in revenue, and the value of that additional bidder is greater than the value of a reserve price. For $N \geq 8$, buyers fully learn and bid their valuations; we are back to a standard framework in which

BK's result holds.

There are also new forces that favor a reserve price over additional bidders. First, additional bidders often fail to learn their valuations fully, and so do not add as much competitive pressure as in the standard auction model. Second, a reserve price is more valuable in our setting. Indeed, the regression to the mean of losing bids leaves a larger expected gap between the highest and second-highest bid, and hence more room for a carefully-designed reserve price to intervene. Combined, these forces seem to dominate when the cost of learning about the competition does not increase too quickly with N (e.g., when $\gamma(N-1) = (N-1)^{0.9}$ as in the right panel of Figure 4).

We cannot characterize the optimal reserve price generally, but it is worth nothing that it has different properties than in the standard auction model.

Remark 2. *The optimal reserve price varies with the number of buyers N.*

When buyers know their valuations ex ante, the optimal reserve price is known to be independent of the number of participants N in the auction (see, e.g., Myerson (1981)).[24] In our framework, the optimal reserve price needs to account for learning incentives and, as a result, varies with N.

5.2 Maintaining Uncertainty over Competition

To mitigate the revenue loss, the seller needs to incentivize buyers to learn their valuations for the good. In this section, we show that by maintaining uncertainty on the set of auction participants, the seller can induce more learning and increase revenue.

(Non-)Disclosure of Competition. We first consider a seller who has direct control over what information each buyer has about the competition. That is, the signal about the competition Π_i^{other} is no longer chosen by buyers but by the seller herself. Buyers only choose how much to learn about themselves. The rest of the model remains unchanged. In particular, we still focus on symmetric equilibria, and so on symmetric disclosure policies, such that $\Pi_i^{other} = \Pi_j^{other}$ for all i, j.

[24] This result technically requires buyers' set of possible valuations to be an interval and the distribution to satisfy an appropriate regularity condition. It holds approximately when our finite set of valuations is sufficiently fine.

Proposition 5. *There exists $\overline{\lambda}$ such that, for all $\lambda \leq \overline{\lambda}$, if partition $\widehat{\Pi}^{other}$ is coarser than partition Π^{other}, then*

$$\mathbb{E}\left[\, \textit{equilibrium revenue} \,\middle|\sigma_\lambda, \Pi^{other}\right] \leq \mathbb{E}\left[\, \textit{equilibrium revenue} \,\middle|\sigma_\lambda, \widehat{\Pi}^{other}\right].$$

Furthermore, the inequality is strict whenever $N > 2$ and the difference between the two partitions is not just that $\widehat{\Pi}^{other}$ reveals whether one's toughest opponent has the lowest possible valuation.[25] Thus expected revenue is monotonically decreasing in how much information is revealed about the competition. If such information is under her control, the seller has an (often strict) incentive not to disclose it.

Our results can explain why sellers sometimes try to keep secret the identity of participants in an auction. For instance, potential bidders in takeover auctions sign a confidentiality agreement that prevents them from revealing, among other things, their participation in the auction and the value of their bids (see Gentry and Stroup (2019) for a description of a typical takeover auction).

Noising up the Auction Process. However, in many settings, the seller may not fully control what information buyers have about the competition, or prevent it from leaking. We now explore an alternative way the seller can maintain uncertainty over the *effective* set of bidders, by adding noise to the auction process.

The set of buyers $N = \{1, \ldots N\}$ is still exogenous, but the seller can now commit to only letting some (random) subset of buyers compete in the auction. That is, the seller can commit to only considering some of the submitted bids. Let $\tilde{M}$ be the random set of buyers who get access to the auction—that is, buyers whose bids are taken into consideration,—whose distribution is chosen by the seller. As before, buyers acquire information before bidding and, in particular, before knowing the realization of $\tilde{M}$.

Consider the following way to randomize access to the auction. With probability $1 - q < 1$, all buyers get access $M = \{1, \ldots, N\}$. All bids are then taken into consideration: the good goes to the highest bidder who pays the second-highest bid. With probability q, one buyer chosen uniformly at random is denied access to the auction: $M = \{1, \ldots, N\} \setminus i$ for some i. In such an event, the seller acts as if buyer i had not submitted a bid. Hence, even if one learns that another buyer has a greater valuation, there is still some strictly positive probability q/N that the other buyer's bid will not

[25] This is the case if either $\{\min\{v \in V\}\} \notin \widehat{\Pi}^{other} \setminus \Pi^{other}$ or $|\widehat{\Pi}^{other} \setminus \Pi^{other}| > 2$.

be accounted for. Information about one's own valuation is then strictly beneficial: for λ sufficiently small, buyers become fully informed.

Proposition 6. *Take any $\varepsilon > 0$. There exists an access rule $\tilde{M}$ such that, for λ small enough,*

$$\mathbb{E}\left[\,\textit{equilibrium revenue}\,\middle|\sigma_\lambda\right] \geq \mathbb{E}\left[v_{(2)}\right] - \varepsilon$$

in any equilibrium σ_λ. In particular, the access rule described above with $q = 1 - \frac{\varepsilon}{\mathbb{E}[v_{(2)}]}$ yields such revenue.

Proposition 6 suggests that randomizing access to the auction is a powerful tool to incentivize information acquisition. By maintaining uncertainty over whose bids a buyer will face, the seller reduces the negative effect of competition on learning incentives. In the proof of Proposition 6, we show that if a buyer's toughest competitor has a non-zero chance of being excluded from the auction, then the buyer has a strict incentive to learn his valuation for the good. For λ small enough, he will then do so.

Even though this intuition relies on buyers only being able to learn about their toughest competitor $\max_{j\neq i} v_j$—instead of *all* their competitors $(v_j)_j$,—Proposition 6 does not. Indeed, even if buyers could learn about the entire vector $(v_j)_j$, any access rule that excludes all *subsets* of buyers with positive probability—instead of one buyer at a time—would give buyers a strict incentive to learn their own valuations. What matters for the result is that there is uncertainty about the auction outcome conditional on realized values. This uncertainty can come from randomness in who is allowed to participate, as modeled here, but other interpretations are that noise is added to buyers' bids, or that the auction format is not fully predictable.

6 Discussion and Robustness of the Model

The key assumptions in our analysis are those imposed on the information acquisition process. Some are required for tractability, while others can be relaxed. We now discuss these assumptions in more detail.

6.1 The Structure of the Learning Process

Order of Signals. We model information acquisition as a two-step process, in which buyers first acquire a signal about their competitors' valuations and then one about their own. The following proposition says that such *ordering* is without loss within the class of two-step learning processes.

Proposition 7 (order of signals)**.** *Consider our main model specification, but suppose that buyers can now choose in which* order *to acquire the two signals.*[26] *There exists* $\overline{\lambda} > 0$ *such that, for all* $\lambda \leq \overline{\lambda}$ *and in any equilibrium* σ_λ*, buyers choose to acquire information on others first, and all results are unchanged.*

The proof of Proposition 7 consists in showing that, for λ small enough, there cannot be an equilibrium in which buyers choose to acquire information about their own value first. If such an equilibrium existed, then buyers would fully learn their valuations. However, they would then strictly benefit from learning about the competition first. Hence imposing that particular ordering is without loss for our results.

Number of Signals. Restricting attention to two-step learning processes is a stronger assumption. In general, there is no reason why buyers would not want to undertake a more elaborate learning process in which they go back and forth between acquiring some information about their own values and some information about others. We show that giving buyers this added flexibility does not change the main results of the paper: buyers learn about the competition, and equilibrium revenue remains bounded below the expected second-highest valuation as information costs vanish.

Proposition 8 (number of signals)**.** *Consider our main model specification, but suppose that buyers can now choose as many signals—i.e., partitions—as they wish as well as in which order to acquire them.*[27] *Proposition 3 and Theorem 2 hold. That is, as information costs vanish, buyers fail to learn their valuations fully which results in a revenue loss.*

If anything, this added flexibility allows buyers to even better condition how much they learn about themselves on the competition they face. To illustrate this, consider our uniform example with $|V| = 20$ possible valuations, $N = 2$ buyers, and entropic cost. In our baseline model, the equilibrium information structure is as depicted in Figure 5 (left panel): buyers first choose $\Pi^{other} = \{\{v_i : v_i \in [0, \frac{1}{3}]\}, \{v_i : v_i \in [\frac{1}{3}, \frac{2}{3}]\}, \{v_i : v_i \in [\frac{2}{3}, 1]\}\}$ and then learn their valuation fully only if it falls into the same interval

[26] That is, they can choose whether to first acquire a signal about their own value v_i and then one about others $\max_{j \neq i} v_j$, or vice versa.

[27] For instance, they can choose to first learn whether $\max_{j \neq i} v_j > v^*$ and then learn whether their own value is above v^*, and then go back and learn more about the competition, etc.

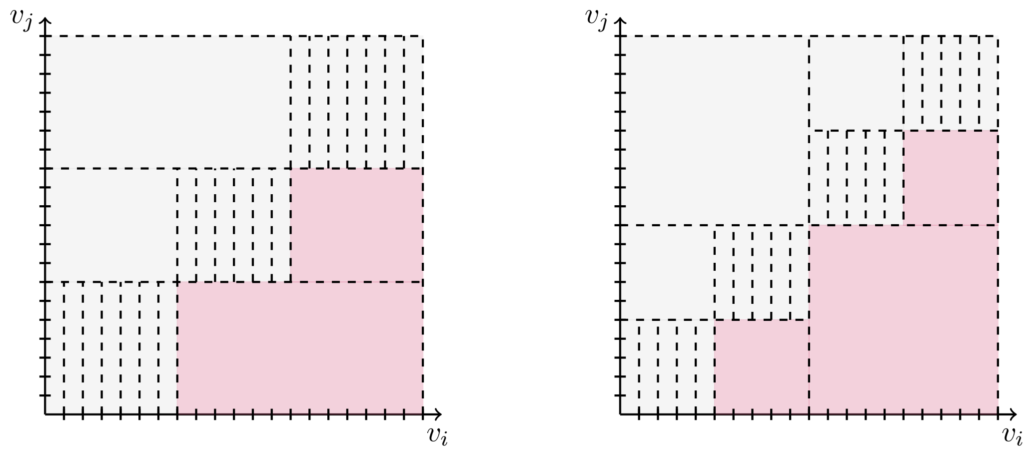


Figure 5: Equilibrium information structure when buyers can only acquire two signals (left panel) vs. as many signals as they wish (right panel).

as the opponent's. When buyers are allowed to go back and forth between learning about self and others, the equilibrium information structure is as depicted on the right panel and resembles a binary search: buyers first learn whether $v_j > 0.5$ and whether $v_i > 0.5$, which divides the state space into four quadrants. If $v_j > 0.5$ and $v_i \leq 0.5$ (upper left quadrant) or if $v_j \leq 0.5$ and $v_i > 0.5$ (lower right quadrant), then they stop acquiring information. Otherwise they keep on learning. The areas shaded in pink are all the states of the world in which buyer i is the losing buyer and fails to learn his valuation fully, which drive the revenue loss. If anything, the shaded area is larger when buyers have more flexibility in how they learn.

Learning about $\max_{j \neq i} v_j$. Another key simplifying assumption is that buyers only learn about the value of their toughest competitor $\max_{j \neq i} v_j$ and not about each of their competitors separately $(v_j)_{j \neq i}$. There is a sense in which this is more of an equilibrium selection criterion than a restriction on buyers' strategies, as we now explain.

Suppose that instead of acquiring a signal about $\max_{j \neq i} v_j$, buyer i can acquire a signal Π_i^j about each of his competitors j at cost $c(\Pi_i^j, \mathbb{P})$. These signals can be acquired simultaneously or sequentially. Afterwards, buyer i chooses a signal about his own value Π_i^{self} and submits a bid, as in our baseline model. Even if buyer i learns about his competitors separately, he can still choose a strategy that treats them symmetrically. For instance, if he learns about them simultaneously, then choosing the same partition $\Pi_i^j = \Pi_i^{j'}$ for all j, j' treats competitors symmetrically. If i learns about the competitors sequentially—e.g., by first learning whether j's value is above some threshold v^* and then only learning about j' if $v_j < v^*$—he can still treat his competitors symmetrically by picking uniformly at random whom he learns about first. Formally, given signal realizations $(\pi_i^j)_{j \neq i}$, let $\pi_i^{max} = \{v : \mathbb{P}(\max_{j \neq i} v_j = v \mid (\pi_i^j)_{j \neq i}) > 0\}$ be the set of values that i deems possible for his toughest competitor. Say strategy σ_i *treats competitors symmetrically* if, under σ_i, (i) the distribution of $(v_j)_{j \neq i} \mid \pi_i^{max}$ is exchangeable for every π_i^{max}, and (ii) i's continuation strategy depends only on what he learned about his toughest competitor $\sigma_i(\cdot \mid (\pi_i^j)_{j \neq i}) = \sigma_i(\cdot \mid \pi_i^{max})$. In words, condition (ii) says that what signal i acquires about himself and how he bids only depend on π_i^{max} (e.g., it depends on the realization of $\max_{j \neq i} v_j$ but not on the identity of the toughest competitor $\arg\max_{j \neq i} v_j$). Condition (i) says that, conditional on π_i^{max}, the values of i's competitors are identically distributed.

Proposition 9. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ of the game in which buyers are restricted to choosing a strategy that treats competitors symmetrically, and let $\sigma^* = \lim_{\lambda \longrightarrow 0} \sigma_\lambda$. There exists $\overline{\lambda}$ such that, for all $\lambda \leq \overline{\lambda}$, strategy $\sigma_i = \sigma^*$ remains a best response to $\sigma_{-i} = \sigma^*$ when buyer i' strategy set is unrestricted.*

Thus, for small information costs, buyers find it optimal to treat their competitors symmetrically in the learning process if others do so in equilibrium. If buyers treat their competitors symmetrically, then their strategies depend only on what they learn about their toughest opponent $\max_{j \neq i} v_j$. One can then interpret our baseline model as capturing such behavior in a reduced-form way. Of course, our model cannot capture equilibria in which buyers do *not* treat their competitors symmetrically. For instance, our analysis does not capture equilibria where all buyers learn the value of some arbitrary buyer i^* and use v_{i^*} as a coordination device to decide who wins the auction (i.e., who bids high when all others bid zero).

Learning Jointly about Self and Others. We have assumed throughout that buyers learn separately about their own value v_i and the value of their toughest opponent $\max_{j \neq i} v_j$. An even more flexible learning process would, however, allow buyers to choose any partition of the overall state space. A natural candidate equilibrium strategy would have each buyer i learn whether $v_i > \max_{j \neq i} v_j$ or not. Ignoring ties, this is only sustainable in equilibrium if the buyer who learns he has the highest value bids the maximal value any buyer can have, while all others bid the lowest value. This trivially leads to a revenue loss; effectively, buyers play a "bidding ring" equilibrium but coordinate such that the highest value bidder wins. Even if such joint learning benefits the buyers, we find it less economically realistic and thus focus on the case where buyers can only learn separately about self and others.

6.2 Noisy Signals and Ex-Ante Private Information

Throughout the paper, we have modeled signals as information partitions. Partitions are a particular type of signals in that they are "deterministic:" the signal can be partially informative but it can never be "wrong." Naturally, this is a simplifying assumption, but one that is needed for tractability. With more noise in the learning process, characterizing equilibrium bidding becomes much more challenging. Recall that in our setting, the second-price auction is no longer strategy-proof: that is, it might not be optimal for a buyer i to bid his expected valuation given his information set. Indeed, other buyers might have acquired information about their competitors—which include i himself—and so their bids might carry information relevant to i. There is no nice structure on how a buyer should update about his value upon tying at a particular bid—e.g., he could update positively at a some bid but negatively at another. By reducing the noise in the learning process we recover tractability: if a buyer ties, he must have a valuation close to that of his toughest opponent, and so must have learned his valuation fully. This is also why we focus on small information cost λ.

In this section, we tweak our baseline model to allow for more general signal structures. Instead of being completely uninformed of their values at the beginning of the game, buyers are now endowed with *some* private information. More precisely, they get to observe for free the realization of a signal s_i where $s_i = \underline{s}$ when $v_i \leq v^*$ and $s_i = \overline{s}$ when $v_i > v^*$ for some $v^* \in V$. In words, they initially know whether their value is "high" (above v^*) or "low" (below v^*.) As in our baseline model, buyers can acquire two additional signals: one about their own value and one about their competitors'. However, we now impose that buyers can only learn about their competitors' *exogenous* information, i.e, they can only learn about $\max_{j \neq i} s_j$ and not about $\max_{j \neq i} v_j$. This assumption makes all the above complications go away as it ensures that a buyer i can only learn things about j that j *already knows*. We can thus allow for more general signals and we give buyers full flexibility in how to design these two signals. That is, they can choose any distribution over posterior beliefs about others ($\max_{j \neq i} s_j$) and about self (v_i) that is consistent with their prior. The cost of information needs to be adapted to allow for this larger set of signals. For simplicity, we set c to be the entropy-based cost from Example 2, which easily accommodates signals other than partitions. The rest of the model is unchanged.

Theorem 3. *Let c be the entropy-based cost. If the prior conditional on s_i is sufficiently uncertain,*[28] *then Proposition 3 and Theorem 2 holds.*

Our main results thus extend to a setting where buyers can choose noisy signals: as information costs vanish, buyers still fail to learn their valuations fully, which results in a revenue loss. The same intuition as in our baseline model holds: for small enough information costs, buyers must learn enough so as not to make non-trivial mistakes at the bidding stage. One way not to make mistakes is to learn one's valuation fully, but that is not cost-efficient. In equilibrium, a buyer who knows his value is above v^* (i.e., a buyer with $s_i = \overline{s}$) will converge to learning almost perfectly whether his toughest opponent is weak ($\max s_j = \underline{s}$), in which case he will not acquire any information about himself. Doing so is cost-efficient whenever the remaining uncertainty that buyer i has about his value (i.e., the uncertainty in the distribution of v_i conditional on s_i) is sufficiently large.

We emphasize a key feature that is important for our results: even though the signal that a buyer acquires about the competition is noisy, it can be made arbitrarily precise. If there were an exogenous bound on how precise the signal about others could be (e.g., the signal could only be correct with probability at most $q < 1$ for some exogenous q), then, for λ small enough, buyers would converge to fully learning their valuations. They would still have an incentive to learn about the competition for $\lambda > 0$ and the intuitions would go through, but the discontinuity at the limit would not.

6.3 Beyond IPV Second-Price Auctions

Finally, we briefly discuss why our analysis focuses on second-price auctions with independent private values.

The assumption of *private* values is necessary to be able to distinguish between learning about self and learning about others. The assumption of *independent* values is too strong for our purposes: we could allow for some correlation in values $(v_i)_i$ across buyers without altering our analysis. With correlation, there is an additional benefit

[28] That is, if $\sum_{v \leq v^*} [\mathbb{P}(v)/(\sum_{v' \leq v^*} \mathbb{P}(v'))]^2$ and $\sum_{v > v^*} [\mathbb{P}(v)/(\sum_{v' > v^*} \mathbb{P}(v'))]^2$ are sufficiently small.

from learning about the competition, which is that it indirectly provides information about one's own value. This is not the economic mechanism we wish to highlight, which is why we assume independence. However, we do acknowledge that our analysis becomes less relevant if buyers' values are very correlated: the main intuition behind our results is that buyers might not undertake proper due diligence when one of their competitors has a much higher value than them, but that possibility becomes less likely when buyers' values are highly correlated.

A perhaps stronger restriction is the focus on second-price auctions. This seems like a natural starting point since second-price auctions are strategy-proof: if buyers knew their values, they would have no incentive to learn about the competition, and thus we can isolate a specific mechanism for such incentive in our setting. Of course, it would be interesting to know whether other auction formats also suffer a revenue loss once buyers can learn about their competitors, and, if yes, how that loss compares across them. Unfortunately, characterizing equilibria with endogenous interdependent types is untractable. That said, our analysis can still teach us something about other auction formats. For instance, for small enough learning costs, if the equilibrium of an auction leads to an efficient allocation of the good, then the same learning incentives as in our paper arise: buyers want to learn about their competitors so as to save on information costs when they do not have the highest value. Thus, for another auction format to outperform the second-price auction, it would have to induce some allocative inefficiencies.

7 Conclusion

This paper develops a tractable model of multidimensional information acquisition in auctions, in which buyers can learn both about the strength of competition and about their own valuations. We first characterize how the competitive pressure between buyers shapes the information that they seek. In our framework, buyers find it cost-efficient to first acquire some information about their competitors so as to only learn their valuations when they have a chance to win. Second, we show that competition between buyers is made less effective by learning incentives, which depresses expected revenue. We then propose market design solutions to mitigate these effects. Overall, we show that the seller benefits from carefully designing the competition that buyers face—either via a reserve price or by maintaining uncertainty over the extent of competition.

Our results suggest that the interactions between information and competition can have large, previously unexplored implications for auction design. We believe it provides yet another justification for robust mechanism design, or at least a careful consideration of informational incentives in the practice of market design.

Appendix A Additional Material

A.1 Additional Details on the Uniform Example

Throughout the paper, we use a uniform example in which $V = \left\{\frac{1}{K}, \ldots, \frac{K}{K}\right\}$ and $\mathbb{P}(v) = \frac{1}{K}$ for all v to illustrate our results. If we add noise to buyers' valuations, we know that, in equilibrium, all buyers choose the cost-minimizing information structure $\left(\Pi^{other}, \Pi^{self}\right)$ that satisfies

$$\Pi^{self}\left(\pi^{other}\right) = \left\{\left\{v_i : v_i < \min_{v \in \pi^{other}} v\right\}, \left\{v_i\right\}_{v_i \in \pi^{other}}, \left\{v_i : v_i > \max_{v \in \pi^{other}} v\right\}\right\},$$

for all $\pi^{other} \in \Pi^{other}$. (See the Online Appendix for more details on the perturbations.) We find this numerically, and depict Π^{other} for several values of N in Figure 6.

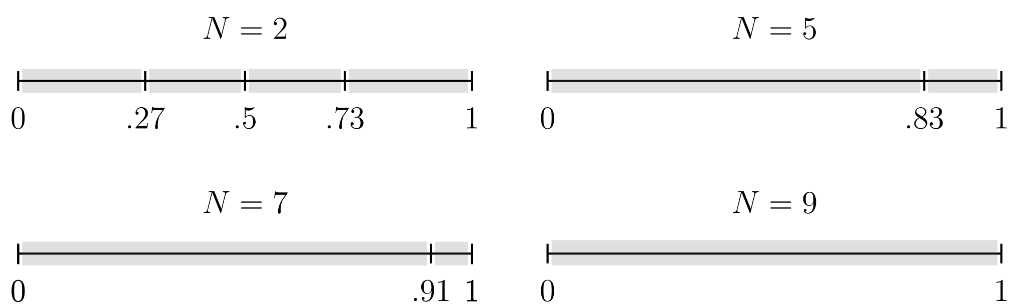


Figure 6: *Parameter $K = 100$ and $\gamma(N-1) = N-1$. No information about the competition is acquired for $N = 9$.*

Appendix B Proofs

B.1 Preliminary Analysis

Throughout the proofs, we label values such that $V = \{v^k\}_{k=1}^K$ with $v^1 < v^2 < \cdots < v^K$.

Proof of Lemma 1. Let $\Pi_0 = \{\{v_i\}_{v_i \in V}\}$. We want to show that, for $\sum_v (\mathbb{P}(v))^2$ low enough, there exists $v^* \in V$ such that

$$\gamma(N-1)c\left(\Pi_{v^*}, \mathbb{P}\right) + \Pr\left(\max_{j\neq i} v_j \leq v^*\right) c\left(\Pi_0, \mathbb{P}\right) + \Pr\left(\max_{j \neq i} v_j > v^*\right) c\left(\Pi_{>v^*}, \mathbb{P}\right) < c\left(\Pi_0, \mathbb{P}\right)$$

$$\iff \left[1 - \mathbb{P}(v \leq v^*)^{N-1}\right]\left[c\left(\Pi_0, \mathbb{P}\right) - c\left(\Pi_{>v^*}, \mathbb{P}\right)\right] - \gamma(N-1)c\left(\Pi_{v^*}, \mathbb{P}\right) > 0.$$

The term in brackets on the LHS is the difference in cost between the fully revealing partition and the partition that bundles all values below v^* and fully reveals all those above v^*. We can bound that term using our assumptions on the cost:

$$\begin{aligned} c\left(\Pi_0, \mathbb{P}\right) - c\left(\Pi_{>v^*}, \mathbb{P}\right) &\geq \left[\sum_{v \leq v^*} \mathbb{P}(v)\right]\left[c(\Pi_0, \mathbb{P}(\cdot \mid v \leq v^*)) - c(\Pi_{>v^*}, \mathbb{P}(\cdot \mid v \leq v^*))\right] \\ &\geq \left[\sum_{v \leq v^*} \mathbb{P}(v)\right] \kappa[H(\mathbb{P}(\cdot \mid v \leq v^*)) - H(\delta_{\{v \mid v \leq v^*\}})] \end{aligned}$$

where $\delta_{\{v|v\leq v^*\}}$ denotes the distribution that puts probability one of the element of the partition $\{v \mid v \leq v^*\} \in \Pi_{>v^*}$. The first inequality comes from weak convexity, and the second from the assumption on how the cost scales with the fineness of a partition. The cost between the fully revealing partition and partition $\Pi_{>v^*}$ thus scales with how uncertain the prior is over values $v \leq v^*$.

It then suffices to show that, for $\sum_v (\mathbb{P}(v))^2$ low enough, there exists v^* such that

(1)
$$\left[1 - \mathbb{P}(v \leq v^*)^{N-1}\right] \mathbb{P}(v \leq v^*)\kappa[H(\mathbb{P}(\cdot \mid v \leq v^*)) - H(\delta_{\pi_{\leq v^*}})] - \gamma(N-1)c\left(\Pi_{v^*}, \mathbb{P}\right) > 0.$$

Given the prior $\mathbb{P}$, set v^* so as to minimize $|\mathbb{P}(v \leq v^*) - 0.5|$. Let $T \equiv \sum_v (\mathbb{P}(v))^2$, and note that no value $v \in V$ can have prior probability greater than $\sqrt{T}$, such that $|\mathbb{P}(v \leq v^*) - 0.5| \leq \sqrt{T}$. Hence, for small T, $\mathbb{P}(v \leq v^*)$ is arbitrarily close to 0.5. The distribution over signals $\mathbb{P}_{\Pi_{v^*}}$ that is induced by partition Π_{v^*} then has two mass points, each arbitrarily close to 0.5. For small T, the associated cost $c\left(\Pi_{v^*}, \mathbb{P}\right)$ varies arbitrarily little with $\mathbb{P}$ when we choose such a v^*, since the distribution $\mathbb{P}_{\Pi_{v^*}}$ varies very little with $\mathbb{P}$ and c is continuous in $\mathbb{P}$ for on the interior of the simplex. Thus, for T small enough, the only term that vary non trivially with the prior $\mathbb{P}$ is $H(\mathbb{P}(\cdot \mid v \leq v^*)) - H(\delta_{\pi_{\leq v^*}})$.

We have left to show that for T small enough, the term $H(\mathbb{P}(\cdot \mid v \leq v^*)) - H(\delta_{\pi_{\leq v^*}})$ can be made arbitrarily large, such that inequality (1) eventually holds. Recall that $\delta_{\pi_{\leq v^*}}$ is the degenerate distribution that assigns probability one to one realization, such that the uncertainty in the signal realization is minimal: $H(\delta_{\pi_{\leq v^*}}) = 0$. So we have left to show that, for T small enough, $H(\mathbb{P}(\cdot \mid v \leq v^*))$ is arbitrarily large. Recall that for any finite $\bar{H}$, there exists a distribution q that has uncertainty $H(q) \geq \bar{H}$. Let $K_{\bar{H}} = |\operatorname{supp} q|$. The fact that the measure of uncertainty H is invariant under permutation implies that, among all distributions with support size $K_{\bar{H}}$, the uniform distribution over that support maximizes H.[29] Furthermore the concavity of H implies that H is continuous over the interior of the simplex. For T small enough, the prior $\mathbb{P}$ is arbitrarily close to a uniform distribution with large support and so is $\mathbb{P}(\cdot \mid v \leq v^*)$. Thus for any $\bar{H}$, there exists $\overline{T}$ such that, if $\sum_v (\mathbb{P}(v))^2 \leq \overline{T}$ then $H(\mathbb{P}(\cdot \mid v \leq v^*)) \geq \bar{H}$. For $\bar{H}$ large enough, the inequality (1) holds. □

Lemma 2. *There exists $\overline{T}$ such that, if $\sum_v [\mathbb{P}(v)]^2 \leq \overline{T}$, then the following is true:*

$$\gamma(N-1)\left[c\left(\left\{\{v^1\},\{v^2,\ldots,v^*\},\{v : v > v^*\}\right\},\mathbb{P}\right) - c\left(\left\{\{v^1\},\{v : v > v^1\}\right\},\mathbb{P}\right)\right]$$
$$< \Pr\left(v^* < \max_{j\neq i} v_j\right)\left[c\left(\{\{v\}_{v\in V}\},\mathbb{P}\right) - c\left(\{v : v \leq v^*\},\{\{v\}_{v>v^*}\},\mathbb{P}\right)\right],$$

for some $v^1 < v^* < v^K$.

Lemma 2 compares two information structures: one with $\Pi^{other} = \{\{v^1\},\{v \mid v > v^1\}\}$ and $\Pi^{self}(\pi^{other}) = \{\{v\}_{v\in V}\}$ for all π^{other}, and one with $\Pi^{other} = \{\{v^1\},\{v \mid v^1 < v \leq v^*\},\{v \mid v > v^*\}\}$ for some v^* and $\Pi^{self}(\pi^{other}) = \{\{v \mid v \leq v^*\},\{v\}_{v>v^*}\}$ for $\pi^{other} = \{v \mid v > v^*\}$. If the prior is sufficiently uncertain, Lemma 2 says that the latter is cheaper. So buyers have an incentive to learn more than just whether $\max_{j\neq i} v_j = v^1$.

Proof. The proof is very similar to that of Lemma 1. If the prior is sufficiently uncertain, we can find v^* such that $\sum_{v\leq v^*}\mathbb{P}(v)$ is arbitrarily close to 0.5. The partitions $\Pi^{other} = \{\{v^1\},\{v \mid v > v^1\}\}$ and $\Pi^{other} = \{\{v^1\},\{v \mid v^1 < v \leq v^*\},\{v \mid v > v^*\}\}$ then

[29] H might have other maximizers as well, but the uniform distribution is one of them.

lead to distributions over signals arbitrarily close to $(0, 1)$ and $(0, 0.5, 0.5)$, respectively, which pin down the LHS. The RHS however can be made arbitrarily large by having a sufficiently uncertain prior $\mathbb{P}$. $\square$

B.2 Proofs of Results of Section 3

B.2.1 Proofs of Proposition 1

Proof of Proposition 1. For Proposition 1, suppose that each buyer can only learn about himself. We first prove that a symmetric equilibrium exists. We then argue by contradiction that buyers must converge to becoming fully informed as λ goes to zero.

The proof of equilibrium existence under one-dimensional learning is much simpler than under multi-dimensional learning, as we can impose that buyers bid their expected value given their information set, i.e., $\beta(\pi_i) = \mathbb{E}[v_i \mid \pi_i]$ for all $\pi_i \in 2^V$. This is indeed weakly optimal: when only buyer i can learn about his value, the second-price auction remains strategy-proof. We can redefine a buyer's strategy to just being his learning strategy $\sigma_i \in \Delta(\mathcal{P} \times \mathcal{P}^{2^V})$. Buyers' objective functions are now continuous in their strategy, such that existence follows from standard arguments.

Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$. We show that $\lim_{\lambda \longrightarrow 0} \Pr(\Pi^{self} = \{\{v_i\}_{v_i \in V}\} \mid \sigma_\lambda) = 1$. By contradiction, suppose that there exists a partition $\widehat{\Pi}^{self} \neq \{\{v_i\}_{v_i \in V}\}$ with $\lim_{\lambda \longrightarrow 0} \Pr(\Pi^{self} = \widehat{\Pi}^{self} \mid \sigma_\lambda) \equiv q > 0$. By definition, that partition must bundle (at least) two values: there exist $\hat{\pi}^{self} \in \widehat{\Pi}^{self}$ such that $|\hat{\pi}^{self}| > 1$. Let $\underline{v}$ and $\overline{v}$ be the smallest and highest elements in $\hat{\pi}^{self}$, respectively. Take the point of view of a buyer i who is at that information set $\pi_i = \hat{\pi}^{self}$. Since we are looking at a symmetric equilibrium, with strictly positive probability all other buyers are at the same information set. Thus buyer i might tie in the auction and must be indifferent between winning and losing at his equilibrium bid: $\beta(\hat{\pi}^{self}) = \mathbb{E}[v_i \mid v_i \in \hat{\pi}^{self}] \in (\underline{v}, \overline{v})$. But then, with strictly positive probability, buyer i faces a bid strictly between $v_i = \overline{v}$ and $v_i = \underline{v}$ and would strictly benefit from learning to distinguish these two values, as he wants to win against such bid if $v_i = \overline{v}$ but lose when $v_i = \underline{v}$. For λ small enough, deviating to $\Pi^{self} = \{\{v_i\}_{v_i \in V}\}$ is strictly profitable. $\square$

B.2.2 Proofs of Proposition 2 and 3

Proof of Proposition 2. We first prove the existence of a symmetric equilibrium building on Corollary 5.3 in Reny (1999) (Step 1). We then argue that the proof extends for the existence of symmetric equilibria that are robust to trembles (Step 2).

Step 1. Without loss, we can restrict agents' possible bids to belong to $[0, \overline{v}]$ for some $\overline{v} > \max\{v \mid v \in V\}$. Agents' pure strategy sets are then compact Hausdorff spaces. Furthermore, their utility functions are bounded, measurable, and symmetric. Corollary 5.3 in Reny (1999) then states that there exists a symmetric mixed strategy equilibrium if (the mixed extension of) our game is better-reply secure along the diagonal. The crux of the proof is then to show that this condition of better-reply security holds in our setting.

Let $w_i(\sigma_i, \sigma_{-i})$ be buyer i's expected utility given strategy profile (σ_i, σ_{-i}), that is

$$w_i(\sigma_i, \sigma_{-i}) \equiv \mathbb{E}_{\sigma_i, \sigma_{-i}} \left[U(v_i, \beta_i(\pi_i), \beta_{-i}(\pi_{-i})) - \lambda \left(\gamma(N-1) c \left(\Pi_i^{other}, p \right) + c \left(\Pi_i^{self} \left(\pi_i^{other} \right), p \right) \right) \right].$$

Let $\sigma^* \in \Delta\Sigma$ denote a (potentially mixed) strategy. By symmetry, $w_i(\sigma^*, \dots, \sigma^*)$ is independent of i. Let $w(\sigma^*) \equiv w_i(\sigma^*, \dots, \sigma^*)$ be the diagonal payoff function, i.e., the payoff of an agent when all agents play the same strategy $\sigma^* \in \Delta\Sigma$. Let $\Gamma \subset \Delta\Sigma \times \mathbb{R}$ be the closure of the graph of the diagonal payoff function w. The game is diagonally better-reply secure if, whenever $(\sigma^*, w^*) \in \Gamma$ and $(\sigma^*, \dots, \sigma^*)$ is *not* an equilibrium, then some buyer i can secure a payoff strictly above w^* along the diagonal at $(\sigma^*, \dots, \sigma^*)$.[30]

We show that our game satisfies this condition. Suppose that $(\sigma^*, \dots, \sigma^*)$ is not an equilibrium and let $(\sigma^*, w^*) \in \Gamma$. By definition, there exists a sequence of strategies $\sigma^{(n)}$ converging to σ^* such that $\lim w(\sigma^{(n)}) = w^*$. There are two cases: either w is continuous at σ^* or it is not. In the first case, $w(\sigma^*) = w^*$. Since σ^* is not an equilibrium, a buyer i has a deviation σ_i' that yields $w_i(\sigma_i', \sigma_{-i}^*) > w(\sigma^*) = w^*$. Note that if w_i is not continuous in σ_{-i} at σ_i', it must be because i faces payoff-relevant ties with positive probability at such strategy profile. But then there exists a deviation σ_i'' under which i breaks all such ties in his favor, such that $w_i(\sigma_i'', \sigma_{-i}^*) \geq w_i(\sigma_i', \sigma_{-i}^*) > w^*$ and buyer i's payoff is continuous in others' strategy σ_{-i} at this deviation $(\sigma_i'', \sigma_{-i}^*)$. Hence i can secure a payoff within ε of $w_i(\sigma_i'', \sigma_{-i}^*) > w^*$, as required by better-reply security.

[30] Buyer i can secure a payoff strictly above w^* along the diagonal at $(\sigma^*, \dots, \sigma^*)$ if there exists $\delta > 0$ and $\bar{\sigma}_i$, such that $w_i(\sigma', \dots, \bar{\sigma}_i, \dots, \sigma') \geq w^* + \delta$ for all σ' in some open neighborhood of σ^*.

Now consider the second case in which w is discontinuous at σ^*. This means relevant ties occur with strictly positive probability at σ^*. Let $\Sigma W(\sigma_i, \sigma_{-i})$ be the sum of buyers' expected payoffs under (σ_i, σ_{-i}) when all relevant ties are broken so as to maximize total buyer surplus. By construction, the function ΣW is upper semi-continuous, and $\lim_{n\to\infty} \Sigma W(\sigma^{(n)}) \leq \Sigma W(\sigma^*, \ldots, \sigma^*)$. Note that $Nw^* \leq \Sigma W(\sigma^*, \ldots, \sigma^*)$. For each buyer i, let $\sigma_i' = \sigma^*$ if i does not face a payoff-relevant tie with positive probability at strategy profile $(\sigma^*, \ldots, \sigma^*)$. If i faces such tie, let σ_i' be a strategy under which he breaks all ties so as to maximize his expected payoff.[31] Since σ_i' breaks ties in favor of agent i for each i, it must be that $\sum_i w_i(\sigma_i', \sigma_{-i}^*) > \Sigma W(\sigma^*, \ldots, \sigma^*) \geq Nw^*$. Thus at least one agent must have some σ_i' that secures a payoff strictly above w^*. Overall, the game is then better-reply secure and must admit a symmetric mixed-strategy equilibrium.

Step 2. We now argue that it must also admit a symmetric equilibrium that is robust to trembles. Consider a perturbed version of our game $G^{(k)}$ in which payoffs are perturbed with the $\varepsilon^{(k)}$, with $\lim \varepsilon^{(k)} = 0$.[32] For each k, the existence of a symmetric mixed-strategy equilibrium $\sigma^{(k)}$ follows from the above argument.

We first show that the equilibrium of the perturbed game $\sigma^{(k)}$ converges to some feasible strategy $\sigma^* \in \Delta\Sigma$ as k grows large. This is true (at least for some subsequence) since the set of strategies $\Delta\Sigma$ to which $\sigma^{(k)}$ belongs is (sequentially) compact. So there exists a feasible strategy σ^* to which $\sigma^{(k)}$ converges to as perturbations vanish.

Finally, we show that the strategy it converges to σ^* forms a symmetric equilibrium of the unperturbed game. By way of contradiction, suppose $(\sigma^*, \ldots, \sigma^*)$ is not an equilibrium. Then, by better-reply security, an agent i can secure a payoff strictly above $\lim w(\sigma^{(k)}) \equiv w^*$ using some strategy σ_i'. However σ_i' would then also be a profitable deviation against $\sigma_{-i} = \sigma^{(k)}$ for k large enough since payoffs are continuous in the perturbation $\varepsilon^{(k)}$, and so $\sigma^{(k)}$ cannot be an equilibrium of the perturbed game for k large enough. □

Proof of Proposition 3. By contradiction, suppose there exists a sequence of equilibria $\{\sigma_\lambda\}_\lambda$ such that $\lim_{\lambda\longrightarrow 0} \Pr\left(\Pi^{self} = \{\{v_i\}_{v_i \in V}\} \mid \sigma_\lambda\right) = 1$. That is, each buyer i might be

[31] Note that σ_i' can be chosen such that w_i is continuous in σ_{-i} at $\sigma_i = \sigma_i'$.

[32] That is, agents have the same strategy space but the payoff associated with a strategy profile equals the expected payoffs if the agent's bid coincides with the one specified by his strategy with probability $1 - \varepsilon^{(k)}$ and is drawn from some exogenous continuous distribution F otherwise.

mixing over partitions in equilibrium, but he must put a probability that tends to one as λ goes to zero on the fully-revealing partition $\{\{v_i\}_{v_i\in V}\} \equiv \Pi_0$.

We construct a profitable deviation. Consider an alternative strategy for buyer i, in which he first acquires information as to whether the maximum valuation among other bidders is above some threshold $v^{k^*} < v^K$, before learning about his own. That is, he chooses $\Pi^{other}_{k^*} = \{\{v^1,\dots,v^{k^*}\},\{v^{k^*+1},\dots,v^K\}\}$. Then, when he learns that his toughest competitor has a value above the threshold $\max_{j\neq i} v_j > v^{k^*}$, buyer i chooses to partition his set of valuations into $\Pi^{self}(\{v^{k^*+1},\dots,v^K\}) = \{\{v_i : v_i \leq v^{k^*}\},\{v_i\}_{v_i>v^{k^*}}\} \equiv \Pi^{self}_{>k^*}$. Intuitively, he does not learn to distinguish all the valuations below the threshold, as he most likely would not win at any of these. When he learns $\max_{j\neq i} v_j \leq v^{k^*}$ buyer i fully learns his value $\Pi^{self}(\{v^1,\dots,v^{k^*}\}) = \Pi_0$.

By Lemma 1, there exists k^* such that this alternative information strategy leads to strictly lower information cost than becoming fully informed about oneself:

$$\Pr(\max_{j\neq i} v_j > v^{k^*})\left[c(\Pi_0,\mathbb{P}) - c(\Pi^{self}_{>k^*},\mathbb{P})\right] - \gamma(N-1)c(\Pi^{other}_{k^*},\mathbb{P}) \equiv \Delta c > 0.$$

However, there is a potential opportunity cost if learning only partially his valuation yields a lower gross payoff to i. (It has to yield a weakly lower payoff to i as information is valuable.) We now show that this opportunity cost must be smaller than $\lambda\Delta c$ for λ small enough.

This alternative learning strategy can only yield a lower gross payoff in the auction when i fails to learn his valuation fully, that is, when i is at information set $(\pi^{other},\pi^{self}) = (\{v \mid v > v^{k^*}\},\{v \mid v \leq v^{k^*}\})$. For i to strictly benefit from having fully learned and bid his valuation, it must be that, with strictly positive probability, the highest bid faced by buyer i lies in (v^1, v^{k^*}). This implies that buyer $j^* \in \arg\max_{j\neq i} v_j$, whom we know has a value $v_{j^*} > v^{k^*}$, must sometimes make a bid $b' < v^{k^*}$. For j^* to make such a low bid in equilibrium, he must fail to learn that he has a high value and bundle his high value with lower ones. Failing to learn his valuation is costly for j^* as it leads him to sometimes lose the auction against i, despite his value being higher than i's winning bid. Note that j^* cannot lose against i's bid with non-vanishing probability, as otherwise it would be profitable for him to learn his valuation and bid it for λ small enough. For such bundle to be optimal for j^*, he must then expect to face a bid weakly above v_{j^*} with a probability that tends to one as λ goes to zero. Hence there must be another buyer, call him j', who bids $b_{j'} \geq v_{j^*}$ with probability close to one in those states. Since by assumption, buyers learn and bid their valuations with a probability that tends to one in equilibrium, it must be that $b_{j'} = v_{j'} = v_{j^*}$. Thus buyer j^* and j' are symmetric in the sense that they have the same value and that their respective toughest opponents have the same value too. By the symmetry of the equilibrium, they must be learning and bidding symmetrically. They must then be randomizing in equilibrium, putting probability close to 1 on learning their value and bidding v_{j^*} and some positive probability on learning less and bidding below v^{k^*}. For j^* to randomize, it must be that the value of getting more information equals the marginal cost of doing so $\lambda \Delta c'$. Note that the value of the more informed strategy for j^* is at least $\Pr(j' \text{ failed to learn}) \Pr(\max_{l \neq j^*, j'} \beta(\pi_l) < b') \mathbb{P}(v_{j^*})[v_{j^*} - b']$ since it allows j^* to win against bid b'. Thus for j^* to be indifferent, $\Pr(j' \text{ failed to learn})$ must be of the order of λ.

Finally, we argue that buyer i cannot find it strictly profitable to fully learn his valuation when it is below v^{k^*}. Indeed the value from that information is bounded above by $\Pr(j' \text{ failed to learn}) \Pr(j^* \text{ failed to learn}) v^{k^*}$, which is of the order of λ^2. Thus for λ small enough, it cannot be greater than the associated cost $\lambda \Delta c$. □

B.2.3 Proof of Theorem 1

To prove Theorem 1, we find necessary conditions that must be satisfied by any information structure $\left(\Pi^{other}, \Pi^{self}\right) \in \mathcal{P} \times \mathcal{P}^{2^V}$ that has non-vanishing weight in some sequence of equilibria $\{\sigma_\lambda\}_\lambda$. Lemmas 3 and 4 show, in a succession of steps, that such an equilibrium information structure $\left(\Pi^{other}, \Pi^{self}\right)$ must satisfy

$$(\star) \qquad \begin{aligned} &\{v_i\} \in \Pi^{self}(\pi^{other}) \quad \forall \min_{v \in \pi^{other}} v < v_i < \max_{v \in \pi^{other}} v \\ &\exists \pi^{self}_{<} \in \Pi^{self}(\pi^{other}) \text{ s.t. } v_i \in \pi^{self}_{<} \quad \forall v_i < \min_{v \in \pi^{other}} v \end{aligned}$$

for all $\pi^{other} \in \Pi^{other}$. Lemma 5 shows that an equilibrium information structure must also have buyers learn something about the competition, which completes the proof of Theorem 1.

Lemma 3. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then, for all $\pi^{other} \in \Pi^{other}$,*

$$\{v_i\} \in \Pi^{self}\left(\pi^{other}\right) \quad \forall \min_{v \in \pi^{other}} v < v_i < \max_{v \in \pi^{other}} v.$$

In words, any information structure that has non-vanishing weight in equilibrium must satisfy the following condition: if agent i learns that his toughest competitor has value in some interval $\pi^{other} \equiv [v^{\underline{k}}, v^{\overline{k}}]$, then i at least learns to distinguish all the values he can have that lie in $(v^{\underline{k}}, v^{\overline{k}})$ as he knows competition will fall into that range.

Proof of Lemma 3. Let $\left(\Pi^{other}, \Pi^{self}\right)$ be any information structure that is chosen with non-vanishing probability such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon > 0$. Take any $\pi^{other} \in \Pi^{other}$, and let $v^{\underline{k}} \equiv \min_{v'_i \in \pi^{other}} v'_i$ and $v^{\overline{k}} \equiv \max_{v'_i \in \pi^{other}} v'_i$.

We prove that, after learning $\max_{j \neq i} v_j \in \pi^{other}$, the equilibrium partition that an agent chooses about himself $\Pi^{self}\left(\pi^{other}\right)$ cannot bundle two values $v', v'' \in [v^{\underline{k}}, v^{\overline{k}}]$.[33] By contradiction, suppose that this is not true: after learning that $\max_{j \neq i} v_j \in \pi^{other}$, an agent chooses a partition that bundles $v', v'' \in [v^{\underline{k}}, v^{\overline{k}}]$. That bundle can be composed of only these two values, or can have other values in it too. Let $\underline{v}$ (resp, $\overline{v}$) be the lowest (resp, highest) element in that bundle, and denote that bundle by $\pi^{self}_{\underline{v},\overline{v}}$.

Let $j^* \in \arg\max_{j \neq i} v_j$ be (any one of) agent i's toughest competitor(s). At information set $\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right)$ agent i knows that his toughest competitor has $\max_{j \neq i} v_j \in \pi^{other}$ and that his own value v_i might also belong to that set, since $\pi^{other} \cap \pi^{self}_{\underline{v},\overline{v}} \neq \emptyset$. By the symmetry of the equilibrium, he hence knows that, with strictly positive probability, j^*'s information set is also $\pi_{j^*} = \left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right)$. Take any bid that is submitted with positive probability at that information set $\beta\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right) \in \operatorname{supp} \sigma_\lambda(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}, \cdot)$. With non-trivial probability, this is the highest bid that agent i faces such that agent i will tie. Indeed, it is for instance the case when all other agents choose the same information structure as i (with happens with probability ε^{N-1}) and have all value $v_j \in \{v', v''\}$. For such equilibrium bid to be optimal, agent i must then be indifferent between losing the auction and winning *at his equilibrium bid* (i.e., winning at a tie, such that he pays his equilibrium bid):

$$\begin{aligned}
\beta\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right) &= \mathbb{E}\left[v_i \,\middle|\, \pi_i = \left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right), \max_{j \neq i} \beta(\pi_j) = \beta(\pi_i)\right] \\
&= \sum_{v_i = \underline{v}}^{\overline{v}} v_i \Pr\left[v_i \,\middle|\, \pi_i = \left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right), \max_{j \neq i} \beta(\pi_j) = \beta(\pi_i)\right].
\end{aligned}$$

[33] This is enough to prove the result as any bundle involving a value $v' \in (v^{\underline{k}}, v^{\overline{k}})$ must involve another value $v'' \in [v^{\underline{k}}, v^{\overline{k}}]$ since partitions must be monotone.

If not, then agent i would have an incentive to marginally increase or decrease his bid, depending on whether or not he wants to win the auction at that price. Note that $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}) \in [\underline{v}, \overline{v}]$ since $\underline{v}$ (resp. $\overline{v}$) is the lowest (resp. highest) value of v_i that agent i deems possible at information set $\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right)$. Furthermore, conditional on $\pi_i = \left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right)$ and $\max_{j\neq i} \beta(\pi_j) = \beta(\pi_i)$, the events $v_i = v'$ and $v_i = v''$ both have strictly positive probability (more specifically, probability at least $\varepsilon^{N-1}\mathbb{P}(v')^N$ and $\varepsilon^{N-1}\mathbb{P}(v'')^N$, respectively).

We now show that agent i has a strict, non-vanishing incentive to acquire more information, and will hence do so for small enough information costs. There are three cases. If $\beta\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right) \leq \min\{v', v''\}$ then agent i would strictly prefer to win the auction whenever $v_i = \max\{v', v''\}$ instead of tying. Unbundling $v_i = \max\{v', v''\}$ from the rest of $\pi^{self}_{\underline{v},\overline{v}}$ would allow i to break the tie in his favor and make a strictly positive gain. If $\beta\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right) \geq \max\{v', v''\}$ then agent i would strictly prefer to lose the auction whenever $v_i = \min\{v', v''\}$ instead of tying. Unbundling $v_i = \min\{v', v''\}$ from the rest of $\pi^{self}_{\underline{v},\overline{v}}$ would allow i to avoid winning when the price is strictly above his value. Finally, if $\beta\left(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}\right) \in (\min\{v', v''\}, \max\{v', v''\})$ then agent i would strictly prefer to lose the auction whenever $v_i = \min\{v', v''\}$ but win whenever $v_i = \max\{v', v''\}$. Unbundling $v_i = \min\{v', v''\}$ from $v_i = \max\{v', v''\}$ would allow i to break the tie in his favor and make a strictly positive gain.

There is a strictly positive cost $\lambda\Delta c$ associated with unbundling these values, as it requires choosing a finer partition. However, for λ small enough, the value of distinguishing these values more than compensates the cost, since ε is independent of λ. $\square$

Lemma 4. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda\longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then, for all $\pi^{other} \in \Pi^{other}$,*

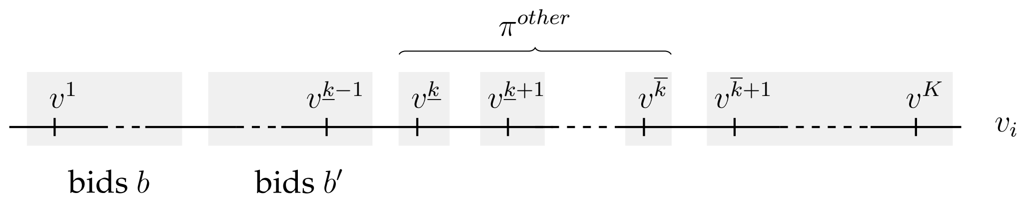
$$\exists \pi^{self}_{<} \in \Pi^{self}(\pi^{other}) \text{ s.t. } v_i \in \pi^{self}_{<} \quad \forall v_i < \min_{v\in\pi^{other}} v$$

Proof of Lemma 4. Let $\left(\Pi^{other}, \Pi^{self}\right)$ be any information structure that is chosen with non-vanishing probability such that $\lim_{\lambda\longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon > 0$. Take any $\pi^{other} \in \Pi^{other}$, and let $v^{\underline{k}} \equiv \min_{v'_i\in\pi^{other}} v'_i$ and $v^{\overline{k}} \equiv \max_{v'_i\in\pi^{other}} v'_i$.

We prove that if i learns $\max_{j\neq i} v_j \in \pi^{other}$, then i chooses a signal about himself that bundles all the values he might have that lie for sure below his toughest competitor's valuation: there exists $\pi^{self}_{<} \in \Pi^{self}(\pi^{other})$ s.t. $v_i \in \pi^{self}_{<}$ for all $v_i < v^{\underline{k}}$. Denote by

$j^* \in \arg\max_{j \neq i} v_j$ (one of) i's toughest competitors.

By contradiction, suppose that the claim is not true, and that in equilibrium buyer i partitions all these values into at least two elements:



The proof is similar to that of Proposition 3. Since choosing such finer partition is costly, it must lead him to make better decision-making: so he must make (at least two) different bids b and b' depending on what he learned, and these two bids lead to different outcomes. Hence it must be that, with some strictly positive probability, the highest bid faced by buyer i lies in between these two bids $\Pr(b \leq \max_{j \neq i} \beta(\pi_j) \leq b' \mid \max_{j \neq i} v_j \in \pi^{other}, v_i \leq v^{\underline{k}}) > 0$, as otherwise these two bids would be completely equivalent. Furthermore, it must be that $b' < v^{\underline{k}}$. Indeed, i ties with positive probability at bid b' since another agent might be at the same information set as him, and so must be indifferent between winning and losing at that equilibrium bid. Since that other agent learned about j^* and not i, that means i's bid b' must equal his expected valuation given his information set, which puts positive probability on values strictly below $v^{\underline{k}}$.

This implies that buyer j^*, whom we know has a value $v_{j^*} \in \pi^{other}$, must sometimes make a bid strictly below $b' < v^{\underline{k}}$. For j^* to make such a low bid in equilibrium, he must fail to learn that he has a high value and bundle his high value with lower ones. Let π_{j^*} be the information set at which j^* acts in such a way, with $\beta(\pi_{j^*}) < b'$. We furthermore know that j^* sometimes has this information set when he is a highest-valuation buyer and his value $v_{j^*} \in \pi^{other}$. Hence $\max\{v \mid v \in \pi_{j^*}^{self}\} \geq v^{\underline{k}}$.

Failing to learn his valuation is costly for j^* as it leads him to sometimes lose the auction against i, despite his value being higher than i's winning bid b'. Note that j^* cannot lose against i's bid with non-vanishing probability, as otherwise it would be profitable for him to learn his valuation and bid it for λ small enough. For such bundle to be optimal for j^*, he must then expect to face a bid weakly above v_{j^*} with a probability that tends to one as λ goes to zero. Hence there must be another buyer, call him j', who bids $b_{j'} \geq v_{j^*}$ with non-vanishing probability in those states. As in the proof of Proposition 3, this implies that j^* and j' are symmetric and must be randomizing between fully learning and bidding their value, and learning less and bidding below

b'. For j^* to randomize, it must be that the value of getting more information equals the marginal cost of doing so $\lambda\Delta c'$. Note that the value of the more informed strategy for j^* is at least $\Pr(j' \text{ failed to learn}) \Pr(\max_{l\neq j^*,j'} b_j(\pi_l) = b')\mathbb{P}(v_{j^*})[v_{j^*} - b']$. Thus for j^* to be indifferent, $\Pr(j' \text{ failed to learn})$ must be of the order of λ.

Finally, we argue that buyer i cannot find it strictly profitable to learn to distinguish some values below $v^{\underline{k}}$. Indeed the value from that information is bounded above by $\Pr(j' \text{ failed to learn}) \Pr(j^* \text{ failed to learn}) v^{\underline{k}}$, which is of the order of λ^2. Thus for λ small enough, it cannot be greater than the cost associated with unbundling these values. $\square$

Lemma 5. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda\longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then buyers must learn something about the competitors:*

$$|\Pi^{other}| > 1.$$

Proof of Lemma 5. Suppose not, such that an equilibrium puts non-vanishing probability $\varepsilon > 0$ on some information structure $\left(\Pi^{other}, \Pi^{self}\right)$ under which buyers learn nothing about the competition: $|\Pi^{other}| = 1$. We know from Lemma 3 that buyers must learn their valuations fully under such information structure: $\Pi^{self}(\pi^{other}) = \{\{v_i\}_{v_i\in V}\}$ for the only $\pi^{other} \in \Pi^{other}$. Furthermore, Lemma 1 says that there exists another information structure $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ that is strictly cheaper than $\left(\Pi^{other}, \Pi^{self}\right)$ and that has the following form: $\widehat{\Pi}^{other} = \{\{v \mid v \leq \overline{v}\}, \{v \mid v > \overline{v}\}\}$, $\widehat{\Pi}^{self}(\widehat{\pi}^{other}) = \{\{v\}_v\}$ if $\widehat{\pi}^{other} = \{v \mid v \leq \overline{v}\}$, and $\widehat{\Pi}^{self}(\widehat{\pi}^{other}) = \{\{v \mid v \leq \overline{v}\}, \{v\}_{v>\overline{v}}\}$ if $\widehat{\pi}^{other} = \{v \mid v > \overline{v}\}$. Let $\lambda\Delta c$ be the difference in information costs between these two information structures. We show that deviating to $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ is a strictly profitable deviation.

Take the point of view of some agent i. For $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ not to be a profitable deviation from $\left(\Pi^{other}, \Pi^{self}\right)$, it has to be that, under the former, agent i sometimes gets a strictly lower gross payoff at the auction stage. This can only happen when i fails to learn his valuation fully under $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$. That is, it can only happen when $\hat{\pi}_i^{self} = \{v \mid v \leq \overline{v}\}$. The same arguments as in the proof of Proposition 3 show that the opportunity cost from not learning his valuation fully are (at most) of the order of λ^2, and so cannot be worth the associated cost $\lambda\Delta c$ for λ small enough. $\square$

Wrapping up, if an information structure has non-vanishing weight in some equilibrium, then it must satisfy ($\star$) and involve acquire some information about $\max_{j\neq i} v_j$.

B.3 Proof of Theorem 2

We know from Theorem 1 that, for λ small enough, the only information structures that have non-trivial probability must satisfy $\Pi^{other} \neq \{V\}$ and

$$\exists \pi_<^{self} \in \Pi^{self}(\pi^{other}) \text{ s.t. } v_i \in \pi_<^{self} \quad \forall v_i < \min_{v \in \pi^{other}} v.$$

The proof of Theorem 2 leverages this to show that, in any equilibrium, expected revenue remains bounded away from the expected second highest valuation as the cost parameter λ goes to zero. We first show that when losing buyers fail to learn their valuations, they bid their expected valuations given their information sets (Lemma 6). We then strengthen Theorem 1 to show that $\Pi^{other} \neq \{\{v^1\}, \{v \mid v > v^1\}\}$ (Lemma 7). Finally, we show that such behavior reduces the second highest bid in expectation (Theorem 2).

Lemma 6. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then, for all $\pi^{other} \in \Pi^{other}$ and $\pi_<^{self} \in \Pi^{self}(\pi^{other})$:*

$$\beta\left(\pi^{other}, \pi_<^{self}\right) = \mathbb{E}\left[v_i \,\middle|\, v_i \in \pi_<^{self}\right].$$

Proof of Lemma 6. Take any sequence of equilibria, any information structure $\left(\Pi^{other}, \Pi^{self}\right)$ that has non-vanishing weight as λ goes to zero, and any $\pi^{other} \in \Pi^{other}$ such that $\min_{v \in \pi^{other}} v > \min_{v \in V} v$.

Step 1. We first show that $\pi_<^{self} = \{v_i \mid v_i < \min_{v \in \pi^{other}} v\}$ and $\{\min_{v \in \pi^{other}} v\} \in \Pi^{self}(\pi^{other})$. Theorem 1 only says that $\{v_i\} \in \Pi^{self}(\pi^{other})$ for all $v_i \in (\min_{v \in \pi^{other}} v, \max v \in \pi^{other} v)$ but it says nothing about the boundary case of $v_i = \min_{v \in \pi^{other}} v$. Such valuation might in principle be bundled with lower ones such that $\pi_<^{self} = \{v_i \mid v_i \leq \min_{v \in \pi^{other}} v\}$. In this first step, we show that it isn't.

By contradiction, suppose that $\pi_<^{self} = \{v_i \mid v_i \leq \min_{v \in \pi^{other}} v\}$. There is then a non-vanishing probability that buyer i at information set $(\pi^{other}, \pi_<^{self})$ ties at any equilibrium bid he submits with positive probability. Indeed, if all agents choose the same information structure and all have values $v_j = \min_{v \in \pi^{other}} v$, then they would all enter the auction at the same information set as i. Any bid submitted at that information set must make a buyer indifferent between winning and losing at that bid:

$$\beta(\pi^{other}, \pi^{self}_<) = \sum_{v_i} v_i \Pr[v_i \mid \pi^{other}, \pi^{self}_<, \max_{j \neq i} \beta(\pi_j) = \beta(\pi^{other}, \pi^{self}_<)].$$

We now argue that the submitted bid $\beta(\pi^{other}, \pi^{self}_<)$ must be bounded below $\min_{v \in \pi^{other}} v$. Note that $\min_{v \in \pi^{other}} v$ is the highest value that a buyer deems possible at $\pi^{self}_<$. Conditioning on a tie does provide additional information about v_i since it means other buyers learn that their toughest opponent has value in π^{other}. Since $\pi^{other} \cup \pi^{self}_< = \{\min_{v \in \pi^{other}} v\}$, if there were only two buyers in the auction, then a tie would only be consistent with $v_i = v_j = \min_{v \in \pi^{other}} v$. However, with $N \geq 3$ buyers this is no longer the case, and all values strictly below $\min_{v \in \pi^{other}} v$ have strictly positive probability even conditional on a tie. Indeed, it could be that two of i's rivals have $v_j = \min_{v \in \pi^{other}} v$ but $v_i < \min_{v \in \pi^{other}} v$. Thus $\beta(\pi^{other}, \pi^{self}_<) < \min_{v \in \pi^{other}} v$. But then buyers have a strict non-vanishing incentive to distinguish $v_i = \min_{v \in \pi^{other}} v$ from $v_i < \min_{v \in \pi^{other}} v$. Indeed, if they do not, then they sometimes tie and lose against a bid strictly below $\min_{v \in \pi^{other}} v$ even though their value might be $v_i = \min_{v \in \pi^{other}} v$. For small enough information cost λ, buyers would acquire more information to prevent this mistake and so this cannot be part of an equilibrium.

Step 2. We show that, when a buyer fails to learn his valuation fully but only learns that he does not have the highest one—i.e., when $\pi^{self}_i = \{v_i \mid v_i < \min_{v \in \pi^{other}} v\} \equiv \pi^{self}_<$,—then he bids his expected valuations given his information set. We already know that, in any tremble-robust equilibrium, a buyer cannot make a bid that lies outside the interval of values he deems possible: $\beta(\pi^{other}, \pi^{self}_<) \in \left[\min_{v \in \pi^{self}_<} v, \max_{v \in \pi^{self}_<} v\right]$. There are two possible cases: either i wins with positive probability at that bid in equilibrium or he does not.

Suppose he does and let $j^* \in \arg\max_{j \neq i} v_j$ be (one of) i's toughest competitors. Buyer i can win at $\beta(\pi^{other}, \pi^{self}_<)$ only if buyer j^* fails to learn his valuation and bundle his high value in $v_{j^*} \in \pi^{other}$ with some lower ones. As argued in the proof of Lemma 4, that can only be the case if j^* learned that some other buyer j' has value $v_{j'} \leq v_{j^*}$, and they both randomize between learning their valuation fully (with probability close to one) or not. By the symmetry of the equilibrium, buyer i sometimes tie at his winning bid $\beta(\pi^{other}, \pi^{self}_<)$, and so must be indifferent between winning and losing given the tie. Since he wins at a tie only when there exist two bidders j^* and j' with strictly higher values than him, other buyers' bid cannot reveal information to i about his *own* value. Thus to be indifferent between winning and losing, i must bid his expected valuation given his information set $(\pi^{other}, \pi_<^{self})$.

If i never wins at bid $\beta(\pi^{other}, \pi_<^{self})$, then that equilibrium bid is disciplined by the trembling-hand-like refinement that we impose. In the perturbed game, all bidders tremble with vanishing probability, such that i only wins the auction when j^* trembles. In that scenario, none of i's competitors' bids reveal any information about i's value: all buyers $j \neq j^*$ learned about their toughest competitors, which is j^*, and j^* is trembling so his bid is drawn at random. Given that bidding truthfully is a weakly dominant strategy in a SPA, and that buyer i faces a distribution of bids that has full support given j^*'s tremble, he has a strict incentive to bid his expected valuation for the good given his information set:

$$\beta\left(\pi^{other}, \pi_<^{self}\right) = \mathbb{E}\left[v_i \,\middle|\, v_i < \min_{v_i' \in \pi^{other}} v_i'\right].$$

$\square$

Lemma 7. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that* $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. *Then* $\Pi^{other} \neq \{\{v^1\}, \{v \mid v > v^1\}\}$.

Proof of Lemma 7. The proof of Lemma 7 is similar to that of Lemma 5. Toward a contradiction, suppose that an equilibrium puts non-vanishing weight on $\Pi^{other} = \{\{v^1\}, \{v \mid v > v^1\}\}$. We know from previous lemmas that $\Pi^{self}(\{v \mid v > v^1\}) = \{\{v\}_{v \in V}\}$. Consider the following deviation for some agent i. Agent i acquires $\widehat{\Pi}^{other} \neq \{\{v^1\}, \{v \mid v^1 < v \leq v^*\}, \{v \mid v > v^*\}\}$ for some $v^* \in (v^1, v^K)$. If i learns that $\max_{j \neq i} v_j = v^1$, he behaves in the same way as under the original strategy. If i learns that $\max_{j \neq i} v_j \in (v^1, v^*]$, he learns his valuation fully. If i learns that $\max_{j \neq i} v_j > v^*$, he chooses $\widehat{\Pi}^{self}(\{v \mid v > v^*\}) = \{\{v \mid v \leq v^*\}, \{v\}_{v > v^*}\}$. By Lemma 2, we know that there exists v^* such that this alternative learning strategy is strictly cheaper. We have left to show that it does not reduce i's payoff at the auction stage, and so must be a profitable deviation.

When buyer i fully learns his valuation, he achieves his full-information optimal payoff from the auction. Thus for buyer i can only be made worse off by the alternative learning strategy when he fails $(\hat{\pi}^{other}, \hat{\pi}^{self}) = (\{v \mid v > v^*\}, \{v \mid v \leq v^*\})$, that is, when he learns that $\max_{j \neq i} v_j > v^*$ and that $v_i \leq v^*$. For i to be made worse off, it has to be that the winning bid is below v^* with strictly positive probability in those states of the world. However, we can then use the same argument as in the proof of Proposition 3 to show that, if this is the case, then some buyer is not optimizing. □

Proof of Theorem 2. Finally, we show that there exist $L > 0$ and $\overline{\lambda} > 0$ such that, for all $\lambda \leq \overline{\lambda}$, the expected second-highest bid is lower than $\mathbb{E}[v_{(2)}] - L$ in any equilibrium.[34] Take any equilibrium, and denote by $q(\lambda)$ the probability that a buyer chooses an information structure satisfying ($\star$). We know from Theorem 1 that $\lim_{\lambda \longrightarrow 0} q(\lambda) = 1$. We focus on the case where all buyers choose such an information structure—the other case has vanishing probability, and induces a revenue that is bounded above by the highest possible valuation. We first show that, *given any realized second-highest bid* $b_{(2)}$, revenue (i.e., the realized second-highest bid) must lie weakly below $\mathbb{E}[v_{(2)} \mid b_{(2)}]$. We then show that, with strictly positive probability, it is bounded strictly below $\mathbb{E}[v_{(2)} \mid b_{(2)}] - L$ for some $L > 0$.

Note that a highest-valuation buyer wins with probability one if all buyers choose an information structure satisfying ($\star$). Let i_1 be a highest-valuation buyer and $v_{(1)}$ his realized valuation. There are two cases: either the price is set by a buyer who learned his valuation fully, or it isn't. The first case is direct: since the second-highest bidder (call him j^*) learned his value, he must have bid truthfully, and revenue then equals $b_{(2)} = v_{j^*} \leq v_{(2)}$. In the second case, the second-highest bidder failed to learn his value, which means that $\pi^{self}_{j^*} = \{v_j \mid v_j \leq \overline{v}\}$ for some $\overline{v} < v_{i_1}$. If j^* wins at such bid, then all other bidders $j \neq j^*, i_1$ must also have had values $v_j \leq \overline{v}$.[35] We know from Lemma 6 that j^* must have bid $\beta(\pi_{j^*}) = \mathbb{E}[v_{j^*} \mid v_{j^*} \leq \overline{v}]$, which always lies weakly below $\mathbb{E}[v_{(2)} \mid b_{(2)} = \beta(\pi_{j^*})] = \mathbb{E}[v_{(2)} \mid v_{i_1} > \overline{v}, v_j \leq \overline{v} \ \forall j \neq i_1]$.[36] Thus, when all agents choose information structures satisfying ($\star$), $b_{(2)} \leq \mathbb{E}[v_{(2)} \mid b_{(2)}]$, and expected revenue is weakly below the expected second-highest valuation.

We now prove that, with strictly positive non-vanishing probability, the second-highest bid $b_{(2)}$ is bounded strictly below the expected second-highest valuation given $b_{(2)}$. In particular, we show that this is the case when all buyers choose the same information structure and the gap between the highest and second-highest valuations is large enough. Let $V = \{v^1, v^2, \ldots, v^K\}$ with $v^{k+1} > v^k$. Take any information structure $(\Pi^{other}, \Pi^{self})$ that has non-vanishing weight in equilibrium. Let $\underline{v} = \min\{v \mid \exists \pi^{other} \in$

[34] $v_{(2)}$ denotes the second highest value for every realization of $(v_i)_i$.

[35] If not, they would have either learned their values and outbid j^*, or just learned that $v_j \leq \overline{v}'$ for some $\overline{v}' > \overline{v}$, which would also have led them to outbid j^*.

[36] Indeed, the latter is the expectation of the highest value among the $N - 1 \geq 2$ losing bidders, which is weakly above the expected value of one specific losing bidder j^*.

Π^{other} s.t. $v \in \pi^{other}, v^K \in \pi^{other}\}$ denote the smallest valuation that Π^{other} bundles with v^K. We know from 7 that $\Pi^{other} \neq \{\{v^1\}, \{v^2, \ldots, v^K\}\}$. Hence $\underline{v} > v^2$. Consider what happens when the highest-valuation bidder has value $v_{i_1} \geq \underline{v}$ while all others $j \neq i_1$ have value $v_j < \underline{v}$. When all buyers choose information structure $(\Pi^{other}, \Pi^{self})$, all buyers $j \neq i_1$ must learn $\max_{j \neq i} v_j \in [\underline{v}, v^K]$ since $v_{i_1} \geq \underline{v}$. Furthermore, all buyers but i_1 must fail to learn their valuations precisely: $\pi_j^{self} = \{v_j \mid v_j < \underline{v}\}$ with $|\pi_j^{self}| \geq 2$. The second-highest bid then equals $\mathbb{E}[v_j \mid v_j < \underline{v}]$.

Overall, we get

$$\begin{aligned}\mathbb{E}\left[\text{ equilibrium revenue } |\sigma_\lambda\right] - \mathbb{E}[v_{(2)}] \leq \left(1 - [q(\lambda)]^N\right)\left(v^K - \mathbb{E}[v_{(2)}]\right) \\ +[q(\lambda)]^N \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right)^N \Pr\left(v_{i_1} \geq \underline{v}, v_j < \underline{v}\ \forall j \neq i_1\right) \times \\ \left(\mathbb{E}[v_i \mid v_i < \underline{v}] - \mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}]\right).\end{aligned}$$

We give an upper bound for the second term. Note that $\mathbb{E}[v_i \mid v_i < \underline{v}] - \mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}]$ is strictly negative. Indeed, it compares the expected value of a buyer conditional on it being lower than some bound $\mathbb{E}[v_i \mid v_i < \underline{v}]$ to the expected *second-highest* value conditional on it being lower than that same bound and the highest-value being higher than this bound $\mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}]$. Hence the latter is just the expected value of the best of these $N-1$ draws, simply truncating the distribution at the bound as we know that all these $N-1$ draws lie below it. Since there are $N \geq 3$ buyers, the latter is strictly positive whenever there is some variance in the distribution of $v_i < \underline{v}$. This is the case as $|\{v_i \mid v_i < \underline{v}\}| \geq 2$. Hence

$$\mathbb{E}[v_{(2)} \mid v_{(2)} < \underline{v}, v_{(1)} \geq \underline{v}] - \mathbb{E}[v_i \mid v_i < \underline{v}] \equiv l > 0.$$

Then

$$\begin{aligned}\mathbb{E}\left[\text{ equilibrium revenue } \mid \left(\Pi^{other}, \Pi^{self}\right)\right] - \mathbb{E}[v_{(2)}] \leq \left(1 - [q(\lambda)]^N\right)\left(v^K - \mathbb{E}[v_{(2)}]\right) \\ - [q(\lambda)]^N \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right)^N \Pr\left(v_{(1)} \geq \underline{v}, v_{(2)} < \underline{v}\right) \Pr\left(v_{(1)} \geq \underline{v}, v_{(2)} < \underline{v}\right) l.\end{aligned}$$

Since $\lim_{\lambda \longrightarrow 0} q(\lambda) = 1$, the first RHS term goes to zero as information costs vanish. Since $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$, the second does not. There exists $\overline{\lambda}$ such that for all $\lambda \leq \overline{\lambda}$, expected revenue is bounded away from the expected second-highest valuation. $\square$

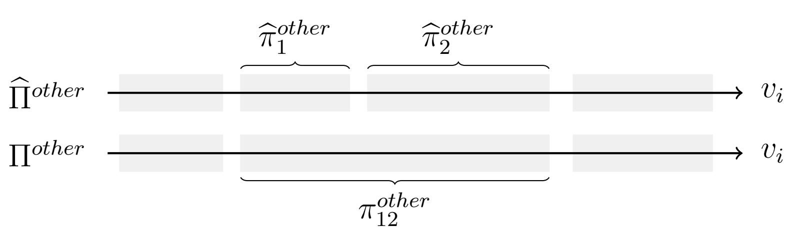
B.4 Proof of Results of Section 5

Proof of Proposition 5. Our characterization of equilibrium behavior applies verbatim, except that the signal about the competition is now fixed and under the discretion of the seller. Given the chosen Π^{other} and any realization $\pi^{other} \in \Pi^{other}$, any signal about that that has non-vanishing weight in equilibrium must fully reveal one's value if it falls into π^{other} and bundle together all values below $\min\{v : v \in \pi^{other}\}$. We know from Lemma 6 that a buyer who knows his value is below that of his toughest opponent but does not know his value fully, bids his expected value given his information set. Thus expected revenue given Π^{other} equals

$$\begin{aligned}\Pr(v_{(2)} \in \pi^{other}(v_{(1)}))\mathbb{E}[v_{(2)} \mid v_{(2)} \in \pi^{other}(v_{(1)})]& \\ + \Pr(v_{(2)} \notin \pi^{other}(v_{(1)}))\mathbb{E}[v \mid v < \min\{v' : v' \in \pi^{other}(v_{(1)})\}],&\end{aligned}$$

where $v_{(1)}$ denotes the highest value among buyers, $v_{(2)}$ the second-highest value, and $\pi^{other}(v_{(1)}) = \{\pi^{other} \in \Pi^{other} : v_{(1)} \in \pi^{other}\}$ is the cell of the partition Π^{other} to which $v_{(1)}$ belongs. Thus, the first line captures the states of the world in which the highest and second-highest value fall in the same cell, in which case both buyers learn and bid their values, and revenue equals the second-highest value. The second line captures states of the world in which they do not fall in the same cell, in which case all losing buyers only learn that their value is below $\min\{v' : v' \in \pi^{other}(v_{(1)})\}$ and bid their expected value given this information.

Suppose Π^{other} is strictly coarser than $\widehat{\Pi}^{other}$. Without loss, we can assume that Π^{other} is constructed by merging two cells of $\widehat{\Pi}^{other}$.[37] Denote by $\widehat{\pi}_1^{other}$ and $\widehat{\pi}_2^{other}$ the two cells that are merged, and let $\pi_{12}^{other} = \widehat{\pi}_1^{other} \cup \widehat{\pi}_2^{other}$, such that:



These two disclosure policies lead to different revenue only when $v_{(1)} \in \pi_{12}^{other}$. Con-

[37]We can always go from a finer to a coarser partition by a sequence of such two-cell mergers, and the claim holds for any such merger.

ditional on such event, expected revenue under Π^{other} equals

$$\Pr(v_{(2)} \in \pi_{12}^{other} \mid v_{(1)} \in \pi_{12}^{other})\mathbb{E}[v_{(2)} \mid v_{(1)}, v_{(2)} \in \pi_{12}^{other}] \\ + \Pr(v_{(2)} \notin \pi_{12}^{other} \mid v_{(1)} \in \pi_{12}^{other})\mathbb{E}[v \mid v < \min\{v' : v' \in \pi_{12}^{other}\}],$$

while expected revenue under $\widehat{\Pi}^{other}$ equals

$$\sum_{k=1,2} \Pr(v_{(1)}, v_{(2)} \in \widehat{\pi}_k^{other} \mid v_{(1)} \in \pi_{12}^{other})\mathbb{E}[v_{(2)} \mid v_{(1)}, v_{(2)} \in \widehat{\pi}_k^{other}] \\ + \sum_{k=1,2} \Pr(v_{(1)} \in \widehat{\pi}_k^{other}, v_{(2)} \notin \widehat{\pi}_k^{other} \mid v_{(1)} \in \pi_{12}^{other})\mathbb{E}[v \mid v < \min\{v' : v' \in \widehat{\pi}_k^{other}\}].$$

Note that when $v_{(1)} \in \widehat{\pi}_1^{other}$, expected revenue is the same under both disclosure policies. Indeed, either $v_{(2)} \in \widehat{\pi}_1^{other}$ as well, and revenue is $v_{(2)}$, or $v_{(2)} \notin \widehat{\pi}_1^{other}$, and revenue is $\mathbb{E}[v \mid v < \min\{v' : v' \in \pi_{12}^{other}\}] = \mathbb{E}[v \mid v < \min\{v' : v' \in \widehat{\pi}_1^{other}\}]$. Thus revenue only differs when $v_{(1)} \in \widehat{\pi}_2^{other}$. Conditional on $v_{(1)} \in \widehat{\pi}_2^{other}$, the difference in revenue between partition Π^{other} and $\widehat{\Pi}^{other}$ is then:

$$\Pr(v_{(2)} \in \widehat{\pi}_1^{other} \mid v_{(1)} \in \widehat{\pi}_2^{other})\mathbb{E}[v_{(2)} \mid v_{(2)} \in \widehat{\pi}_1^{other}, v_{(1)} \in \widehat{\pi}_2^{other}] \\ + \Pr(v_{(2)} \notin \pi_{12}^{other} \mid v_{(1)} \in \widehat{\pi}_2^{other})\mathbb{E}[v \mid v < \min\{v' : v' \in \pi_{12}^{other}\}] \\ - \Pr(v_{(2)} \notin \widehat{\pi}_2^{other} \mid v_{(1)} \in \widehat{\pi}_2^{other})\mathbb{E}[v \mid v < \min\{v' : v' \in \widehat{\pi}_2^{other}\}].$$

Note that $\mathbb{E}[v_{(2)} \mid v_{(2)} \in \widehat{\pi}_1^{other}, v_{(1)} \in \widehat{\pi}_2^{other}] \geq \mathbb{E}[v \mid v \in \widehat{\pi}_1^{other}]$ and that

$$\mathbb{E}[v \mid v < \min\{v' : v' \in \widehat{\pi}_2^{other}\}] = \Pr(v \in \widehat{\pi}_1^{other} \mid v < \min\{v' : v' \in \widehat{\pi}_2^{other}\})\mathbb{E}[v \mid v \in \widehat{\pi}_1^{other}] \\ + \Pr(v \notin \pi_{12}^{other} \mid v < \min\{v' : v' \in \widehat{\pi}_2^{other}\})\mathbb{E}[v \mid v < \min\{v' : v' \in \pi_{12}^{other}\}].$$

Thus the difference is always weakly positive. Intuitively, in states of the world where the two disclosure policies lead to difference revenue, the finer policy always has losing buyers fail to learn their values (last line) while the second-highest bidder sometimes learns and bid his value under the coarser policy (first line). If there is only one losing buyer, the losing bid averages out to the same value, and the difference is zero. With $N > 2$ buyers and so more than one losing buyer, the coarser disclosure policy can do strictly better as revenue sometimes equal to highest-value among losing buyers instead of the expected value. □

Proof of Proposition 6. Consider the following way to randomize access to the auction:

$$\Pr(M = N) = 1 - q \quad \text{and} \quad \Pr(M = N \setminus i) = \frac{q}{N} \text{ for all } i,$$

for some $q \in (0, 1)$. That is, with probability $1 - q$ all buyers get access to the auction. With remaining probability, one buyer chosen uniformly at random is excluded. Take any information partition about self Π^{self} that has non-vanishing weight in equilibrium. We show that, for any $q > 0$, there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, buyers must always become fully informed of their valuations: $\Pi^{self} = \{\{v\}_{v \in V}\}$.

By contradiction, suppose not: Π^{self} bundles some valuations together. Take any such bundle and let $\underline{v}$ and $\overline{v}$ be the lowest and highest element in that bundle, respectively. Denote that bundle by $\pi^{self}_{\underline{v},\overline{v}}$ and let $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}})$ a buyer's equilibrium bid at that bundle. We know that $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}) \in [\underline{v}, \overline{v}]$.

Consider deviating to $\widehat{\Pi}^{self} = \{\{v\}_{v \in V}\}$. Since that partition is finer, it is costlier and increases buyer i's information costs by $\lambda \left[c\left(\widehat{\Pi}^{self}, \mathbb{P}\right) - c\left(\Pi^{self}, \mathbb{P}\right) \right] > 0$. Such a finer partition must (weakly) increase buyer i's gross payoff. We show that it strictly increases i's gross payoff. Whatever information i acquired about his toughest opponent's valuation is irrelevant with probability q/N. Indeed, with probability q/N, i's toughest opponent will be excluded from the auction. In such event, buyer i is left competing with buyers about whose valuations i has no information. In particular, with non-trivial probability, all these buyers chose the same signal about self Π^{self}, all have values $v_j \in \pi^{self}_{\underline{v},\overline{v}}$, and all bid $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}})$. Because there is a positive probability of tie, buyer i must be indifference between winning and losing at his equilibrium bid, such that $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}}) \in (\underline{v}, \overline{v})$. But buyer i then has a strict incentive to learn to distinguish $v_i = \overline{v}$ from $v_i = \underline{v}$. Indeed, he wants to win against $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}})$ when $v_i = \overline{v}$, and lose against $\beta(\pi^{other}, \pi^{self}_{\underline{v},\overline{v}})$ when $v_i = \underline{v}$.

These gains from distinguishing $v_i = \overline{v}$ and $v_i = \underline{v}$ are strictly positive and independent of λ. Hence, for λ small enough, the increase in information cost must be strictly smaller than the gains, and buyers must become fully informed about their valuations in equilibrium: $\Pi^{self} = \{\{v\}_{v \in V}\}$.

In any equilibrium, fully informed buyers must bid their valuations for the good: $\beta((\pi^{other}, \{v_i\})) = v_i$. Expected revenue is then at least $q\mathbb{E}[v_{(2)}]$. Given any $\varepsilon > 0$, set $q = 1 - \frac{\varepsilon}{\mathbb{E}[v_{(2)}]}$. For λ small enough, expected revenue from such randomized access is at least $\mathbb{E}[v_{(2)}] - \varepsilon$, which completes the proof. $\square$

B.5 Proof of Results in Section 6.1

Proof of Proposition 7. We show that there cannot exist an equilibrium in which buyers first choose to learn about their own values and then about others.

By contradiction, suppose such an equilibrium exists, and let Π^{self} be an information partition about self that has non-vanishing probability in equilibrium. We first argue that, for sufficiently small information costs λ, $\Pi^{self} = \{\{v\}_{v\in V}\}$. Suppose not, such that there exists $\pi^{self} \in \Pi^{self}$ with $|\pi^{self}| > 1$. Let $\underline{v}$ and $\overline{v}$ be the smallest and highest values in π^{self}, respectively. In states of the world where $v_j \in \pi^{self}$ for all buyers j, all buyers are at the same information set π^{self} with non-vanishing probability. Following a similar argument as in the proof of Lemma 3, they then tie for the good with non-vanishing probability, and must be indifferent between losing and winning at their equilibrium bid. Their equilibrium bid then lies strictly in between $\underline{v}$ and $\overline{v}$, and given that they face such bid with non-vanishing probability, they have a strict incentive to learn to distinguish value $v_i = \underline{v}$ from value $v_i = \overline{v}$. Therefore a buyer learns his value fully $\Pi^{self} = \{\{v_i\}_{v_i\in V}\}$. We however know from Proposition 3 that such an equilibrium cannot exist. □

Proof of Proposition 8. The proof of Proposition 3 is identical. Indeed, it proceeds by contradiction: we assume that an equilibrium in which buyers converge to fully learning their values exists and then show that they have a strict incentive to deviate to an alternative information structure. That deviation still exists in this extended model since we are only expanding the set of information structures available to buyers, and so the conclusion follows directly.

The proof of Theorem 2 extends almost verbatim as well. During the information acquisition stage, buyers can acquire any sequence of partitions about their values and others'. Irrespective of the exact form this process takes, their information set upon entering the auction is fully described by an interval of values they deem possible for themselves π_i^{self} and an interval of values they deem possible for their toughest opponent π_i^{other}. Take any information structure that has non-vanishing weight in equilibrium. The same arguments as in the proof of Lemma 3 yield that if these two intervals overlap, then the belief about self must be degenerate ($|\pi_i^{self}| = 1$). This ensures buyers make no non-vanishing mistakes in equilibrium as they learn their valuations fully when the auction is "close." Furthermore, the same arguments as in the proof of Lemma 4 yield that if at any stage in the information acquisition process, a buyer knows for sure that his value is below that of his toughest opponent, then that buyer cannot learn more about his value. Finally, the same argument as in the proof of Lemma 5 then yields that buyers must acquire some information about others at some point in the learning process. So any non-vanishing information structure must have losing buyers fail to learn their valuation with positive probability, and the proof of Theorem 2 applies. □

Proof of Proposition 9. The result is trivial when $N = 2$, since any strategy treats competitors symmetrically when there is only one competitor. Thus, from now on, let $N \geq 3$. Take any sequence of (symmetric, tremble robust) equilibria $\{\sigma_\lambda\}_\lambda$ of the game where buyers are restricted to choosing strategies that treat competitors symmetrically, and let $\sigma^* = \lim_{\lambda \longrightarrow 0} \sigma_\lambda$. Note that, under σ^*, buyer j's bid only carries information about i's value through what j learned about his toughest opponent, precisely because σ^* treats competitors symmetrically. The analysis in Lemmas 3, 4, and 6 then applies verbatim: for any π^{max} that arises with positive probability in equilibrium and any $\Pi^{self} \in \operatorname{supp} \sigma^*(\cdot \mid \pi^{max})$, it must be that $\{v\} \in \Pi^{self}$ for all $v \in \pi^{max}$ and $\{v : v < \min\{v' \in \pi^{max}\}\} \in \Pi^{self}$. Furthermore, buyers bid their values when they know them—i.e., $\sigma^*(\pi^{self}, \pi^{max}) = v_i$ when $\pi^{self} = \{v_i\}$ for some v_i—and bid their expected value given their information set when they know someone else has a greater value than them $\pi^{self} = \{v : v < \min\{v' \in \pi^{max}\}\}$. Finally, for this to form an equilibrium, anyone who knows that his value is higher than everyone else's must submit a bid high enough to prevent deviations: $\sigma^*(\pi^{self}, \pi^{max}) \geq \max\{v : v \in \pi^{max}\}$ when $\min\{v : v \in \pi^{self}\} \geq \max\{v : v \in \pi^{max}\}$.

By contradiction, suppose the claim is not true: there exists a strategy σ_i' that does not treat competitors symmetrically and that yields a strictly higher payoff for buyer i against $\sigma_{-i} = \sigma^*$ for vanishing λ. Note that, against $\sigma_{-i} = \sigma^*$, strategy $\sigma_i = \sigma^*$ ensures buyer i his full information gross payoff from the auction. Indeed, he gets the same gross payoff as if he had learned his valuation fully, since he does so whenever information is instrumental. So for σ_i' to be strictly better than $\sigma_i = \sigma^*$ for arbitrarily small information cost λ, it must be that (i) buyer i also gets his full information gross payoff under σ_i', and (ii) σ_i' leads to strictly lower information costs than σ^*. Thus, given any $(\pi_i^j)^j$ that can arise under σ_i', buyer i must learn enough about himself and bid so as to achieve his full information gross payoff. But π^{max} is a sufficient statistic to achieve his full information gross payoff since it is a sufficient statistic for the highest bid that i will face. In particular, buyer i achieves his full information optimal payoff if the partition he chooses about himself fully reveals his value when it falls into the interval π^{max}. Any additional information is irrelevant, so, given any realized $(\pi_i^j)^j$, there exists an optimal continuation strategy that only conditions on π^{max} (condition (ii)). For σ_i' to do strictly better than any strategy that treats competitors symmetrically, it must then fail condition (i). That is, under σ_i', the distribution of $(v_j)_{j\neq i} \mid \pi^{max}$ is not exchangeable. We construct a new strategy σ_i'' that yields the same expected payoff as σ_i' but that satisfies condition (i). Strategy σ_i' specifies a process through which i acquires signals (Π_i^j) for all $j \neq i$. That process might involve up to $N-1$ steps if buyer i learns about competitors sequentially. Change the process as follows: before any step, permute uniformly at random the indices of all competitors that buyer i has not learned about yet. The rest of the strategy is unchanged. By construction, the distribution over π^{max} is unchanged: the alternative learning process is equally information about i's toughest opponent. Since this is a sufficient statistic for i's continuation strategy, i's continuation payoff has not changed. Note that the cost of the learning strategy has not changed either, since the same partitions are acquired, we just randomized about whom they are acquired, and the cost is symmetric across competitors. Furthermore, by construction, this alternative strategy satisfies condition (i). To conclude, against $\sigma_{-i} = \sigma^*$, there exists an optimal strategy that treats competitors symmetrically. Given that σ^* forms an equilibrium of the restricted game, $\sigma_i = \sigma^*$ must be an optimal strategy. □

Proof of Theorem 3. The following two facts simplify the equilibrium analysis. First, buyers must bid their expected valuation given their information set in any equilibrium. Indeed, doing so is a weakly dominant strategy: others' bids no longer carry any relevant information to them and so the standard strategy-proofness of the second price auction applies. Furthermore, bidding truthfully is the only bidding strategy that survives our tremble refinement. Second, it is without loss to look at pure learning strategies. Any mixing over distribution over posterior beliefs can be replicated by a single distribution over posterior beliefs that has the same cost. Taken together, these two facts imply that any equilibrium can be fully described by the chosen signal about others $\Pi^{other} : \{\underline{s}, \overline{s}\} \longrightarrow \Delta\Delta(\{\underline{s}, \overline{s}\})$ and about self $\Pi^{self} : [0,1] \times \{\underline{s}, \overline{s}\} \longrightarrow \Delta\Delta V$.[38]

Take any sequence of equilibria $\{(\Pi_\lambda^{other}, \Pi_\lambda^{self})\}_\lambda$ and denote the limiting information structure by $(\Pi^{other}, \Pi^{self}) \equiv \lim_{\lambda \longrightarrow 0}(\Pi_\lambda^{other}, \Pi_\lambda^{self})$.

Step 1: We show that $\operatorname{supp} \Pi^{other}(\underline{s}) = \{\underline{\mu}, 1\}$ and $\operatorname{supp} \Pi^{other}(\overline{s}) = \{0, \overline{\mu}\}$. In words,

[38]Recall that buyers get to observe s_i for free at the beginning of the game so they can condition their learning strategy on the realization of that signal.

a buyer who knows his value is low ($s_i = \underline{s}$) will choose a signal about others that either reveals the competition is strong for sure (posterior probability that $\max_{j \neq i} s_j = \overline{s}$ is 1) or not. Similarly, a buyer who knows his value is high ($s_i = \overline{s}$) will choose a signal about others that either reveals the competition is weak for sure (posterior probability that $\max_{j \neq i} s_j = \overline{s}$ is 0) or not. To that end, we first argue that the good must be allocated to the highest valuation bidder under the limiting information structure $(\Pi^{other}, \Pi^{self})$. Indeed, if the equilibrium allocation were not efficient, then some buyer would have a strict incentive to deviate: since buyers bid their expected value given their information set, any misallocation must be driven by some buyer having failed to learn his valuation fully. But then that buyer would be leaving money on the table, as he would either win at a price above his value or lose against a price below his value. In the limit, learning his value fully and avoiding this mistake must be strictly profitable.

For the good to be efficiently allocated in the limit, it must be that $\operatorname{supp} \Pi^{self}(\overline{s})[\mu] = \{\delta_v\}_{v > v^*}$ whenever $\mu > 0$ and $\operatorname{supp} \Pi^{self}(\underline{s})[\mu] = \{\delta_v\}_{v \leq v^*}$ whenever $\mu < 1$, where δ_v denotes the "corner" distribution that assigns probability one to $v_i = v$. In words, for the good to be allocated efficiently, it must be that a buyer with $s_i = \overline{s}$ fully learns his value whenever he assigns positive probability to the event $\max_{j \neq i} s_j = \overline{s}$. Similarly, it must be that a buyer with $s_i = \underline{s}$ fully learns his value whenever he assigns positive probability to the event $\max_{j \neq i} s_j = \underline{s}$. A direct implication is that, if $1 \notin \operatorname{supp} \Pi^{other}(\underline{s})$, then a buyer with $s_i = \underline{s}$ would *always* learn his valuation fully at the second stage of the information acquisition process. But then learning anything about the competition would be pointless and buyer i would simply learn nothing about the competition and everything about his own value. However, if his prior is sufficiently uncertain this would not be cost-efficient, and he would strictly reduce his overall information cost by learning fully the realization of $\max_{j \neq i} s_j$ and his value only if $\max_{j \neq i} s_j = s_i$. (The argument is similar as in the proof of Proposition 3, though even more straightforward as we know that a buyer with $s_i = \overline{s}$ (resp, $s_i = \underline{s}$) always makes a bid below v^* (resp, strictly above v^*).) Thus $1 \in \operatorname{supp} \Pi^{other}(\underline{s})$. Finally, it cannot be that $\mu', \mu'' \in \operatorname{supp} \Pi^{other}$ with $\mu' < 1$ and $\mu'' < 1$. Indeed, at both of these posterior buyer i would fully learn his valuation. Merging these two posteriors would strictly reduce his information costs without changing his bidding strategy, and so doing it is a profitable deviation. Overall that means $\operatorname{supp} \Pi^{other}(\underline{s}) = \{\underline{\mu}, 1\}$ for some $\underline{\mu} < 1$. The proof of $\operatorname{supp} \Pi^{other}(\overline{s}) = \{0, \overline{\mu}\}$ is virtually identical.

Step 2: We show that expected revenue under $(\Pi^{other}, \Pi^{self})$ is strictly below the expected second highest valuation. Recall that whenever buyers learn their valuations fully, they bid their valuations, and revenue equals to second-highest value. Hence realized revenue can only differ from the realized second-highest value when some losing buyer(s) failed to learn their valuation(s) fully. This can only happen when the second-highest bid is set by a buyer i with $s_i = \underline{s}$ who learned that his toughest opponent has $\max_{j \neq i} s_j = \overline{s}$ and, as a result, chose to learn nothing about his value. The second-highest bid is then $\mathbb{E}[v \mid v \leq v^*]$. Following the same argument as in the proof of Theorem 2, that second-highest bid is strictly lower than if i had learned his value fully and bid his value as long as $N > 2$. $\square$

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Online Appendix to *How Competition Shapes Information in Second-Price Auctions*[*]

Agathe Pernoud Simon Gleyze

Appendix C A Perturbed Model

This was the main model in a previous version of the paper. It is less parsimonious than our current baseline model, but has the benefit of sharpening the characterization of equilibrium information structures (Theorem 1).

C.1 Adding Noise to Buyers' Valuations

The setup is identical to our baseline model, except for the fact that each buyer's valuation is now the sum of two components $\nu_i = v_i + u_i$, where $v_i \in V$ should be interpreted as the main component—we sometimes abuse language and refer to v_i as a buyer's valuation—and $u_i \in U$ as small mean-zero noise. Both components are identically and independently distributed across buyers. As in our baseline model, main components $(v_i)_i$ are drawn i.i.d. from a finite set $V \subset \mathbb{R}_+$ according to a probability distribution $\mathbb{P} \in \Delta V$. Noise terms $(u_i)_i$ are drawn from a compact interval $U \equiv [\underline{u}, \overline{u}] \subset \mathbb{R}$ according to a strictly positive and continuous density, with $\mathbb{E}[u_i] = 0$. They are small, in the sense that $\min_{v_i' \neq v_i''} |v_i' - v_i''| > \overline{u} - \underline{u}$. Hence, if a buyer has a strictly greater v_i than another, then he must necessarily have a strictly greater overall valuation ν_i.

The learning technology about $(v_i)_i$ is the same as in our baseline model. The only difference is that buyers also learn their own (and only their own) noise terms u_i for free at the end of the information acquisition process. A buyer's information set upon

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entering the auction is then $\pi_i = (\pi_i^{other}, \pi_i^{self}, u_i)$ and a bidding strategy is a measurable function $\beta_i : 2^V \times 2^V \times U \longrightarrow \mathbb{R}_+$. The rest of the model remains unchanged.

The noise terms allow us to address issues arising from the discreteness of V, but serve no other purpose. To some extent, they can be interpreted as perturbations à la Harsanyi (1973): they allow us to dispense from randomization and guarantee that buyers cannot predict others' preferences and equilibrium bids *perfectly*. That said, their purpose goes beyond equilibrium purification as they allow us to say more about equilibrium information structures. We take them to be sufficiently small so as not to interfere with the rest of our analysis.

C.2 Equilibrium Information Structures

Propositions 0-3 extend directly to this perturbed model. In particular, there cannot exist a sequence of equilibria such that buyers converge to fully learning their valuations as information costs vanish. Theorem 1′ characterizes the set of information structures that can have non-vanishing weight in equilibrium.

Theorem 1′. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then it solves*

$$\min_{\widehat{\Pi}^{other}, \widehat{\Pi}^{self}} \quad \gamma(N-1)c\left(\widehat{\Pi}^{other}, \mathbb{P}\right) + \mathbb{E}_{\hat{\pi}^{other}}\left[c\left(\widehat{\Pi}^{self}(\hat{\pi}^{other}), \mathbb{P}\right)\right]$$

s.t. $\forall \hat{\pi}^{other} \in \widehat{\Pi}^{other}$,

$$(\star') \qquad \widehat{\Pi}^{self}(\hat{\pi}^{other}) = \left\{\left\{v_i \mid v_i < \min_{v \in \hat{\pi}^{other}} v\right\}, \left\{v_i\right\}_{v_i \in \hat{\pi}^{other}}, \left\{v_i \mid v_i > \max_{v \in \hat{\pi}^{other}} v\right\}\right\}.$$

Note the difference between condition ($\star$) from our baseline model (Theorem 1) and condition ($\star'$). The latter not only requires that a buyer bundles all values below that of his toughest opponent (such that $\{v_i \mid v_i < \min_{v \in \hat{\pi}^{other}} v\} \in \Pi^{self}(\pi^{other})$), but also that a buyer bundles all values *above* that of his toughest opponent ($\{v_i \mid v_i > \max_{v \in \hat{\pi}^{other}} v\} \in \Pi^{self}(\pi^{other})$). Figure 1 illustrates condition ($\star'$).

There exist many information structures satisfying ($\star'$),[1] but Theorem 1′ goes one step further: an equilibrium information structure must furthermore minimize total

[1] For instance, not acquiring any information about the competition $\Pi^{other} = \{V\}$ and fully learning one's own value $\Pi^{self}(\{V\}) = \{\{v\}_{v \in V}\}$.

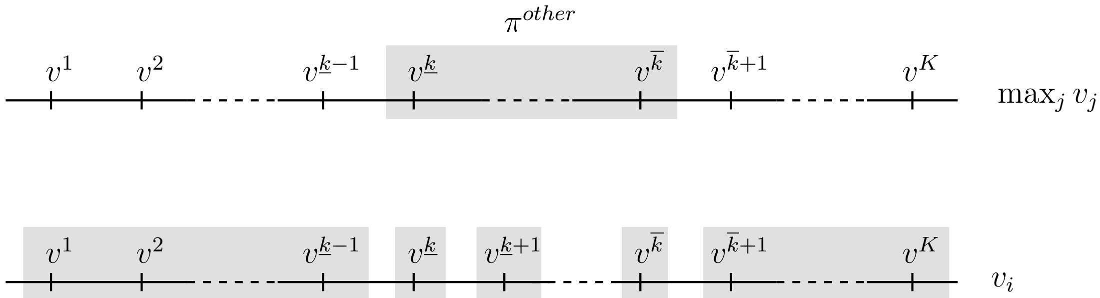


Figure 1: *Let $V = \{v^1, v^2, \ldots, v^K\}$ and order the valuations in increasing order, i.e., $v^k < v^{k+1}$. If buyer i learns that $\max_j v_j \in \pi^{other} \equiv \{v : v^{\underline{k}} \leq v \leq v^{\overline{k}}\}$ (top line), he chooses to fully learn his valuation if it belongs to the set π^{other}, but fails to distinguish all valuations that are for sure lower or higher than $\max_j v_j$ (bottom line).*

information costs. Intuitively, there is a certain amount of information that guarantees buyers make no mistake at the bidding stage (condition ($\star'$)) and buyers choose the cheapest way to achieve it. Lemma 1 from the main text then implies $\Pi^{other} \neq \{V\}$, as fully learning one's own valuation is not cost-efficient, and buyers acquire *some* information about others in equilibrium.

C.3 Proof of Theorem 1′

To prove Theorem 1′, we find necessary conditions that must be satisfied by any information structure $\left(\Pi^{other}, \Pi^{self}\right) \in \mathcal{P} \times \mathcal{P}^{2^V}$ such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. The proof of Theorem 1 directly extends to this perturbed setting, so we know that an equilibrium information structure $\left(\Pi^{other}, \Pi^{self}\right)$ must satisfy

$$(\star) \qquad \begin{aligned} \{v_i\} \in \Pi^{self}(\pi^{other}) \quad & \forall \min_{v \in \pi^{other}} v < v_i < \max_{v \in \pi^{other}} v \\ \exists \pi^{self}_< \in \Pi^{self}(\pi^{other}) \text{ s.t. } v_i \in \pi^{self}_< \quad & \forall v_i < \min_{v \in \pi^{other}} v \end{aligned}$$

for all $\pi^{other} \in \Pi^{other}$. Lemmas 8 and 9 show that this condition can be strengthened to ($\star'$). Finally, Lemma 10 shows that an equilibrium information structure must also minimize total information costs.

Lemma 8. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that*

$\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. *Then:*

$$\left\{\min_{v_i' \in \pi^{other}} v_i'\right\} \in \Pi^{self}\left(\pi^{other}\right) \text{ and } \left\{\max_{v_i' \in \pi^{other}} v_i'\right\} \in \Pi^{self}\left(\pi^{other}\right) \quad \text{for all } \pi^{other} \in \Pi^{other}.$$

Proof of Lemma 8. Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon > 0$. We first argue that $\left\{\min_{v_i' \in \pi^{other}} v_i'\right\} \in \Pi^{self}\left(\pi^{other}\right)$ for all π^{other}. Suppose not, and let $\overline{v} = \max_{v_i' \in \pi^{other}} v_i'$ and $\underline{v} = \min_{v_i' \in \pi^{other}} v_i'$. We know from Theorem 1 that $\Pi^{self}\left(\pi^{other}\right)$ cannot bundle $v_i = \underline{v}$ with higher values since a buyer must learn to distinguish all values $v_i \in (\underline{v}, \overline{v})$. Thus, $v_i = \underline{v}$ is bundled with lower values: $\{v_i | v_i \leq \underline{v}\} \equiv \pi_{\leq} \in \Pi^{self}\left(\pi^{other}\right)$. With non-vanishing probability, buyer i knows that all his opponents are at the same information set as him. This is for instance the case if they all choose the same information structure and all have values $v_j = \underline{v}$. Thus buyer i must be indifferent between winning and losing at any bid he submits at that information set in equilibrium since he might tie at that bid: $\beta(\pi_i) = \mathbb{E}[v_i | \pi_i, \max_j \beta(\pi_j) = \beta(\pi_i)] + u_i \leq \underline{v} + u_i$. Because of the noise terms, with positive probability that bid must lie strictly in between the maximal and minimal values he can have: $\beta(\pi^{other}, \pi_{\leq}, u_j) \in (v^1 + \underline{u}, \underline{v} + \overline{u})$ for a positive mass of u_j. But then buyer i strictly benefits from learning to distinguish $v_i = \underline{v}$ from lower values (in particular, from the lowest value v^1), and for λ small enough, he must find it strictly profitable to do so.

We now argue that $\left\{\max_{v_i' \in \pi^{other}} v_i'\right\} \in \Pi^{self}\left(\pi^{other}\right)$ for all π^{other}. Suppose not, and let $\overline{v} = \max_{v_i' \in \pi^{other}} v_i'$ and $\underline{v} = \min_{v_i' \in \pi^{other}} v_i'$. We know from Theorem 1 that $\Pi^{self}\left(\pi^{other}\right)$ cannot bundle $v_i = \overline{v}$ with lower values since a buyer must learn to distinguish all values $v_i \in \pi^{other} \setminus \overline{v}$. Hence $\overline{v}$ must be bundled with even greater values, and denote this bundle by $\pi^{self}_{\geq \overline{v}}$. Let $\hat{v}$ denote the maximum valuation in $\pi^{self}_{\geq \overline{v}}$, which by definition satisfies $\hat{v} > \overline{v}$.

Step 1. We first show that, at information set $\pi_i = (\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i)$, a buyer must bid arbitrarily close to $\overline{v} + u_i$ for sufficiently small λ. That is, for any $\eta > 0$ there exists $\bar{\lambda}$ such that, for all $\lambda \leq \bar{\lambda}$, $|\beta(\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i) - \overline{v} - u_i| \leq \eta$. Note that, at this information set, buyer i knows that $\max_j v_j \in [\underline{v}, \overline{v}]$ and $v_i \in [\overline{v}, \hat{v}]$. Since the equilibrium is symmetric, he also knows that, with non-vanishing probability, his toughest opponent has the same information set and makes the same equilibrium bid as i. Indeed, this happens whenever $v_i = \overline{v}$, $\max_j v_j = \overline{v}$, and i's toughest opponent(s) chooses the same information structure as i. Note first that an equilibrium bid at $\pi_i = (\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i)$ cannot be bounded below the lowest possible valuation at that information set $\overline{v} + u_i$. Indeed, since an buyer sometimes ties at that bid, he could slightly increase his bid and make a strict gain. More importantly, $\beta(\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i)$ cannot be bounded above $\overline{v} + u_i$ either. If it were the case, then the buyer would have an incentive to learn whether $v_i = \overline{v}$ or $v_i > \overline{v}$, since he would not want to win against $\beta(\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i)$ in the former case. For sufficiently small information cost λ, he would do so.

Step 2. We then show that such bid at information set $\pi_i = (\pi^{other}, \pi^{self}_{\geq \overline{v}}, u_i)$ cannot be part of an equilibrium. If it were, then a i buyer would make such a bid with probability at least ε in all states of the world consistent with information set π_i. In particular, he would make such a bid in states where his toughest opponent has value $\max_j v_j = \overline{v}$ and he has value $v_i = \hat{v}$. But then his toughest opponent would have a strictly positive, non-vanishing incentive to learn that his valuation is $v_j = \overline{v}$, as he then wants to outbid i if $u_j > u_i$ and to lose if $u_j < u_i$. He would do so in equilibrium, and agent i would have a strict incentive to deviate and learn that his value is $v_i = \hat{v}$. $\square$

Lemma 9. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then, for all $\pi^{other} \in \Pi^{other}$,*

$$\left\{ v_i \mid v_i > \max_{v_i' \in \pi^{other}} v_i' \right\} \in \Pi^{self}\left(\pi^{other}\right).$$

Proof of Lemma 9. Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) \geq \varepsilon > 0$. Take any $\pi^{other} \in \Pi^{other}$, and let $v^{\underline{k}} \equiv \min_{v_i' \in \pi^{other}} v_i'$ and $v^{\overline{k}} \equiv \max_{v_i' \in \pi^{other}} v_i'$. That is, upon learning $\max_j v_j \in \pi^{other}$, buyer i knows that his toughest competitor has $v_j \in [v^{\underline{k}}, v^{\overline{k}}]$.

Towards a contradiction, suppose that $\left\{ v_i \mid v_i > v^{\overline{k}} \right\} \notin \Pi^{self}\left(\pi^{other}\right)$ and take the perspective of some buyer i. Buyer i can only find it worthwhile to learn to distinguish some of the values $v_i > v^{\overline{k}}$ if he sometimes faces a bid in that interval, and sometimes loses at that bid. That means that with positive probability, one of i's competitors, all of whom have a value at most $v^{\overline{k}} + \overline{u}$, makes a bid strictly above this and wins with non-vanishing probability at that bid. However, if they win they must be paying a price weakly higher than i's bid, and i's bid must lie above $v^{\overline{k}} + \overline{u}$. Hence making such a bid leads that buyer to have a strictly negative payoff with non-vanishing probability.

However, that buyer could ensure himself zero by learning his valuation fully and making a lower bid. This cannot be optimal for small λ, and hence cannot be part of an equilibrium. $\square$

Lemma 10. *Take any sequence of equilibria $\{\sigma_\lambda\}_\lambda$ and any information structure such that $\lim_{\lambda \longrightarrow 0} \Pr\left(\Pi^{other}, \Pi^{self} \mid \sigma_\lambda\right) > 0$. Then it must be cost-minimizing, that is:*

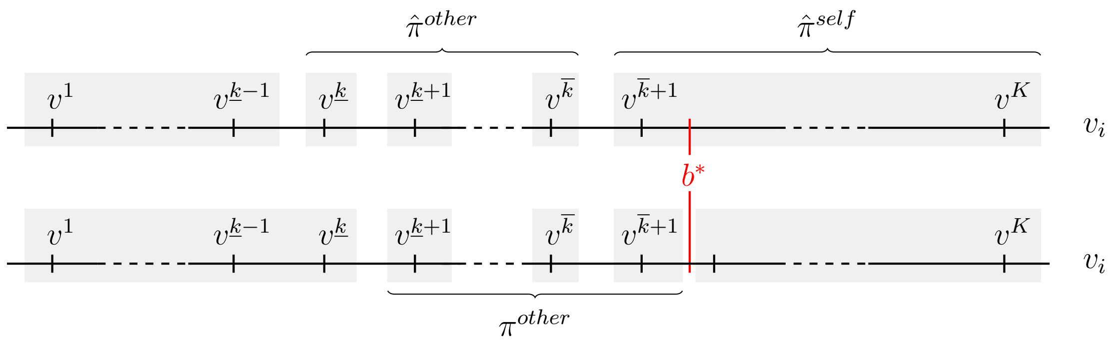
$$\left(\Pi^{other}, \Pi^{self}\right) \in \arg\min_{\widehat{\Pi}^{other}, \widehat{\Pi}^{self}} \quad \gamma(N-1)c\left(\widehat{\Pi}^{other}, \mathbb{P}\right) + \mathbb{E}_{\hat{\pi}^{other}}\left[c\left(\widehat{\Pi}^{self}(\hat{\pi}^{other}), \mathbb{P}\right)\right]$$

$$\text{s.t.} \quad \left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right) \text{ satisfies } (\star').$$

Proof of Lemma 10. Suppose not, such that an equilibrium puts non-vanishing probability $\varepsilon > 0$ on some information structure $\left(\Pi^{other}, \Pi^{self}\right)$ that is not cost-minimizing. That is, there exists another information structure $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ satisfying ($\star'$) that is strictly cheaper than $\left(\Pi^{other}, \Pi^{self}\right)$. Let $\lambda\Delta c$ be the difference in information costs between these two information structures.

Take the point of view of some buyer i. For $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ not to be a profitable deviation from $\left(\Pi^{other}, \Pi^{self}\right)$, it has to be that, under the former, buyer i sometimes gets a strictly lower gross payoff at the auction stage. Since $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$ satisfies ($\star'$), this can only happen when i fails to learn his valuation fully under $\left(\widehat{\Pi}^{other}, \widehat{\Pi}^{self}\right)$. That is, it can only happen when either $\hat{\pi}_i^{self} = \{v_i \mid v_i > \max_{v'_j \in \hat{\pi}_i^{other}} v'_j\}$ or $\hat{\pi}_i^{self} = \{v_i \mid v_i < \min_{v'_j \in \hat{\pi}_i^{other}} v'_j\}$.

Consider the first case, which we illustrate in the figure below. For buyer i not to get his full-information optimal payoff at $(\hat{\pi}_i^{other}, \hat{\pi}_i^{self} = \{v_i \mid v_i > \max_{v'_j \in \hat{\pi}_i^{other}} v'_j\})$, it has to be that the highest bid i faces at this information set sometimes falls strictly within $[\min_{v_i \in \hat{\pi}_i^{self}} v_i + \underline{u}, \max_{v_i \in \hat{\pi}_i^{self}} v_i + \overline{u}]$. Call b^* such a bid and let j^* be an opponent that submits it. For buyer i to be strictly better off under $\left(\Pi^{other}, \Pi^{self}\right)$ because of it, it has to be that, under $\left(\Pi^{other}, \Pi^{self}\right)$, he better discriminates whether he should win against b^*. That is, for some values of $v_i \in \hat{\pi}_i^{self}$ with $v_i < b^*$, i learns his value and bids below b^*. (In the figure, this corresponds to i learning to distinguish $v_i = v^{\overline{k}+1}$ from $v_i > v^{\overline{k}+1}$ under $\left(\Pi^{other}, \Pi^{self}\right)$, as illustrated in the bottom partition.) However, that means buyer j^* is making a strict loss winning against that bid, since we know $v_{j^*} \leq \max_{v_j \in \hat{\pi}_i^{other}} v_j < v_i$. Since information structure $\left(\Pi^{other}, \Pi^{self}\right)$ has non-vanishing



probability, buyer j^* is making a non-vanishing loss, and for small enough λ, he must find it profitable to learn enough so as to avoid this costly mistake.

The reasoning for the second case is very similar. $\square$

Wrapping up, if an information structure has non-vanishing weight in some equilibrium, then it must satisfy ($\star'$) and be cost-minimizing. The information structure under which agents acquire no information about others and fully learn their own valuation does satisfy ($\star'$). However, it is not cost-minimizing (Lemma 1). Hence if an information structure has non-vanishing weight, it must involve acquire some information about $\max_j v_j$.

References

Harsanyi, J. C. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International journal of game theory*, 2(1):1–23.