332: 417 Control System Design - Project 2 Fall 2021

Design of Full- and Reduced-Order Linear Observers for a Proton Exchange Membrane Fuel Cell

10 points = 10% of the course grade.

Project assigned Friday October 29, 2021. Project due on Friday Nov. 12, 2021.
Upload a single pdf file of your project report on Sakai.

Introduction to Fuel Cells

Fuel cells are electro-chemical-mechanical devices that produce electricity from hydrogen rich fuels (natural gas, gasoline, …) without generating carbon dioxide (without burning fuels). The process is reversed to water electrolysis (generates hydrogen and oxygen from water and electricity). A fuel cell is a triode anode-membrane-cathode. Depending on the type of membrane there are several types of fuel cells. The most developed and used are the proton exchange membrane (also called polymer exchange membrane) fuel cells, simply called PEM fuel cells.

In fuel cells hydrogen and oxygen (from the air) produce water and electricity. It is interested to mention that in the sixties the Apollo astronauts used fuel cells to produce water. Electric power generated by fuel cells is huge and a small 10cm-cube fuel cell can power a house and a 20cm-cube fuel cell can power a car. Several companies have already produced electric commercially available cars powered by fuel cells.

Students interested in fuel cells can learn a little bit more by reading the notes from a lecture on fuel cells that Professor Gajic delivered several years ago in our Sustainable Energy class (332:402/585) posted on Sakai in the directory Resources/Articles.

Proton Exchange Membrane (PEM) Fuel Cell Linearized Mathematical Model

This model is considered in detail in a series of journal papers and the book: J. Pukrushpan, A. Stefanopoulou, and H. Peng, *Control of Fuel Cell Power Systems*, Springer, 2004, researchers at the University of Michigan. The corresponding 9^{th}-order nonlinear mathematical model and its state space variables are given by

$$\frac{dx(t)}{dt} = f(x(t), u(t), w(t))$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \end{bmatrix}^T =$$
$$= \begin{bmatrix} m_{O_2} & m_{H_2} & m_{N_2} & \omega_{cp} & p_{sm} & m_{sm} & m_{H_2O_A} & m_{H_2O_C} & p_{rm} \end{bmatrix}^T$$

where subscripts cp, sm, rm stand respectively for the compressor, supply manifold, and return manifold. $\omega_{cp}(t)$ is the compressor (that blows air (oxygen) on the cathode side) angular velocity. The control variable is the compressor motor voltage, $u(t) = v_{cm}(t)$. $w(t)$ is the disturbance and it represents the stack current, that is, $w(t) = I_{st}(t)$. It is assumed that the hydrogen comes from a pressurized tank, so that a simple proportional controller for the anode hydrogen molar flow rate can be used as $W_{H_2}^{in} = K_1(K_2 p_{sm} - p_{anode})$. The main task is this model to determine the cathode air molar flow rate, or more precisely the compressor angular velocity that determines the compressor molar flow rate (equal to the cathode molar flow rate).

The output (measured) variables are given by

$$y(t) = \begin{bmatrix} W_{cp} & p_{sm} & v_{st} \end{bmatrix}^T = h_y(x,u,w)$$

where $W_{cp} = W_{cp}(x_4, x_5)$ is the compressor air molar flow rate, $p_{sm} = x_5$, and v_{st} is the fuel cell stack voltage.

The linearized system matrices at steady state corresponding to the following values: $P_{net}^{ss} = 40$ kW, $I^{ss} = 191$ A, $v_{cm}^{ss} = 164$ V, $\lambda_{O_2}^{ss} = W_{O_2}^{in} / W_{O_2}^{reacted} = 2$, can be found on page 145 (posted on class website in the file projects) of the book. In the following, *for the reason of simplicity, it is assumed that* $\delta w(t) = 0$. The data are given in matrices A, B, C, D, respectively, corresponding to our standard state space matrices A, B, C, D, that is

$$\frac{\delta x(t)}{dt} = A\delta x(t) + B\delta u(t)$$
$$\delta y(t) = C\delta x(t) + D\delta u(t)$$
$$x = x_{ss} + \delta x, \quad u = u_{ss} + \delta u, \quad y = y_{ss} + \delta y$$

It is interesting to observe that the linearized model is of the 8th-order. Namely, the state variable corresponding to the mass of water in the cathode is eliminated from the model (most likely, it is both weakly controllable and weakly observable).

The linearized system matrices (according to page 145 of the book) are given below:

$$A = \begin{bmatrix} -6.30908 & 0 & -10.9544 & 0 & 83.74458 & 0 & 0 & 24.05866 \\ 0 & -161.083 & 0 & 0 & 51.52923 & 0 & -18.0261 & 0 \\ -18.7858 & 0 & -46.3136 & 0 & 275.6592 & 0 & 0 & 158.3741 \\ 0 & 0 & 0 & -17.3506 & 193.9373 & 0 & 0 & 0 \\ 1.299576 & 0 & 2.969317 & 0.3977 & -38.7024 & 0.105748 & 0 & 0 \\ 16.64244 & 0 & 38.02522 & 5.066579 & -479.384 & 0 & 0 & 0 \\ 0 & -450.386 & 0 & 0 & 142.2084 & 0 & -80.9472 & 0 \\ 2.02257 & 0 & 4.621237 & 0 & 0 & 0 & 0 & -51.2108 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 3.946683 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 5.066579 & -116.446 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 12.96989 & 10.32532 & -0.56926 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To simplify notation in your report use

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

having in mind that $x(t), u(t), y(t)$ in fact represent deviations from the corresponding steady state values.

PROJECT TASKS:

(a) Design of a Full-Order Observer. Design a full order observer to estimate all state variables, see Hint 1. Place the closed-loop system eigenvalues via the eigenvalue technique using the MATLAB function place as F=place(A, B, eigenvalue_desired) with the goal to bring the state space variables to their steady state values. Note that in order to be able to run simulation you must specify the system initial conditions x0 even though they are unknown. The observer initial condition is your choice and in general x0hat is different from x0.

Plot the difference between the actual output signal $y(t)$ and observed (estimated) output signal $\hat{y}(t)$. That signal should go to zero pretty quickly. Plot the system output response using the MATLAB function plot. Plot the actual and observed state space variables and the deference between them, that is plot >> plot(t, out.x); plot(t, out.xhat); plot(t, out.e), where e=x-xhat. See Hint 2.

(b) Design of a Reduced-Order Observer. Design a reduced-order observer to estimate all state variables using information coming from the system measurements $y(t)$, see Hint 1. As a crucial starting step the students will have to choose an arbitrary matrix C_1 such that the augmented matrix [C; C1] is invertible. Then, [L1 L2]=inv[C;C1]. The reduced-order observer gain K_1 is calculated using the eigenvalues placement technique via a short MATLAB program given in *IEEE Control System Magazine* paper posted on Sakai. Note that the observer must be much faster than the system, roughly five times so that the observer closed-loop eigenvalues must be appropriately placed further to the left from the system closed-loop eigenvalues (the eigenvalues of A-B*F). Plot the difference between the actual state space variables $x(t)$ (see hint below) and the observed (estimated) state space variables $\hat{x}(t)$ with $\hat{x}(t) = L\hat{q}(t) + (L_1 + L_2K_1)y(t)$ and the reduced observer as $\dot{\hat{q}}(t) = A_q\hat{q}(t) + B_q u(t) + K_q y(t)$. Compare the results with the results obtained in Part (a).

Hint 1: Read first the IEEE Control Systems Magazine paper, and learn the material presented in Lectures 18 and 19 with the corresponding videos on implementation of linear full- and reduced-order observers using MATLAB/Simulink, posted on Sakai.

Hint 2: To get $x(t)$ run in parallel (must have the same input signal) with the system Simulink state space block another Simulink state space block whose matrices are set as A, B, eye(n), zeros(n,m), x0, where n is the order of the system and m is the dimension of the system input. The output signal of such a block is $x(t)$.

Project Reports Format:

Each report should be typed and contain the following parts:

1) Project formulation (you may use the word document of the project assignment and cut and paste its parts).
2) Provide a brief introduction of the techniques used, including all relevant math formulas (**do not copy text or formulas from any source**; use any software to generate them).
3) Plot all signals and figures as required by the project.
4) Include block diagrams (generated by Simulink or any other software) in the main text.
5) Provide comments on the obtained results.
6) Write conclusions.
7) Give a list of some references, books, journal and conference papers. *Do not put among references links to websites*.
8) Appendix: Put here the MATLAB code, and anything else that you consider relevant to the project.