4) Basic solution
A set of n linear independent solutions of a homogeneous n-order linear ODE $Ly=0$ is called the **basic solution** of $Ly=0$ on I: $\{y_1(x), y_2(x),\dots, y_n(x)\}$.

5) Superposition principle
The fundamental theorem for the homogeneous linear ODE is the **Superposition Principle**:
For a homogeneous linear ODE $Ly=0$, any linear combination of two basic solutions on interval I is again its solution.

6) If y_1 is a solution of $y^{(n)} + P_1(x)y^{(n-1)} + \cdots + P_n(x)y = f_1(x)$
and y_2 is a solution of $y^{(n)} + P_1(x)y^{(n-1)} + \cdots + P_n(x)y = f_2(x)$,
then $y_1 + y_2$ should be the solution of equation $y^{(n)} + P_1(x)y^{(n-1)} + \cdots + P_n(x)y = f_1(x) + f_2(x)$.

7) Structure of the general solution of homogeneous linear ODE and nonhomogeneous linear ODE.
General solution of homogeneous linear ODE:
A linear combination of n basic solutions of the homogeneous n-order linear ODE $Ly=0$, such as

$$Y(x) = c_1y_1(x) + c_2y_2(x) + \cdots + c_ny_n(x)$$

General solution of nonhomogeneous linear ODE (y):
The general solution of homogeneous linear ODE $Ly=0$ (Y) +
One particular solution of its corresponding nonhomogeneous linear ODE $Ly=R(x)$ (y^*):

$$y = Y + y^*$$

11-2. 2nd-order linear ODE with constant coefficients

1) 2nd-order homogeneous linear ODE with constant coefficients
If a and b are constant, the following equation is a 2-order linear ODE with constant coefficients:

$$Ly = y'' + ay' + by = 0 \tag{11-2}$$

To solve this equation, using $y = e^{\lambda x}$ (λ: constant) and substitute it into above equation, we obtain the **characteristic equation**:

$$\lambda^2 + a\lambda + b = 0$$

Suppose the solution of the characteristic equation is λ_1 and λ_2. From algebra we know that above quadratic equation may have 3 kinds of roots, according to the sign of $a^2 - 4b$.

<u>Case 1.</u> $\lambda_1 \neq \lambda_2$ and both are real numbers. $e^{\lambda_1 x}$ and $e^{\lambda_2 x}$ are the basis of the $Ly=0$, so the general solution of $Ly=0$ is

$$y = c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x}$$

<u>Case 2.</u> $\lambda_1 = \lambda_2$, $e^{\lambda_1 x}$ and $x\,e^{\lambda_1 x}$ are the basis of the $Ly=0$, so the general solution of $Ly=0$ is

$$y = (c_1 + c_2x)e^{\lambda_1 x}$$

<u>Case 3.</u> λ_1 and λ_2 are complex conjugate roots, if, $\lambda_1 = \alpha + i\beta$, $\lambda_2 = \alpha - i\beta$, $e^{\alpha x}\cos\beta x$ and $e^{\alpha x}\sin\beta x$ are the basis of $Ly=0$, so that the general solution is,

$$y = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$$

2) 2nd-order nonhomogeneous linear ODE with constant coefficients
The 2nd-order nonhomogeneous linear ODE with constant coefficients is like