Colorado COVID-19 Model Equations

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Here are the model equations for age category i with a total of 4 age cateogries {1,2,3,4}

$$\begin{aligned}
\frac{dS_i}{dt} &= -\frac{\beta}{N} \cdot \lambda \cdot (1 - \text{mask} \cdot 0.03) \cdot (1 - \text{siI}) \cdot (1 - \text{SD}_i) \cdot S_i \cdot \sum_{j=1}^{4} I_j \\
&\quad - \frac{\beta}{N} \cdot (1 - \text{mask} \cdot 0.2667) \cdot (1 - \text{SD}_i) \cdot S_i \cdot \sum_{j=1}^{4} A_j \\
\frac{dE_i}{dt} &= -\frac{1}{\alpha} \cdot E_i + \frac{\beta}{N} \cdot \lambda \cdot (1 - \text{mask} \cdot 0.03) \cdot (1 - \text{siI}) \cdot (1 - \text{SD}_i) \cdot S_i \cdot \sum_{j=1}^{4} I_j \\
&\quad + \frac{\beta}{N} \cdot (1 - \text{mask} \cdot 0.2667) \cdot (1 - \text{SD}_i) \cdot S_i \cdot \sum_{j=1}^{4} A_j \\
\frac{dI_i}{dt} &= \frac{\text{pS}_i}{\alpha} \cdot E_i - \gamma \cdot I_i - \text{pID} \cdot \text{pCT} \cdot \kappa \cdot \pi \cdot \omega \cdot I_i \\
\frac{dII_i}{dt} &= \text{pID} \cdot \text{pCT} \cdot \kappa \cdot \pi \cdot \omega \cdot (I_i + A_i) - \gamma \cdot II_i \\
\frac{dIh_i}{dt} &= -\gamma \cdot \text{hosp}_i \cdot I_i + \gamma \cdot \text{hosp}_i \cdot \text{pS}_i \cdot II_i - \frac{1}{\text{losh}_i} \cdot Ih_i \\
\frac{dIc_i}{dt} &= -\gamma \cdot \text{cc}_i \cdot I_1 + \gamma \cdot \text{cc}_i \cdot pS_i \cdot II_i - \frac{1}{\text{losc}_i} \cdot Ic_i \\
\frac{dA_i}{dt} &= -\frac{1 - pS_i}{\alpha} \cdot E_i - \gamma \cdot A_i - \text{pID} \cdot \text{pCT} \cdot \kappa \cdot \pi \cdot \omega \cdot A_i \\
\frac{dR_i}{dt} &= \gamma \cdot (1 - \text{hosp}_i - \text{cc}_i - \text{dnh}_i) \cdot (I_i + \text{pS}_i \cdot II_i) + \gamma \cdot A_i \\
\frac{dRh_i}{dt} &= \frac{1 - \text{dh}_i}{\text{losh}_i} \cdot Ih_i \\
\frac{dRc_i}{dt} &= \frac{1 - \text{dc}_i}{\text{losc}_i} \cdot Ic_i \\
\frac{dD_i}{dt} &= \frac{\text{dc}_i}{\text{losc}_i} \cdot Ic_i + \frac{\text{dh}_i}{\text{losh}_i} Ih_i + \gamma \cdot \text{dnh}_i \cdot I_i
\end{aligned}$$

Variable	Description
S	Susceptibles
E	Exposed
I	Infectious
II	Isolated Infectious
Ih	Hospitalized non-ICU Infectious
Ic	Hospitalized ICU Infectious
A	Asymptomatic Infectious
R	Recovered
Rh	Recovered from non-ICUHospitalization
Rc	Receovered from ICU Hospitalization
D	Deceased

Table 1: Variables used in the model.

Parameter	Description	Value	Source
α	Incubation period	4	
β	Transmission rate	0.4793	
γ	Recovery rate	$1/9$	
κ	Average number of contacts per detected case.	varies	
λ	Difference in infectiousness symptomatic / asymptomatic	1.395	
π	Probability a contact traced infected individual is isolated before infecting other susceptibles	varies	
ω	Probability a contact traced individual is infected.	0.0609	
cc_1	Fraction of age category 1 symptomatic cases requiring ICU	0.00486	
cc_2	2	0.0114	
cc_3	3	0.02153	
cc_4	4	0.05656	
pCT	Fraction of identified cases with contacts traced	0.4	
dc_1	Hospitalization death fraction for age category 1 in ICU	0.0417	
dc_2	2	0.0392	
dc_3	3	0.1543	
dc_4	4	0.3956	
dh_1	Hospitalization death fraction for age category 1 in non-ICU	0	
dh_2	2	0	
dh_3	3	0.0045	
dh_4	4	0.0923	
dnh_1	Non-hospitalization death fraction for age category 1	0.000007	
dnh_2	2	0.000013	
dnh_3	3	0.000113	
dnh_4	4	0.003028	
$hosp_1$	Fraction of age category 1 with symptomatic cases hospitalized	0.01108	
$hosp_2$	2	0.03139	
$hosp_3$	3	0.04711	
$hosp_4$	4	0.05825	
$losc_1$	Length of stay non-ICU for age category 1	4.4	CDPHE
$losc_2$	2	7.3	CDPHE
$losc_3$	3	11.4	CDPHE
$losc_4$	4	9.9	CDPHE
$losh_1$	Length of stay ICU for age category 1	3.6	CDPHE
$losh_2$	2	4.6	CDPHE
$losh_3$	3	6.8	CDPHE
$losh_4$	4	9.7	CDPHE
mask	Fraction wearing masks	varies	
pID	Fraction of infections identified.	0.4	
pS_1	Fraction of individuals symptomatic in age category 1	0.110023	
pS_2	2	0.35705	
pS_3	3	0.561205	
pS_4	4	0.774879	
siI	Fraction self-isolating	0.438	
SD_1	Social distancing impact for age cateogry 1	varies over time	fit
SD_2	2		
SD_3	3		
SD_4	4		
N	Colorado population	5,840,795	US Census 2016 data w/9% growth assumed

Table 2: Parameters used in the model