Announcements

- Happy Valentines Day!
- Homework 3 due tonight!
- I'll aim to get Homework 4 posted sometime tomorrow or early Sunday
- Will be wrapping up with Chapter 3 on Monday, maybe starting a smidge of Chapter 4
- No lab today! I'll be sticking around for a while if you just need to get work done on the homework though!
- Polling: `rembold-class.ddns.net`

Review Question

The height of a particular thrown ball is given by

$$h = 2 + 10t - \frac{1}{2}gt^2$$

where $g = 9.81$. Suppose you wanted to solve the time at which the ball struck the ground ($h = 0$), with an ε value of 0.1 m. Which of the below potential solutions would meet your criteria?

A) 2.14 s

B) 2.22 s

C) 2.26 s

D) 2.30 s

Solution: 2.22 s

Multiplying Rabbits

Suppose we have a breed of rabbit where each pairing of rabbits produces 4 offspring each year. We also know a certain constant percentage of the rabbit population dies each year due to predators or natural causes. If we start with 10 rabbits and have 1000 rabbits 10 years later, what is the yearly rabbit death percentage? We are ok with an error of ± 2 rabbits.

Intersection of Two Lines on Interval

Suppose you have two lines given by the equations:

$$y = Ax + B$$
$$y = Cx + D$$

Write a program to return the intersection point of the two lines with an epsilon of 0.01 on the interval between 0 and 10. If no point is found or exists, print a message stating as much.

When 1 != 1

```
x = 0
for i in range(10):
    x += 0.1
if x == 1.0:
    print('x is 1!')
else:
    print('x is not 1!')
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Crisis!

What is happening to the computer's ability to do basic math?!

10 Types of Numbers

Decimal

- Each digit represents different powers of 10

$$123 = (1 \times 10^2) + (2 \times 10^1) + (3 \times 10^0)$$

- Sig Digit and Exponent:

$$5.62 = 562 \times 10^{-2} = (562, -2)$$

Binary

- Each digit represents different powers of 2

$$101 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$$

- Sig Digit and Exponent:

$$10100 = 20 = 5 \times 2^2 = (101, 1)$$

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  - We can't express the denominator nicely as a power of 2!
  - We have to resort to fractions that are close but have a perfect power of 2 in the denominator.
    - $\frac{3}{32} = 0.09375$
    - $\frac{25}{256} = 0.09765625$

Repercussions

- Best we can do:

$$\frac{3602879701896397}{2^{55}} = 0.1000000000000000055511151231257827 0$$

- When doing operations on these numbers, extra decimals get rounded off
- Rounding errors mean you'll occasionally see values, close, but not exactly equal to what you would expect

Floating Takeaways

- You can't get around this sort of rounding error. It is intrinsic to how the computers work with numbers.
- Be careful using == for number comparisons. Rounding might cause unexpected falsehoods!
  - Far better to check if two numbers are within some small margin of one another
- Rounding errors will usually average out between being too big or too little
  - Means even after many calculations they should be small
  - Repeated calculations of certain types though accumulate errors in the same direction and cause decided issues.