CRN2: 
$$\emptyset \rightarrow A + B$$
,  $2A + B \rightarrow 2A$ ,  $5A + 2B \rightarrow 3B \rightarrow 2A$ . (1.6)

For each network, we number the reactions from left to right, obtaining  $\mathcal{R} = \{r_1, r_2, r_3, r_4\}$ . The networks are displayed in Fig. 1.

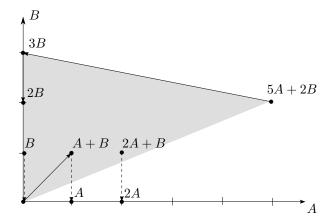


Figure 1: The complex graph of the networks in (1.4)–(1.6). The common reactions in the networks are displayed as solid arrows, while the reactions that are different in the three examples are dashed. The existence of the reaction  $\emptyset \rightharpoonup A+B$  makes the networks asiphonic, and the fact that all the arrows starting on the faces of the reaction polytope (the grey triangle) point inwards makes the network Strongly Endotactic.

**Behavior in the fluid limit regime** Under the law of mass action, as the number of molecules in (1.4)–(1.6) goes to infinity the dynamics of their appropriately rescaled density obeys the system of ordinary differential equations (1.3), *i.e.*,

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_1^5 x_2^2 \begin{pmatrix} -5 \\ 1 \end{pmatrix} + (x_2^3 + x_1^\# x_2) \begin{pmatrix} 0 \\ -1 \end{pmatrix} , \tag{1.7}$$

where # corresponds to the number of the CRN under consideration (e.g. #=0 for CRN0) and without loss of generality we have assumed that  $\kappa_r=1$  for all  $r\in \mathcal{R}$ . The latter assumption will continue to hold throughout the paper.

The CRNs defined above belong to the class of Asiphonic Strongly Endotactic networks, introduced in [1, 2]. This class of CRNs is defined exclusively on structural properties of the networks. These properties on one hand that no

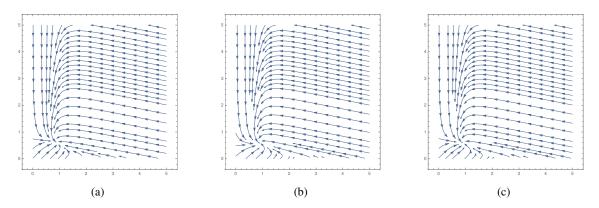


Figure 2: Stream lines of the vector fields in (1.7) for the networks CRNO (a), CRNO (b) and CRNO (c). The vector fields are very similar, with the only noticeable differences close to the horizontal  $(x_1)$  axis in the three figures. Asymptotically, these differences vanish.