

# A Light Intro To Boosting

# Machine Learning

- Not as cool as it sounds
  - Not iRobot
  - Not Screamers (no Peter Weller 😞 )
- Really just a form of
  - Statistics
  - Optimization
  - Probability
  - Control theory
  - ...
- We focus on classification

# Classification

- A subset of machine learning & statistics
- Classifier takes input and predicts the output
- Make a classifier from a *training* dataset
- Use the classifier on a *test* dataset (different from the training dataset) to make sure you didn't just memorize the training set
- A good classifier will have low *test* error

# Classification and Learning

- Learning classifier learns how to predict after being shown many input-output examples
- Weak classifier is slightly correlated with correct output
- Strong classifier is highly correlated with correct output
- (See the PAC learning model for more info)

# Methods for Learning Classifiers

- Many methods available
  - Boosting
  - Bayesian networks
  - Clustering
  - Support Vector Machines (SVMs)
  - Decision Trees
  - . . .
- We focus on boosting

# Boosting

- Question: Can we take a bunch of weak hypotheses and create a very good hypothesis?
- Answer: Yes!

# Brief History of Boosting

- 1984 - Framework developed by Valiant
  - Probably approximately correct (PAC)
- 1988 - Problem proposed by Michael Kearns
  - Machine learning class taught by Ron Rivest
- 1990 - Boosting problem solved (in theory)
  - Schapire, recursive majority gates of hypotheses
  - Freund, simple majority vote over hypotheses
- 1995 - Boosting problem solved (in practice)
  - Freund & Schapire, AdaBoost adapts to error of hypotheses

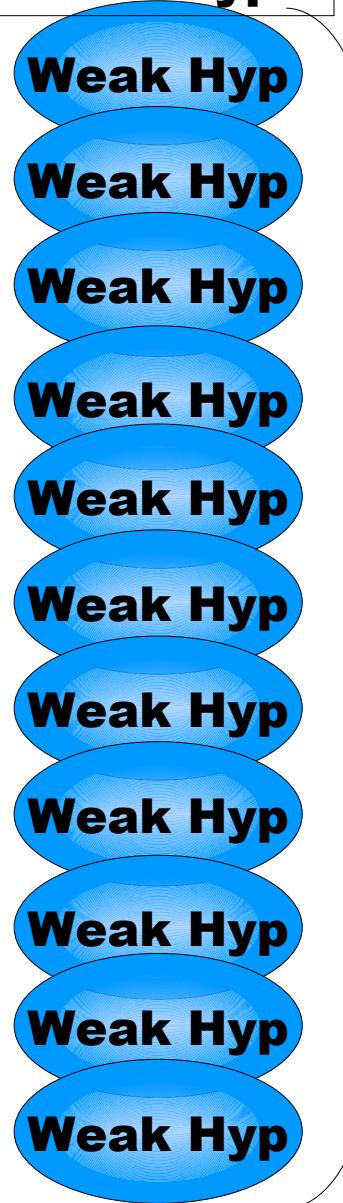
# T Weak Hyps = 1 Strong Hyp

Try Many  
Weak Hyps

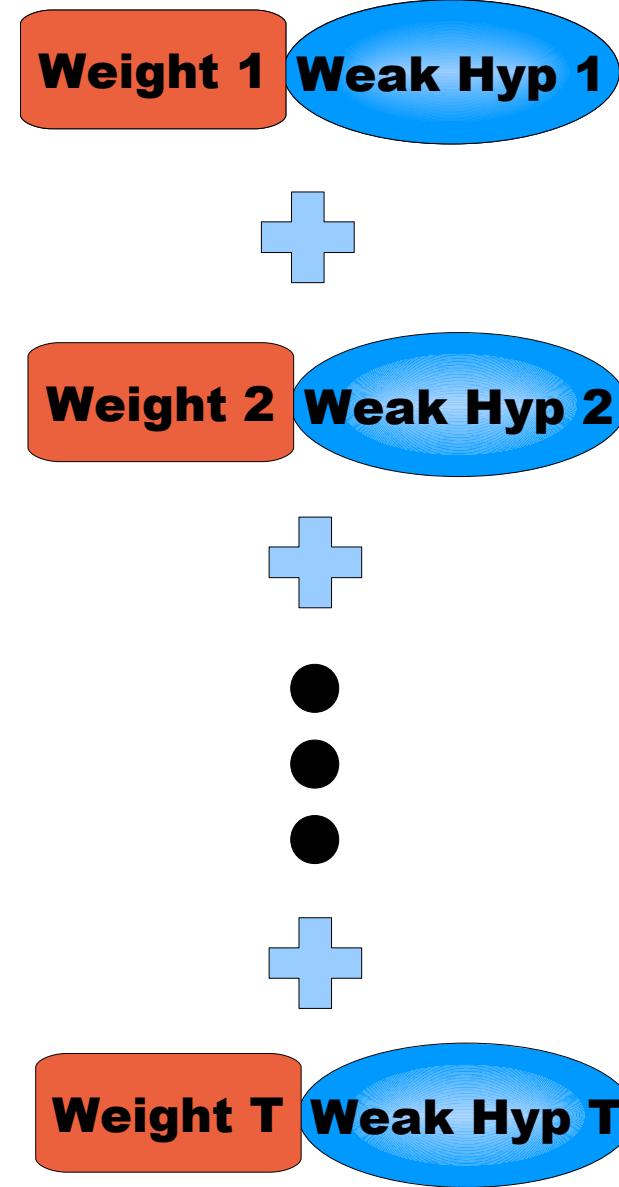
Weak Hyp

# T Weak Hyps = 1 Strong Hyp

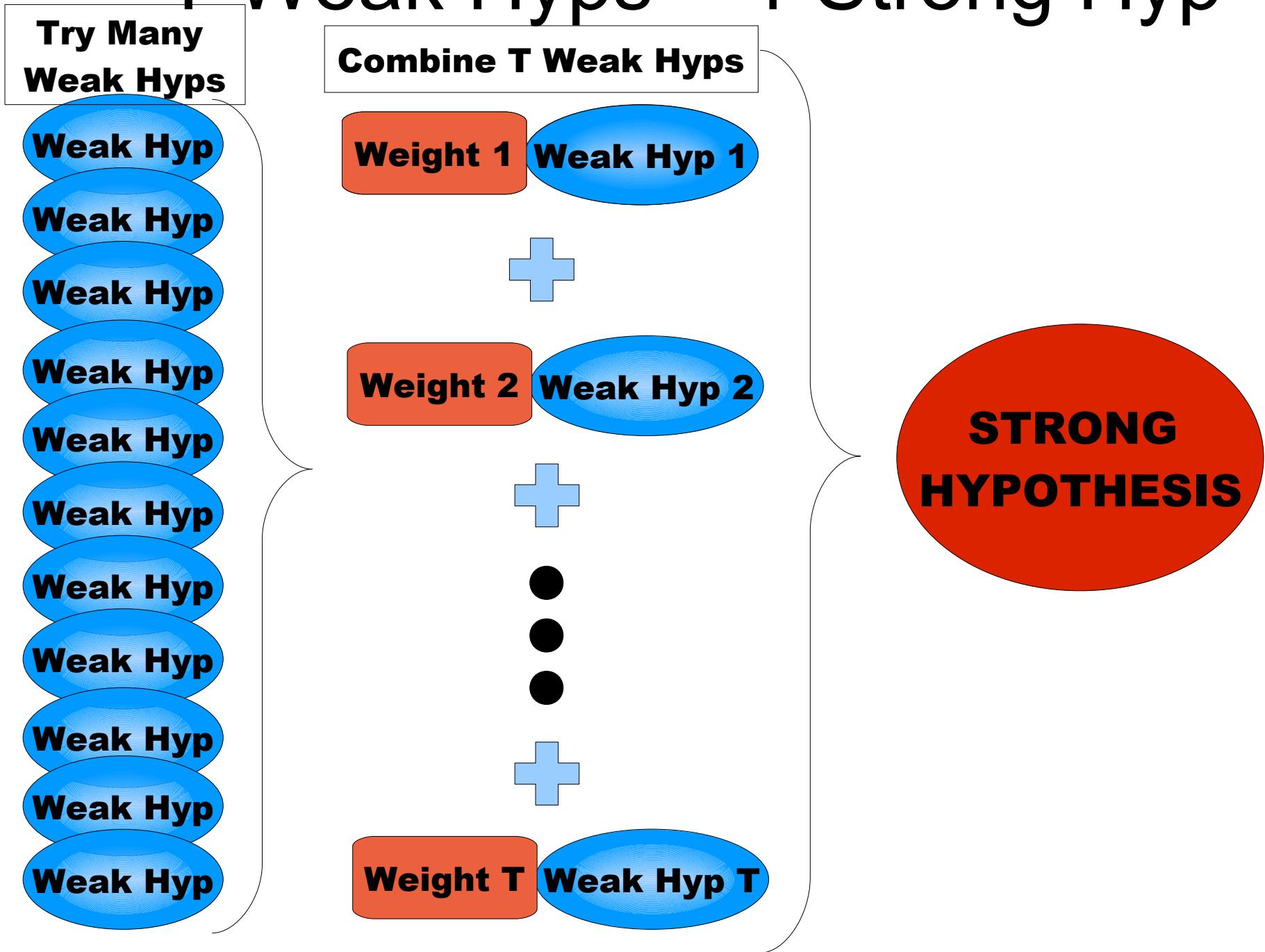
## Try Many Weak Hyps



## Combine T Weak Hyps

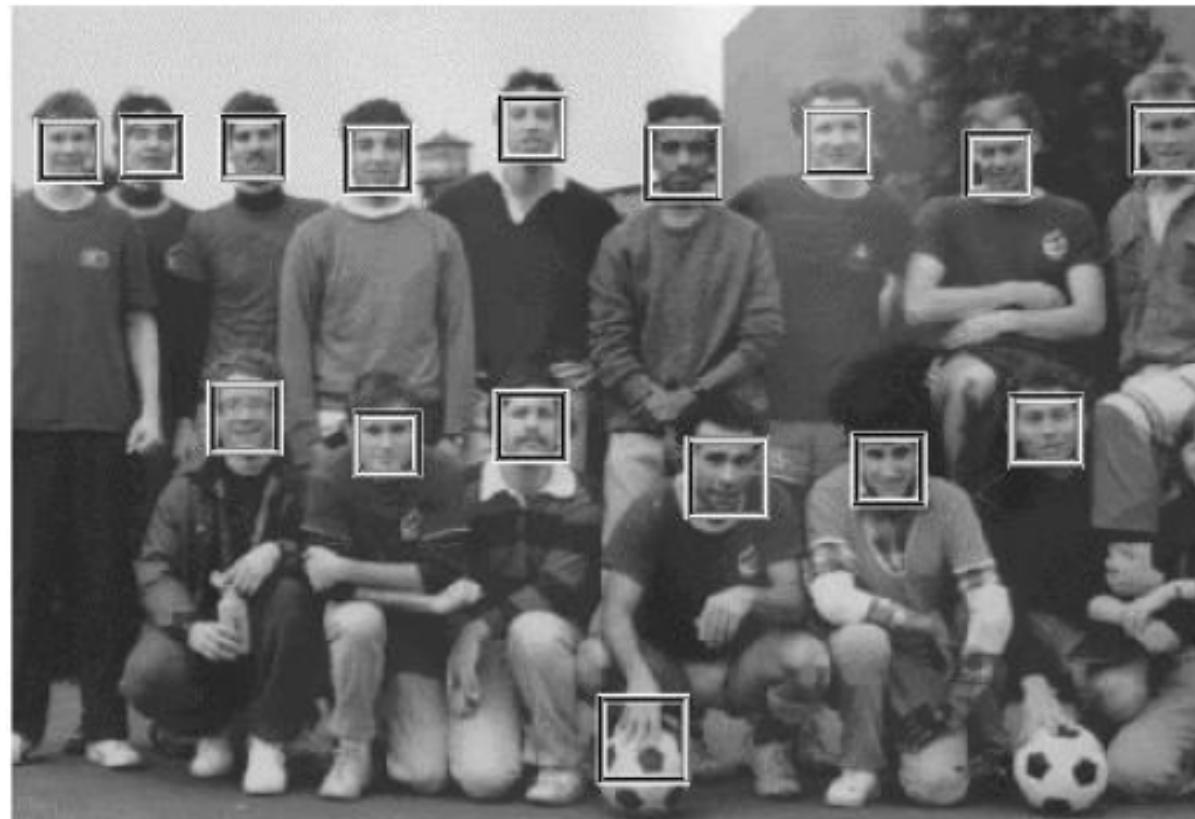


# $T$ Weak Hyps = 1 Strong Hyp



# Example: Face Detection

- We are given a dataset of images
- We need to determine if there are faces in the images



# Example: Face Detection

- Go through each possible rectangle
- Some weak hypotheses might be:
  - Is there a round object in the rectangle?
  - Does the rectangle have darker spots where the eyes should be?
  - Etc.
- Classifier =  $2.1 * (\text{Is Round}) + 1.2 * (\text{Has Eyes})$
- Viola & Jones 2001 solved face detection problem in similar manner

# Algorithms

- Many boosting algorithms have two sets of weights
  - Weights on all the training examples
  - Weights for each of the weak hypotheses used
- It is usually clear from context which set of weights is being discussed

# Basic Boosting Algorithm

- Initial Conditions:
  - Training dataset  $\{ (x_1, y_1), \dots (x_i, y_i) \dots, (x_n, y_n) \}$
  - Each  $x$  is an example with a label  $y$
- Learn a pattern
  - Use  $T$  weak hypotheses
  - Combine them in an “intelligent” manner
- See how well we learned the pattern
  - Did we just memorize training set?

# An Iterative Learning Algorithm

Let  $w_i^t$  be the weight of example  $i$  on round  $t$

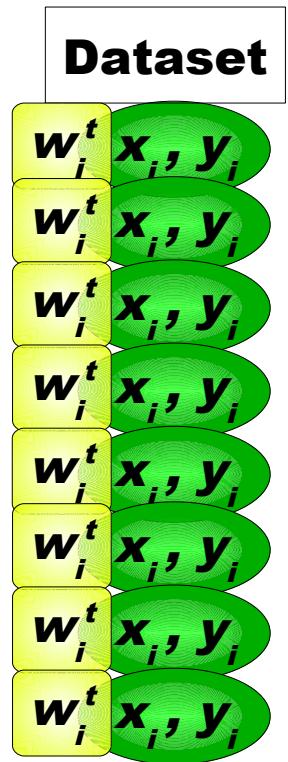
$$w_i^0 = 1/n$$

For  $t = 1$  to  $T$ :

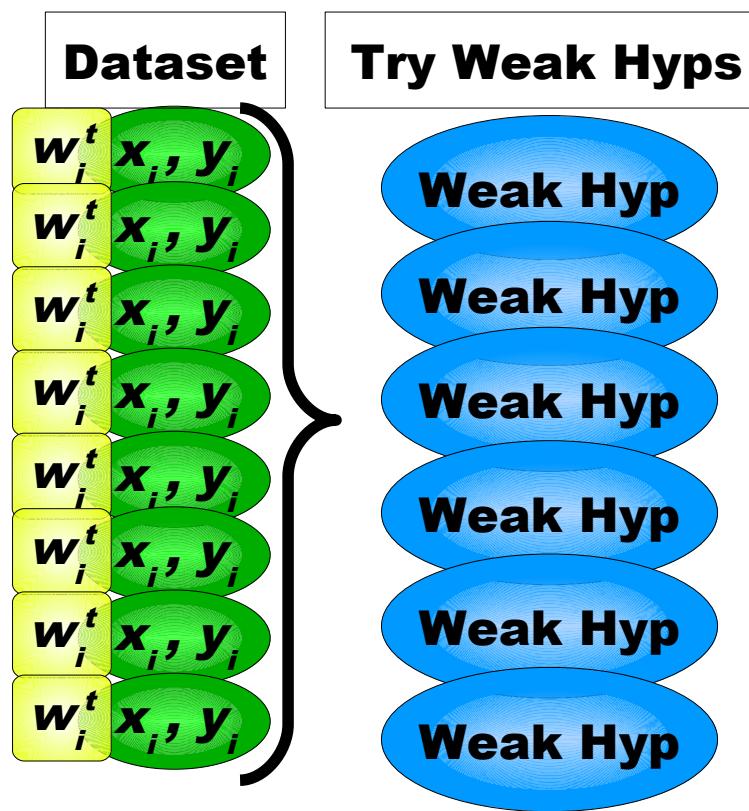
- 1) Try many weak hyps, compute error  $\sum_i w_i^t [[h(x_i) \neq y_i]]$
- 2) Pick the best hypothesis:  $h_t$
- 3) Give  $h_t$  a weight  $\alpha_t$
- 4) More weight to examples that  $h_t$  misclassified
- 5) Less weight to examples that  $h_t$  classified correctly

Return a final hypothesis of  $H_t(x) = \sum_t \alpha_t h_t(x)$

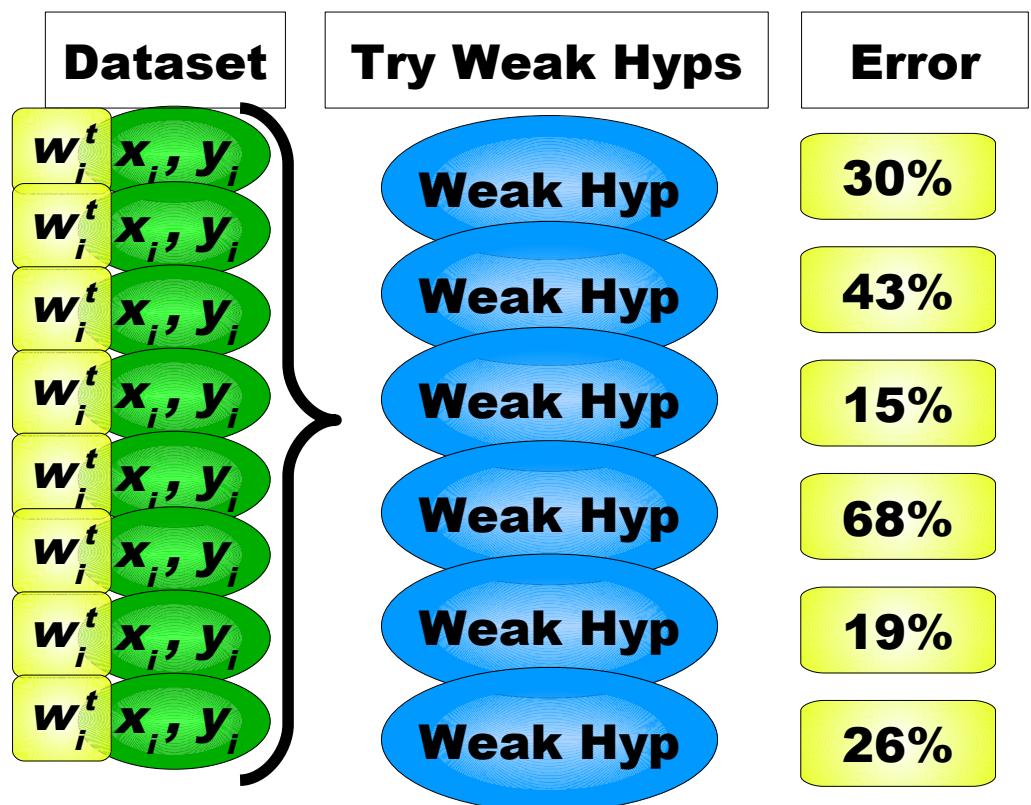
# One Iteration



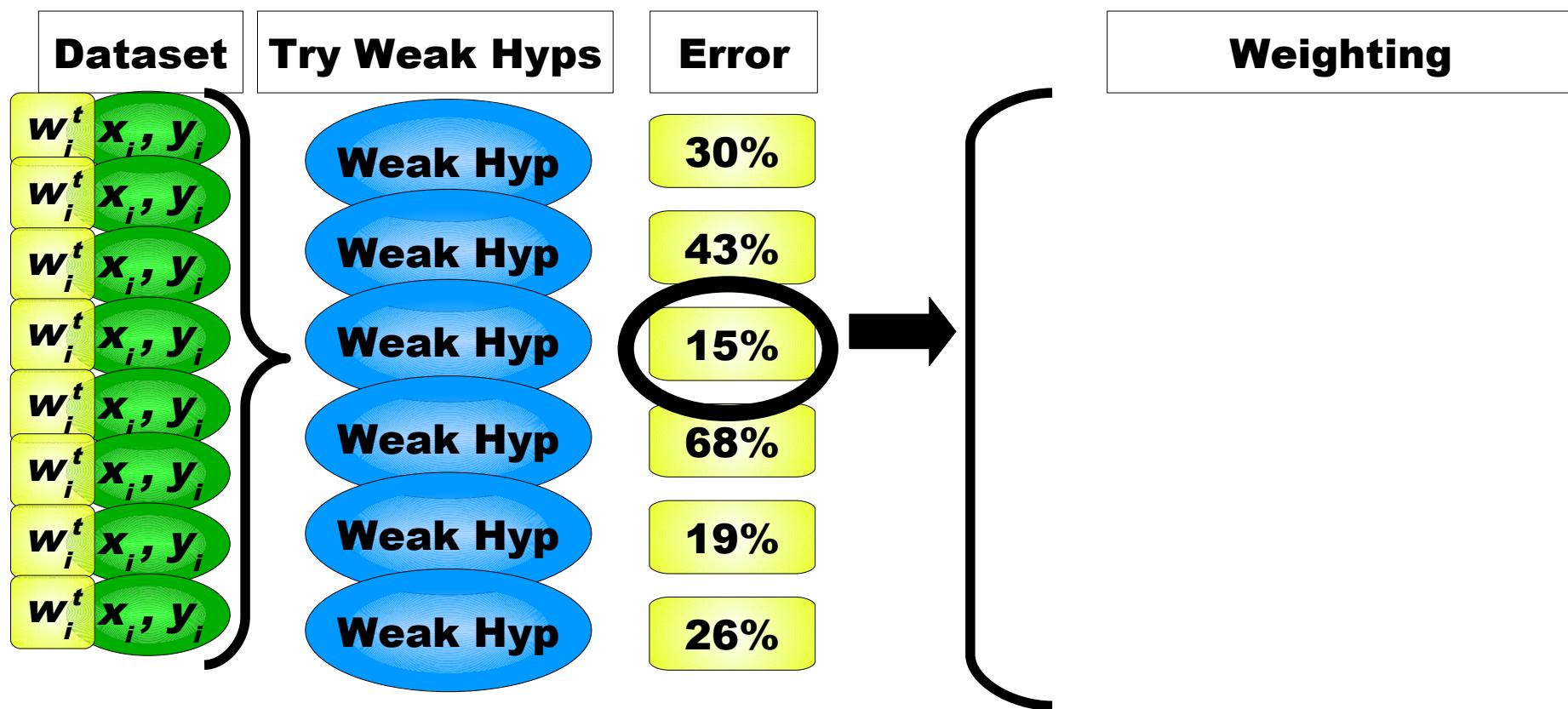
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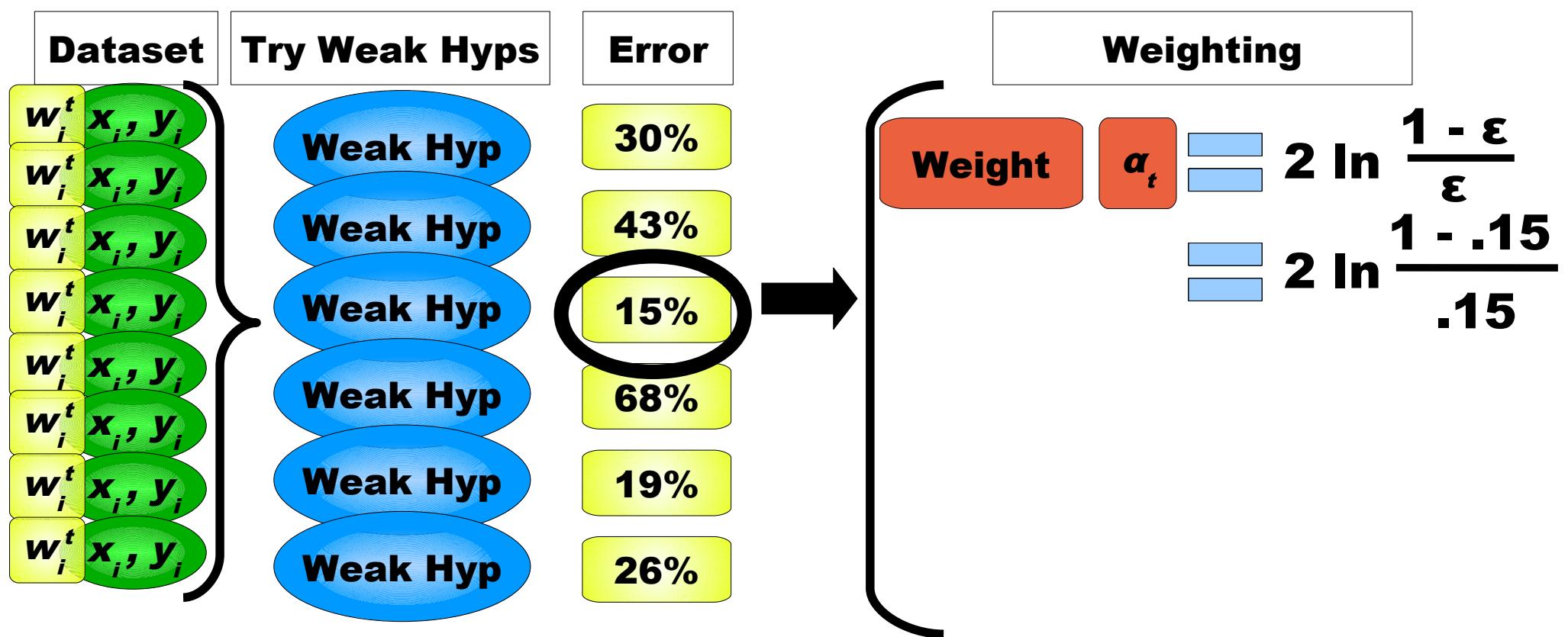
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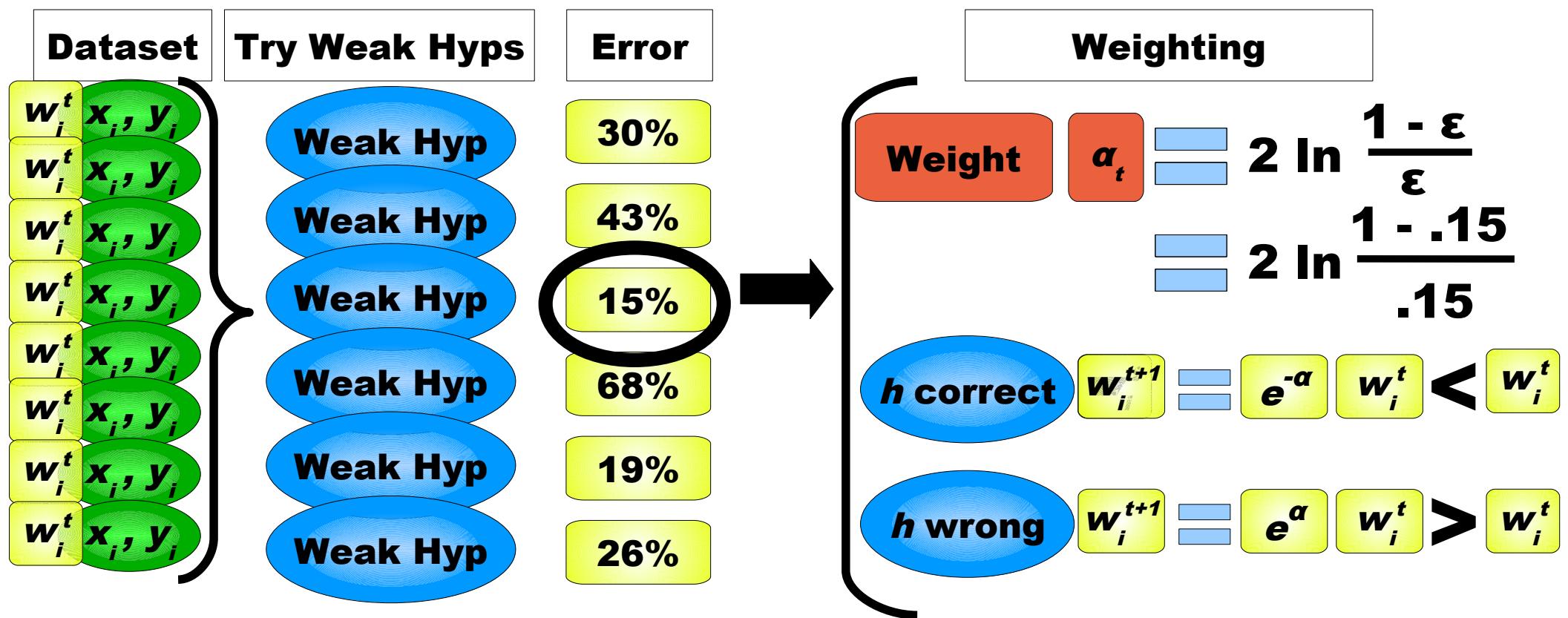
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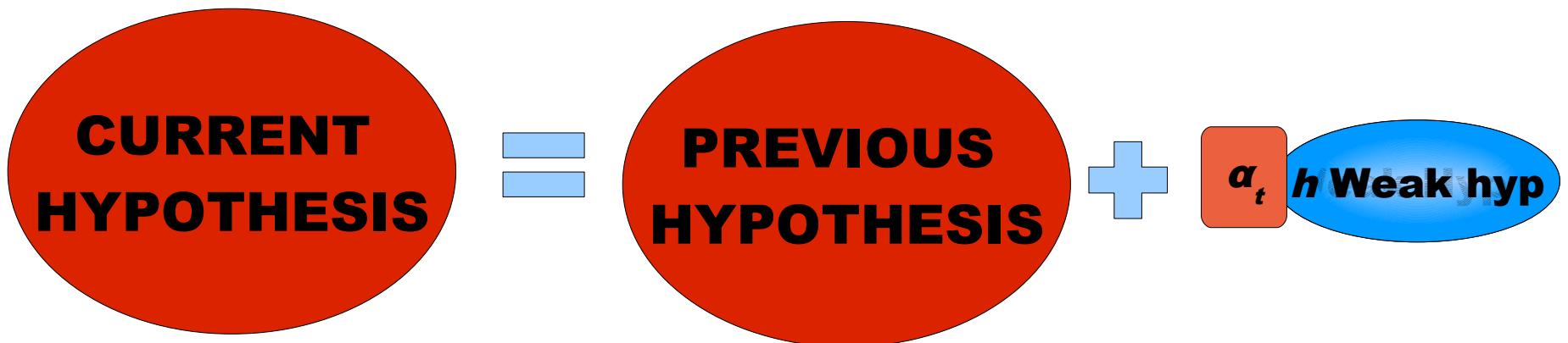
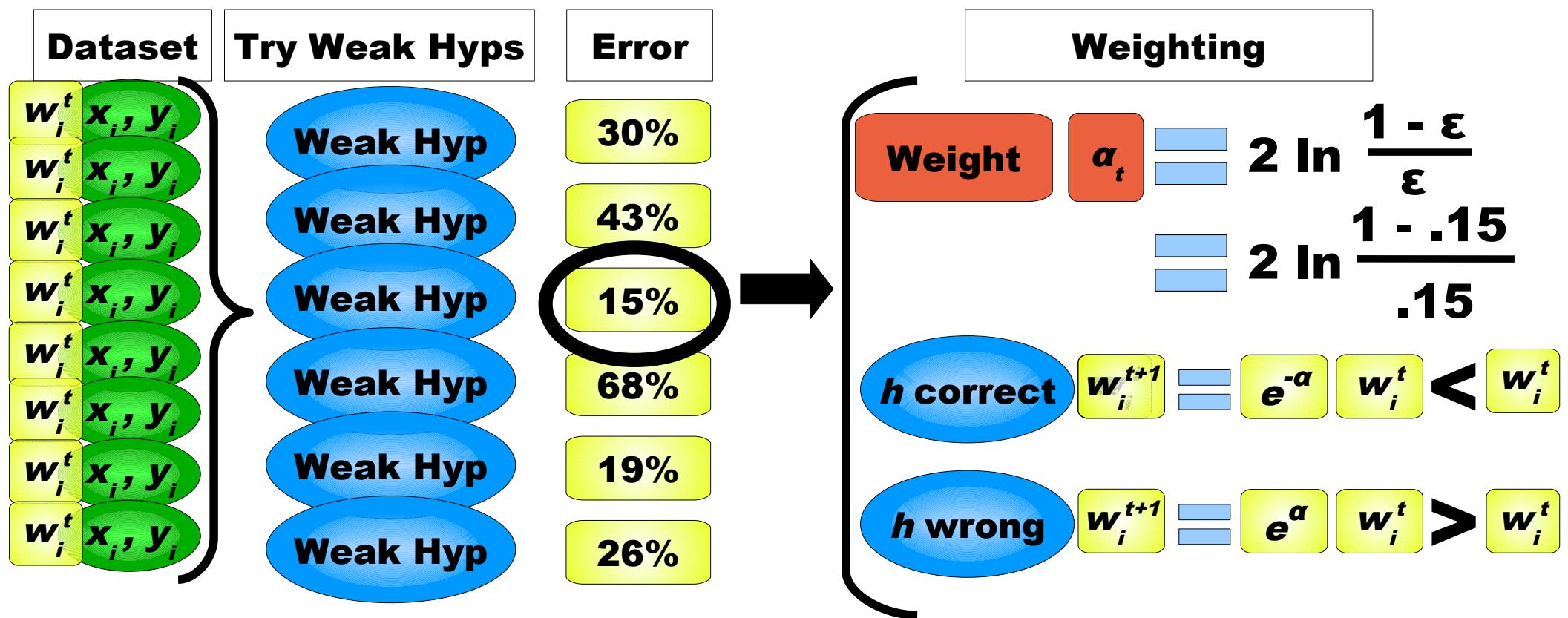
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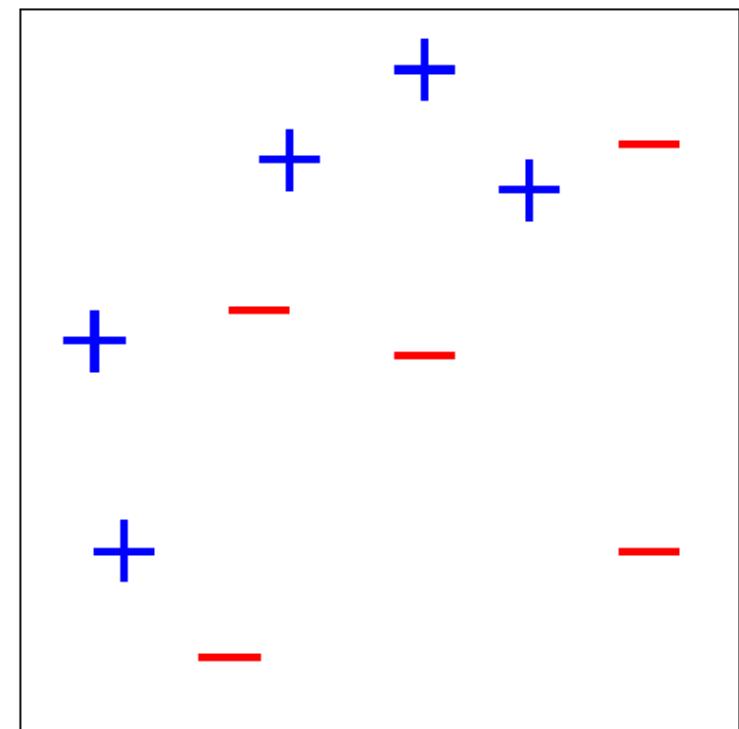


# One Iteration

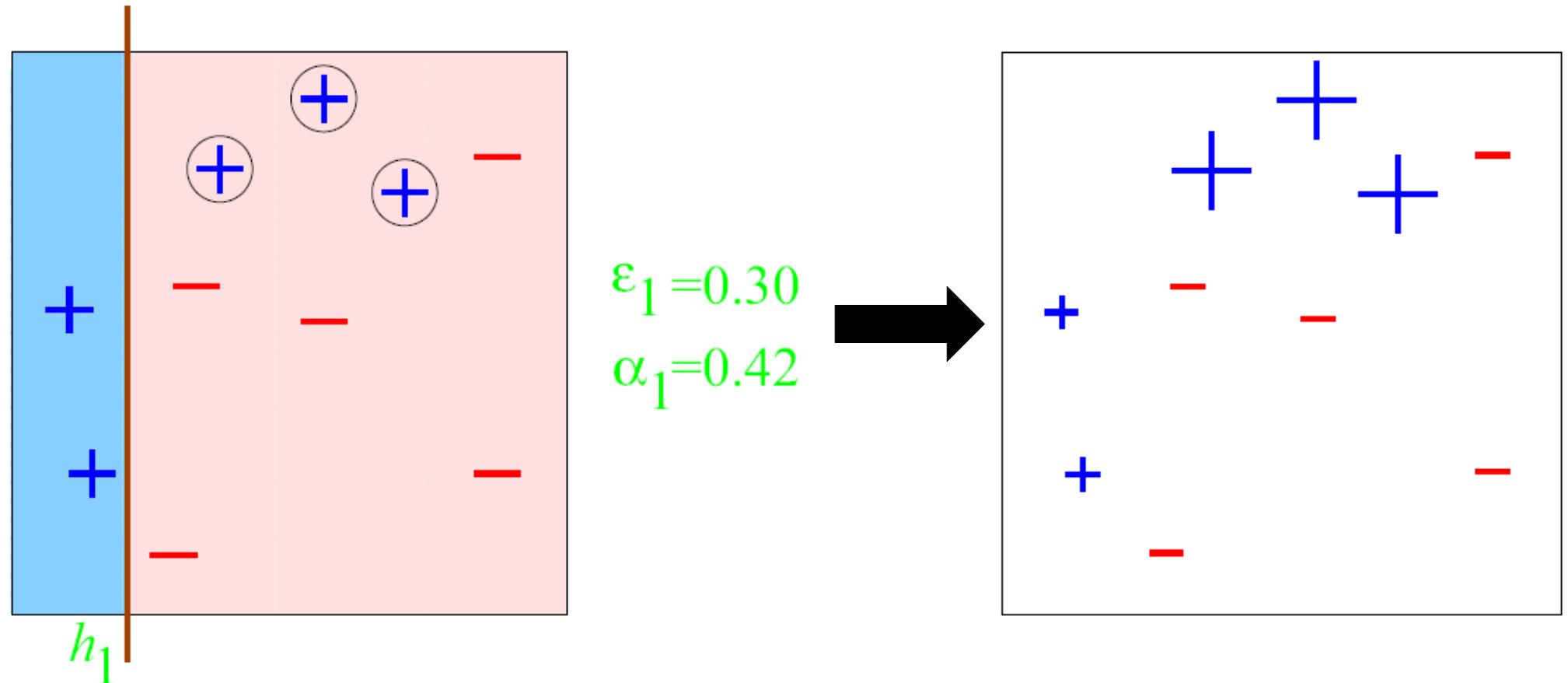


# Toy Example

- Positive examples
- Negative examples
- 2-Dimensional plane
- Weak hyps: linear separators
- 3 iterations

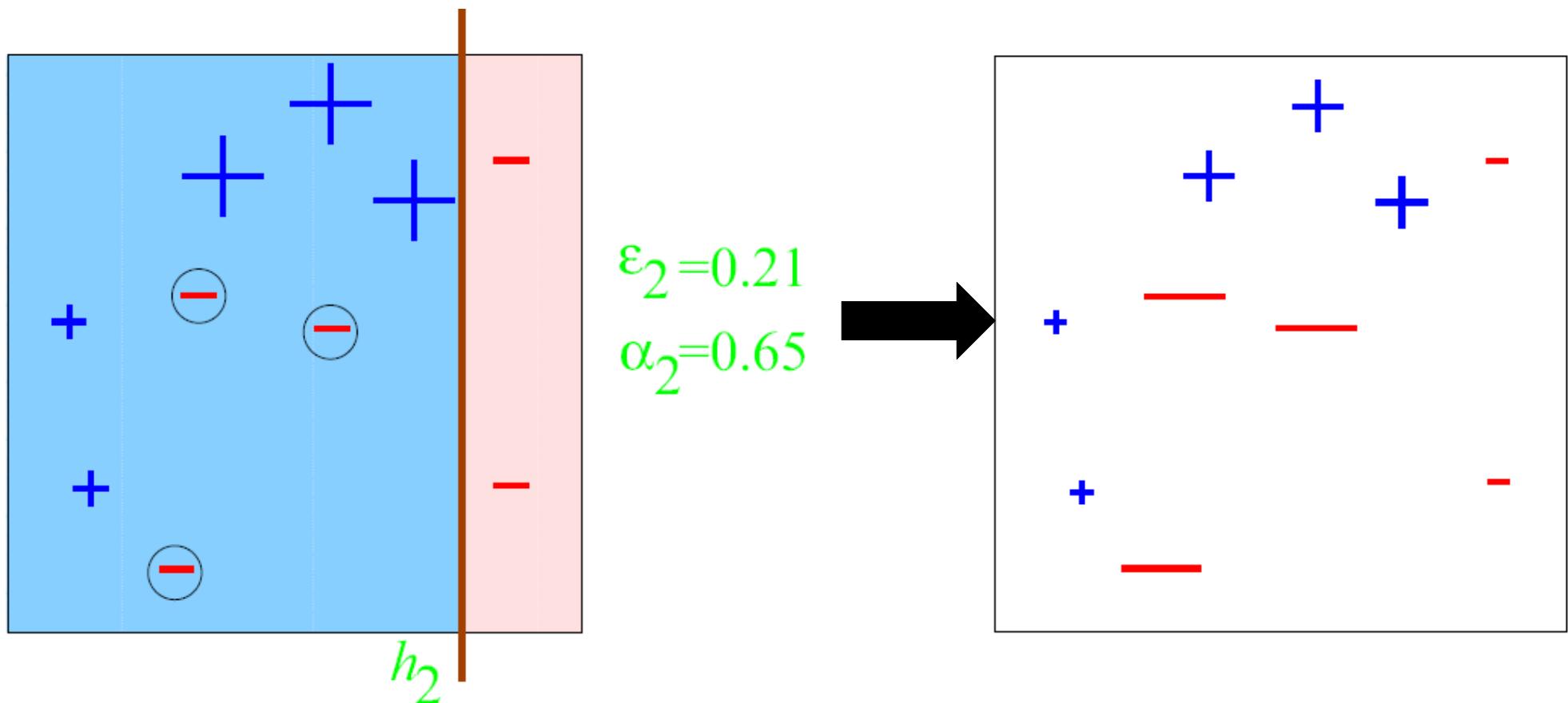


# Toy Example: Iteration 1



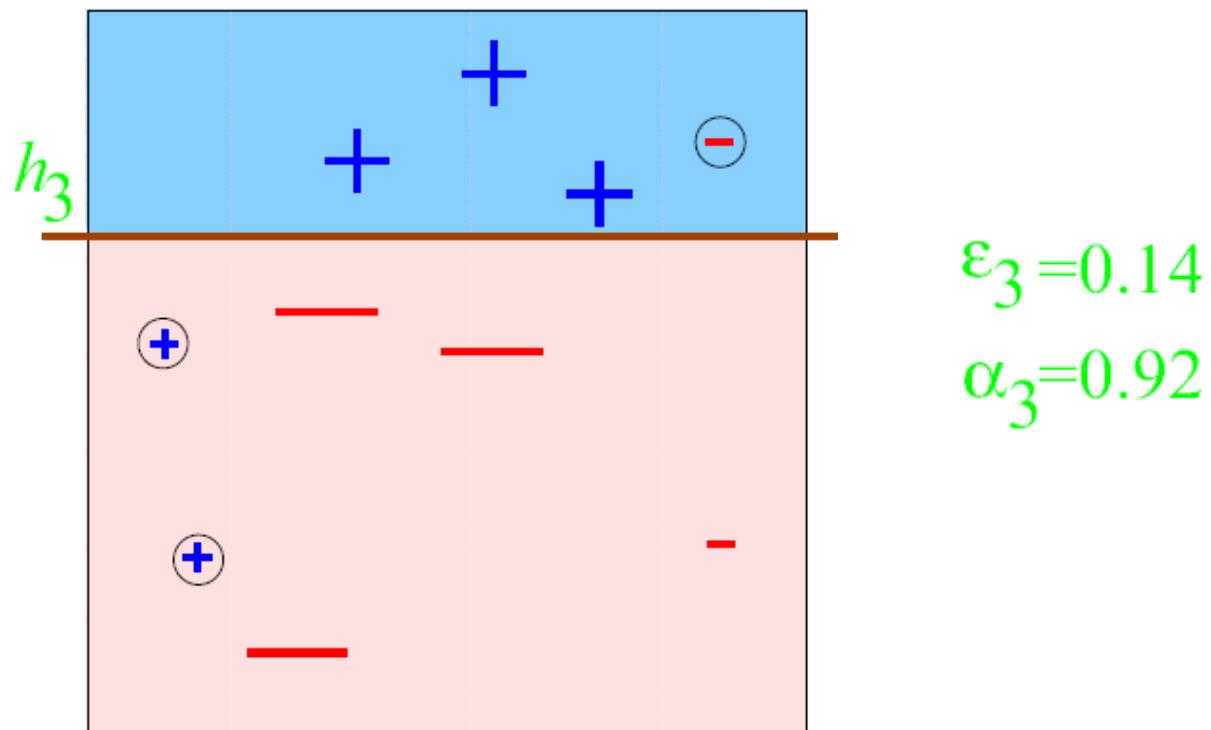
Misclassified examples are circled, given more weight

# Toy Example: Iteration 2



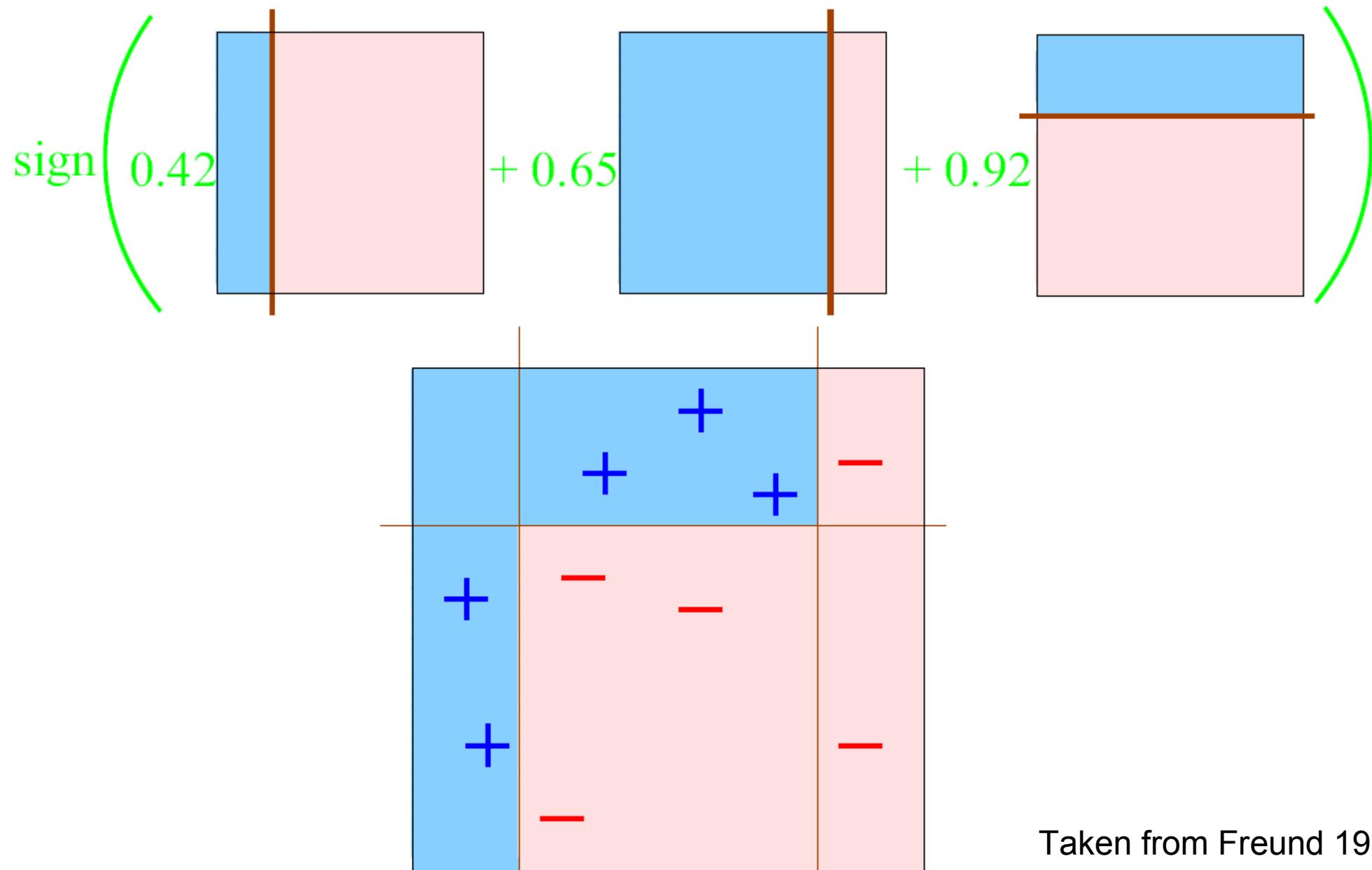
Misclassified examples are circled, given more weight

# Toy Example: Iteration 3



Finished boosting

# Toy Example: Final Classifier



# Questions

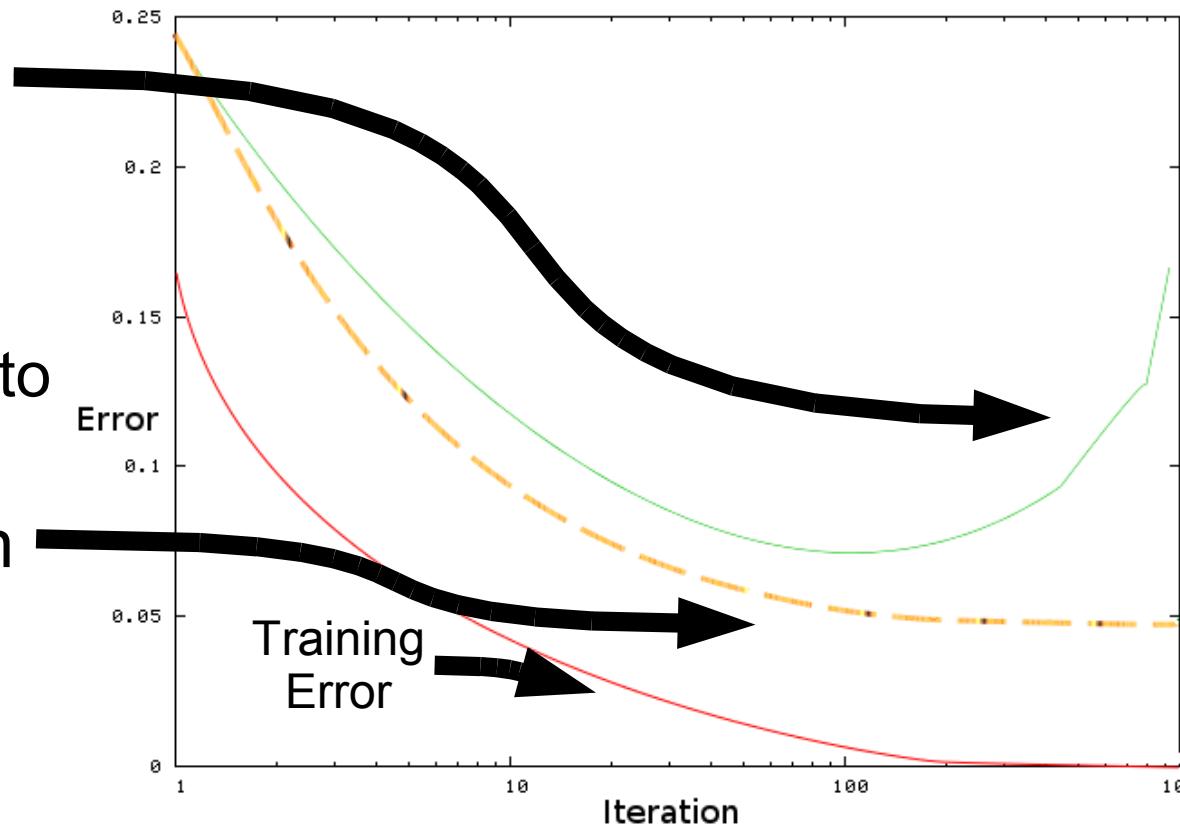
- How should we weight the hypotheses?
- How should we weight the examples?
- How should we choose the “best” hypothesis?
- How should we add the new (this iteration) hypothesis to the set of old hypotheses
- Should we consider old hypotheses when adding new ones?

# Answers

- There are many answers to these questions
- Freund & Schapire 1997 – AdaBoost
- Schapire & Singer 1999 – Confidence rated AdaBoost
- Freund 1995, 2000 – Noise resistant via binomial weights
- Friedman *et al* 1998 and Collins *et al* 2000 – Connections to logistic regression and Bregman divergences
- Warmuth et al 2006 – “Totally corrective” boosting
- Freund & Arvey 2008 – Asymmetric cost, boosting the normalized margin

# What's the big deal?

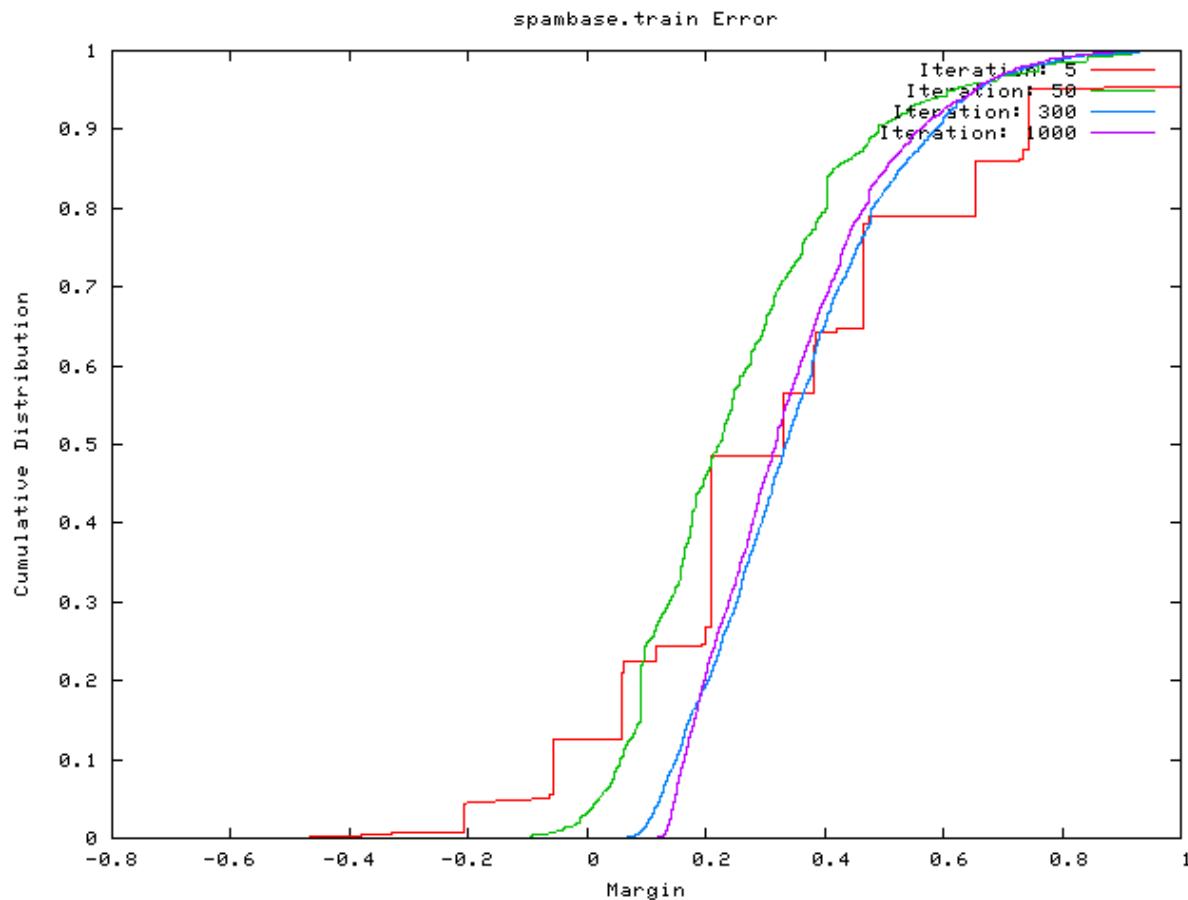
- Most algorithms start to memorize the data instead of learning patterns
- Most test error curves
  - Train decreases
  - Test starts to increase
  - Increase in test is due to “overfitting”
- Boosting continues to learn
  - Test error plateaus
- Explanation: margin



# What's the big deal?

- One goal in machine learning is “margin”
  - “Margin” is a measure of how correct an example is
  - If all hypotheses get an example right, we'll probably get a similar example right in the future
  - If 1 out of 1000 hypotheses get an example right, then we'll probably get it wrong in the future
  - Boosting gives us a good margin

# Margin Plot



- Margin frequently converges to some cumulative distribution function (CDF)
- Rudin et al. show that CDF may not always converge

End Boosting Section

Start Final Classifier Section

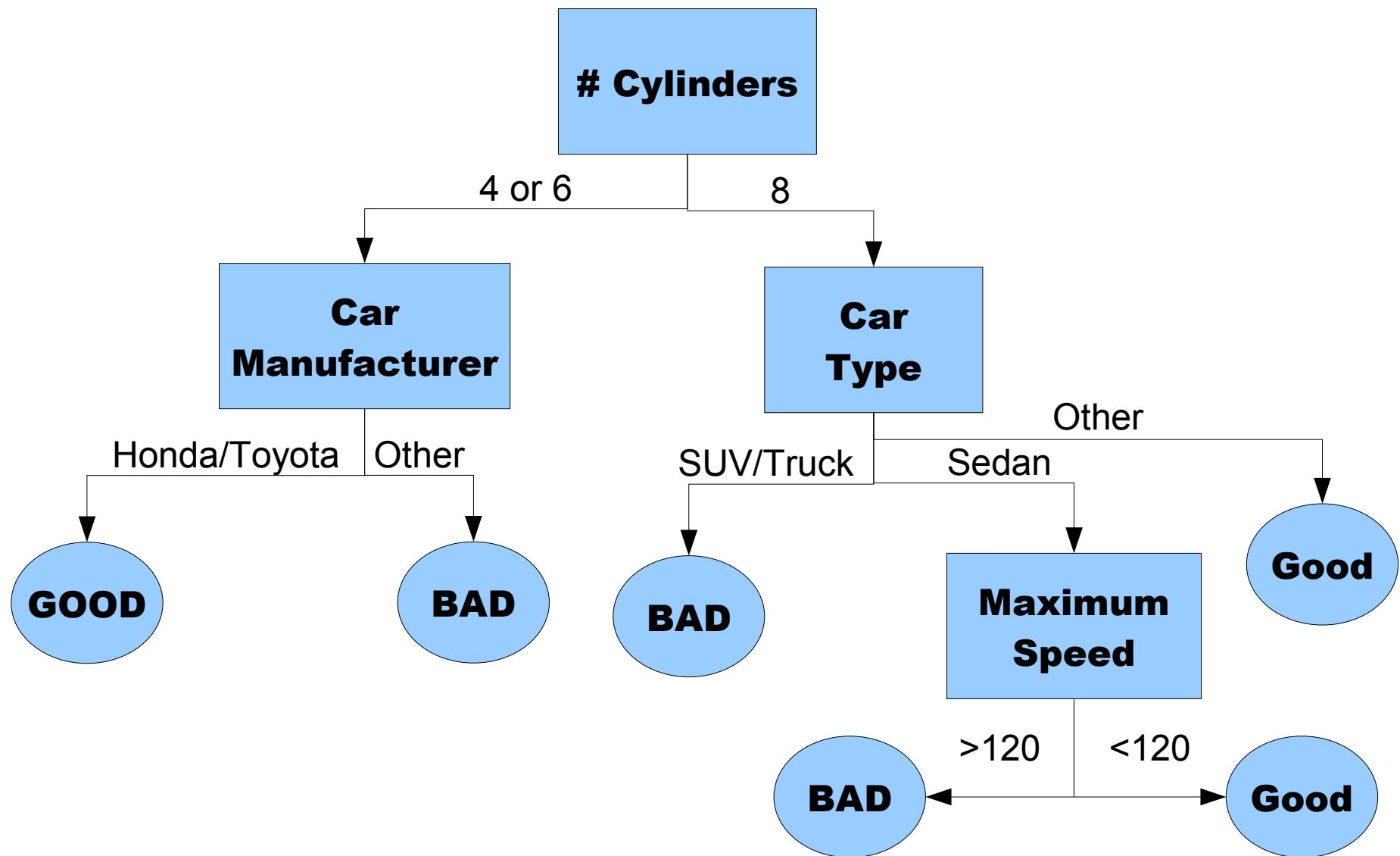
# Final Classifier: Combination of Weak Hypotheses

- Original usage of boosting was just adding many weak hypotheses
- Adding weak hyps could be improved
  - Some of the weak hypotheses may be correlated
  - If there are a lot of weak hypotheses, the decision can be very hard to visualize
- Why can't boosting be more like decision trees
  - Easy to understand and visualize
  - A classic approach used by many fields

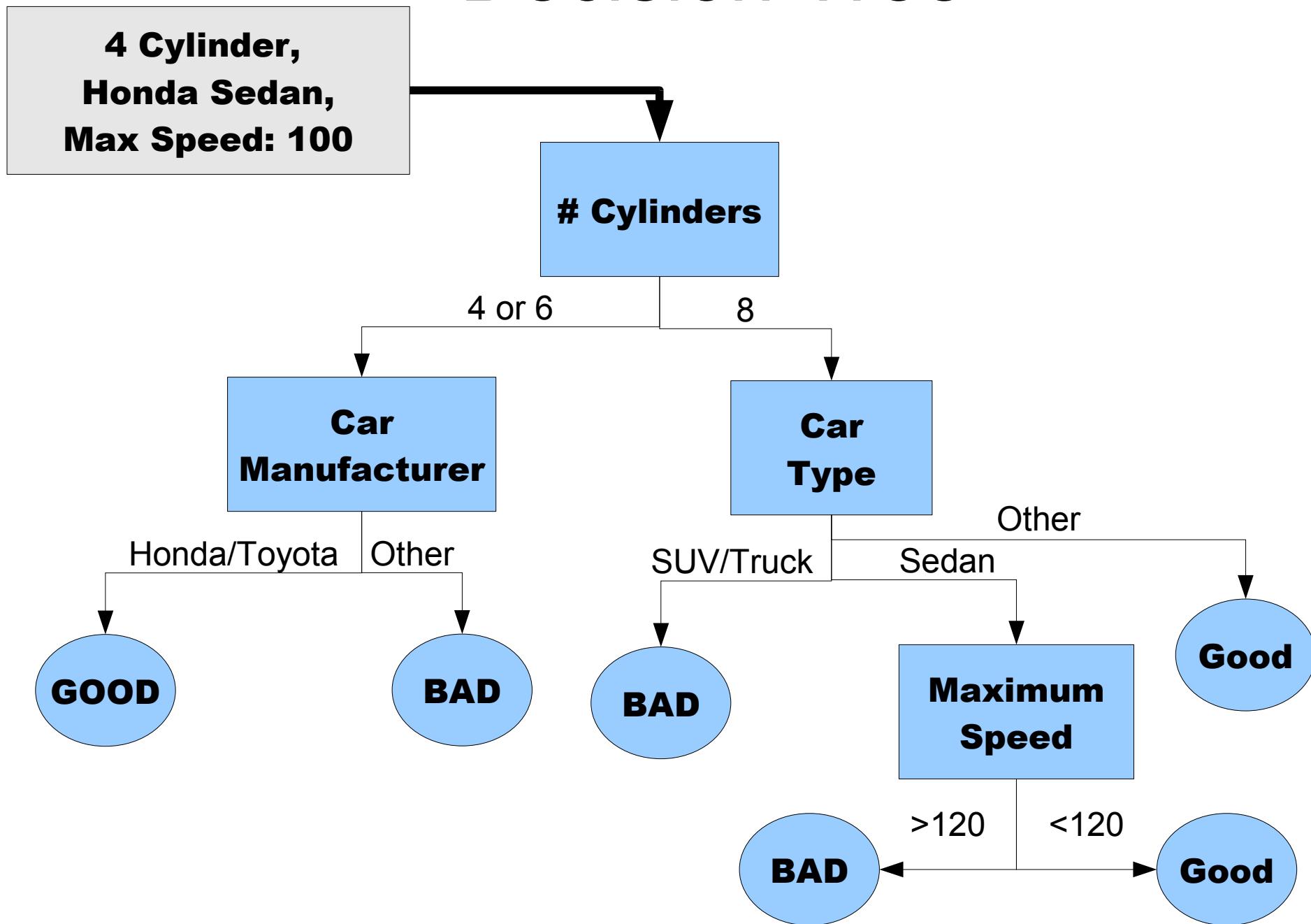
# Final Classifier: Decision Trees

- Follow a series of questions to a single answer
- Does the car have 4 or 8 cylinders?
  - If #cylinders=4 or 8, then was the car made in Asia?
    - If Yes then you get good gas mileage
    - If no then you get bad gas mileage
  - If #cylinders=3,5,6, or 7 then poor gas mileage

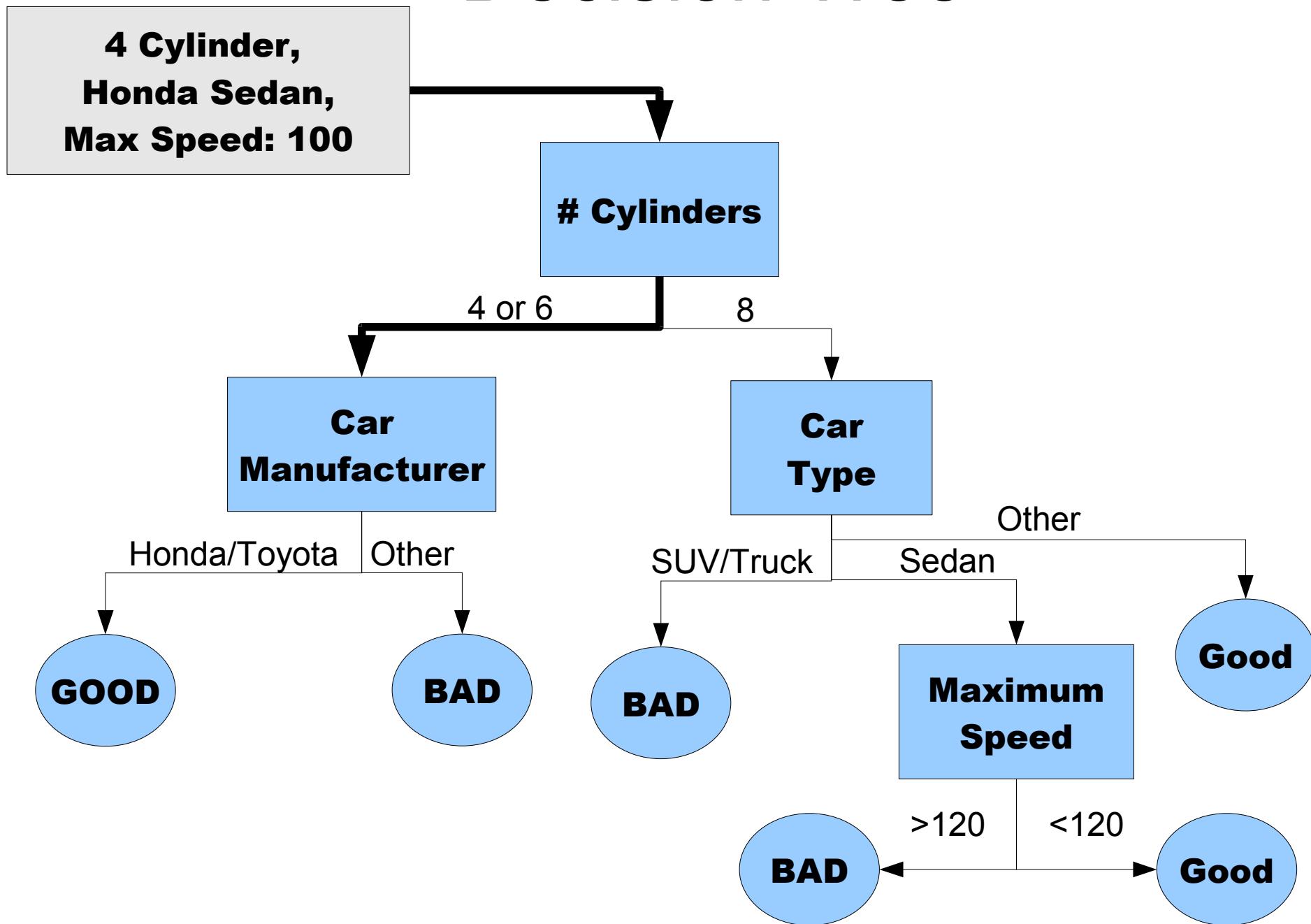
# Decision Tree



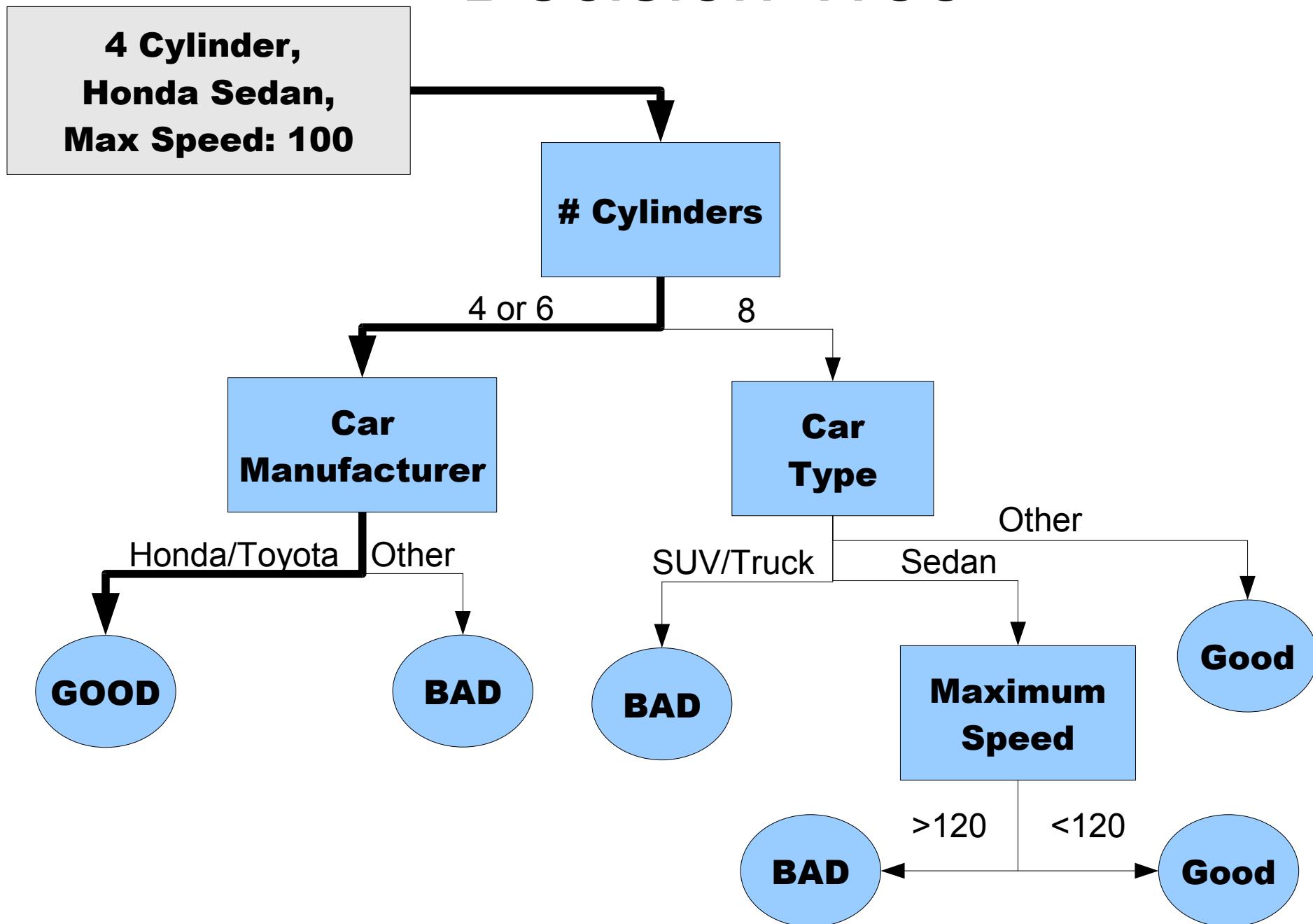
# Decision Tree



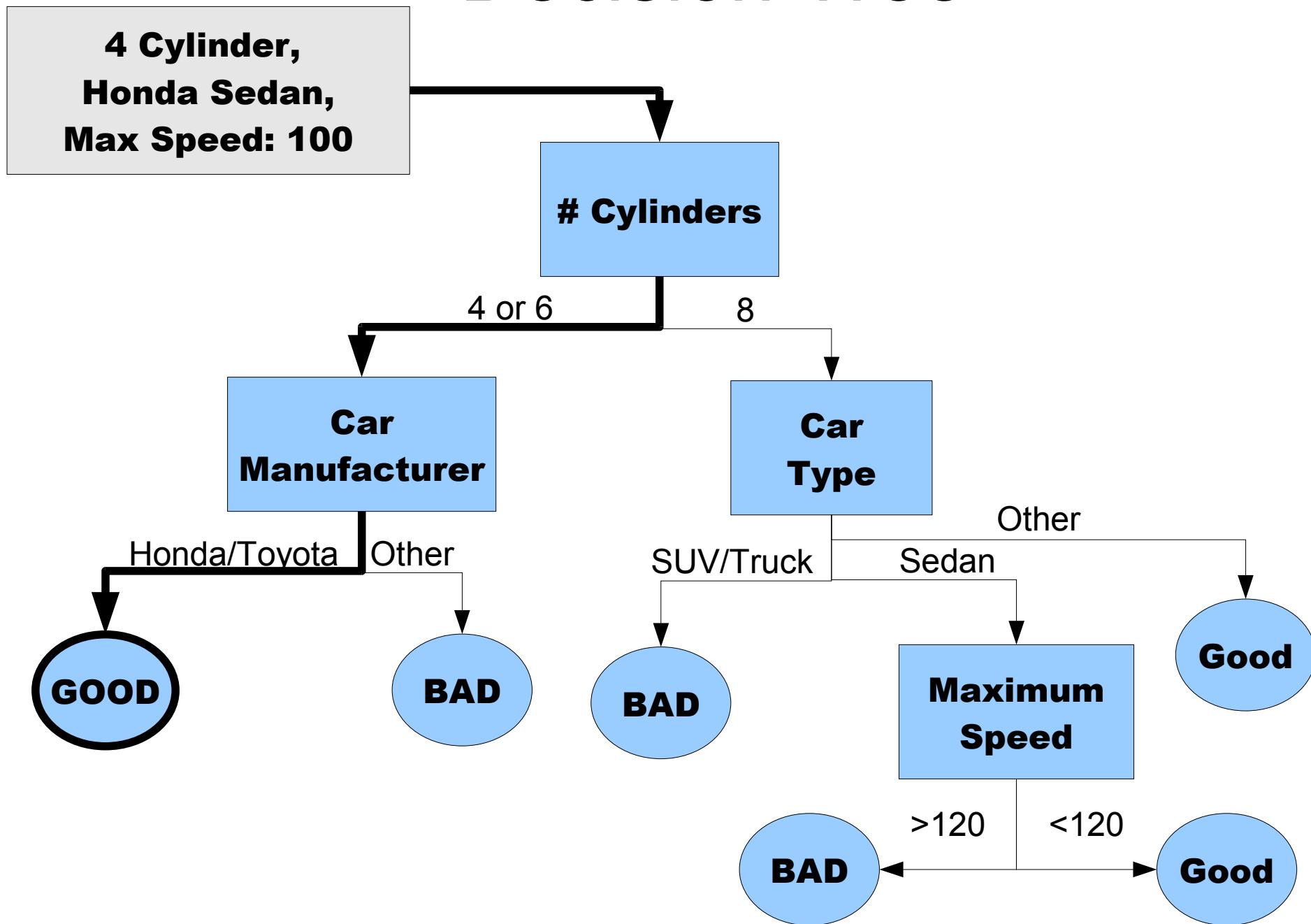
# Decision Tree



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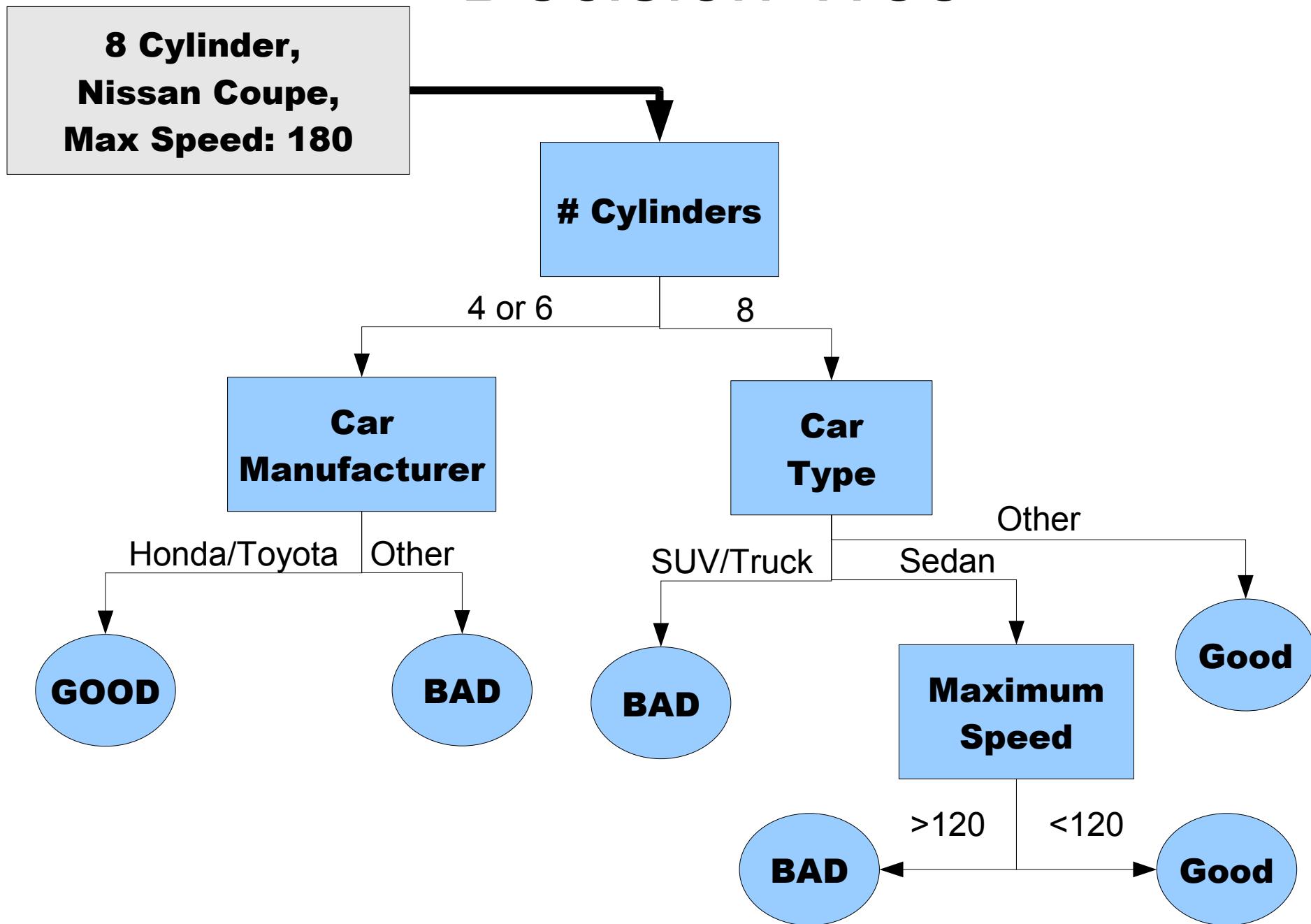


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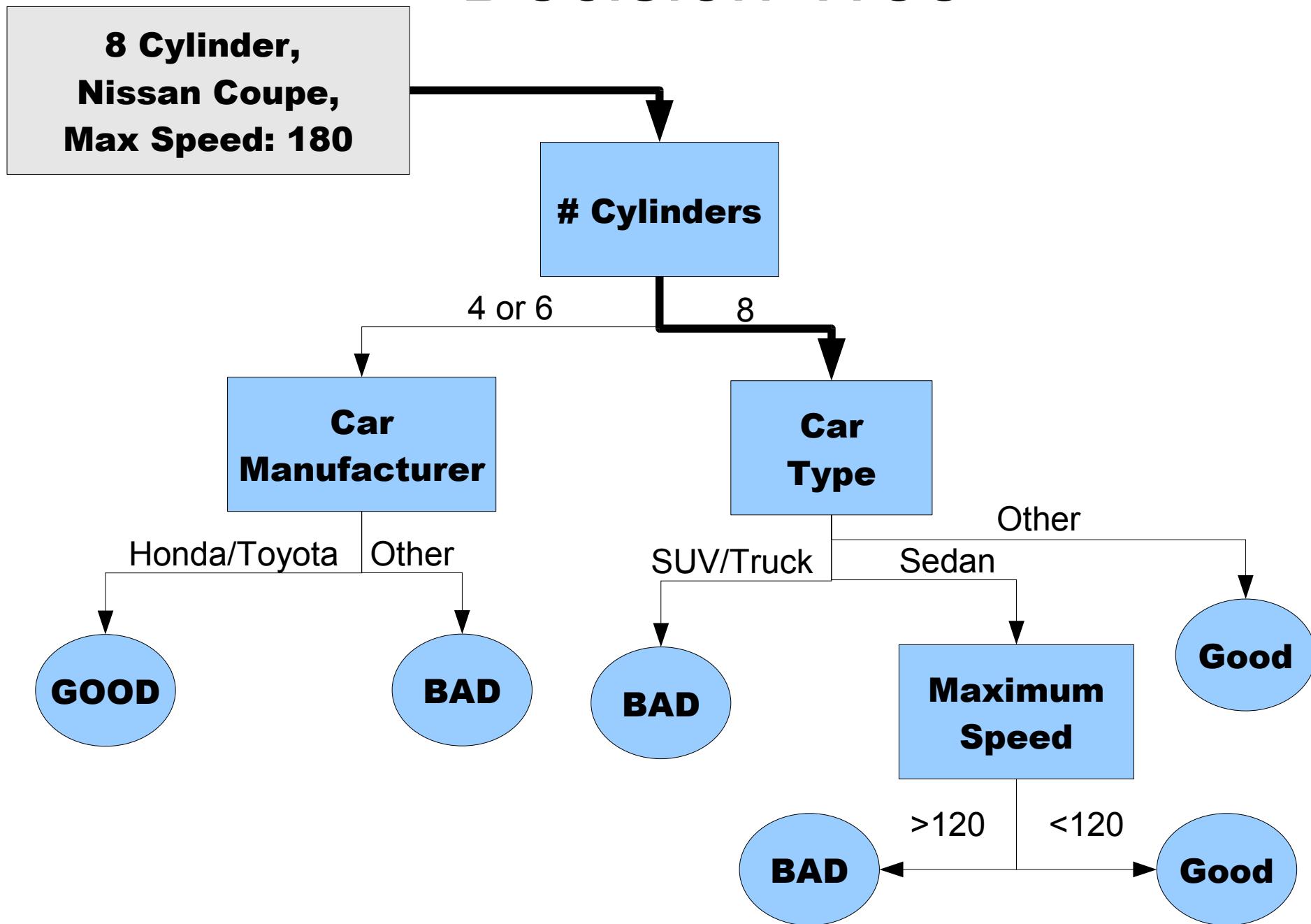


# Another Example

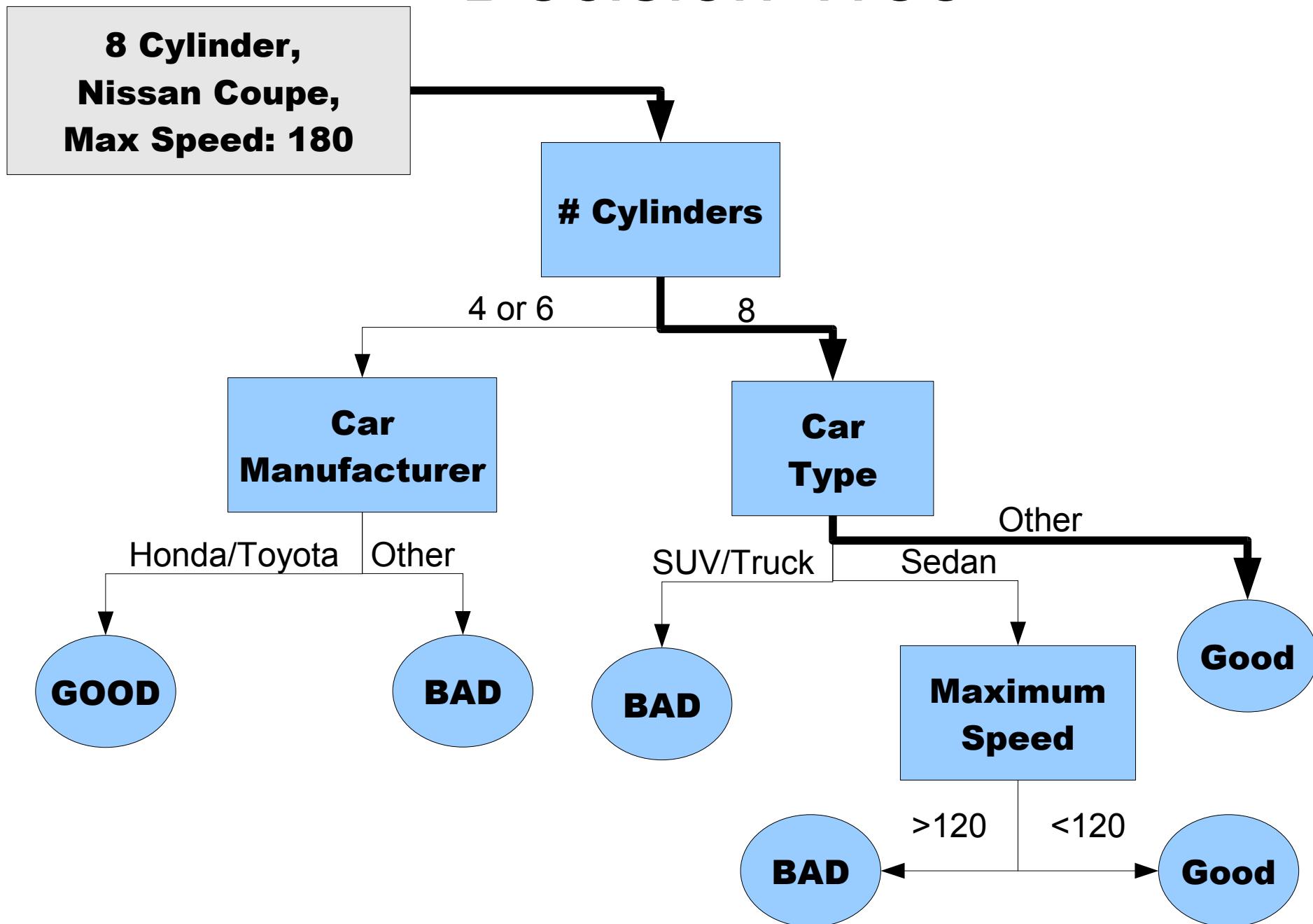
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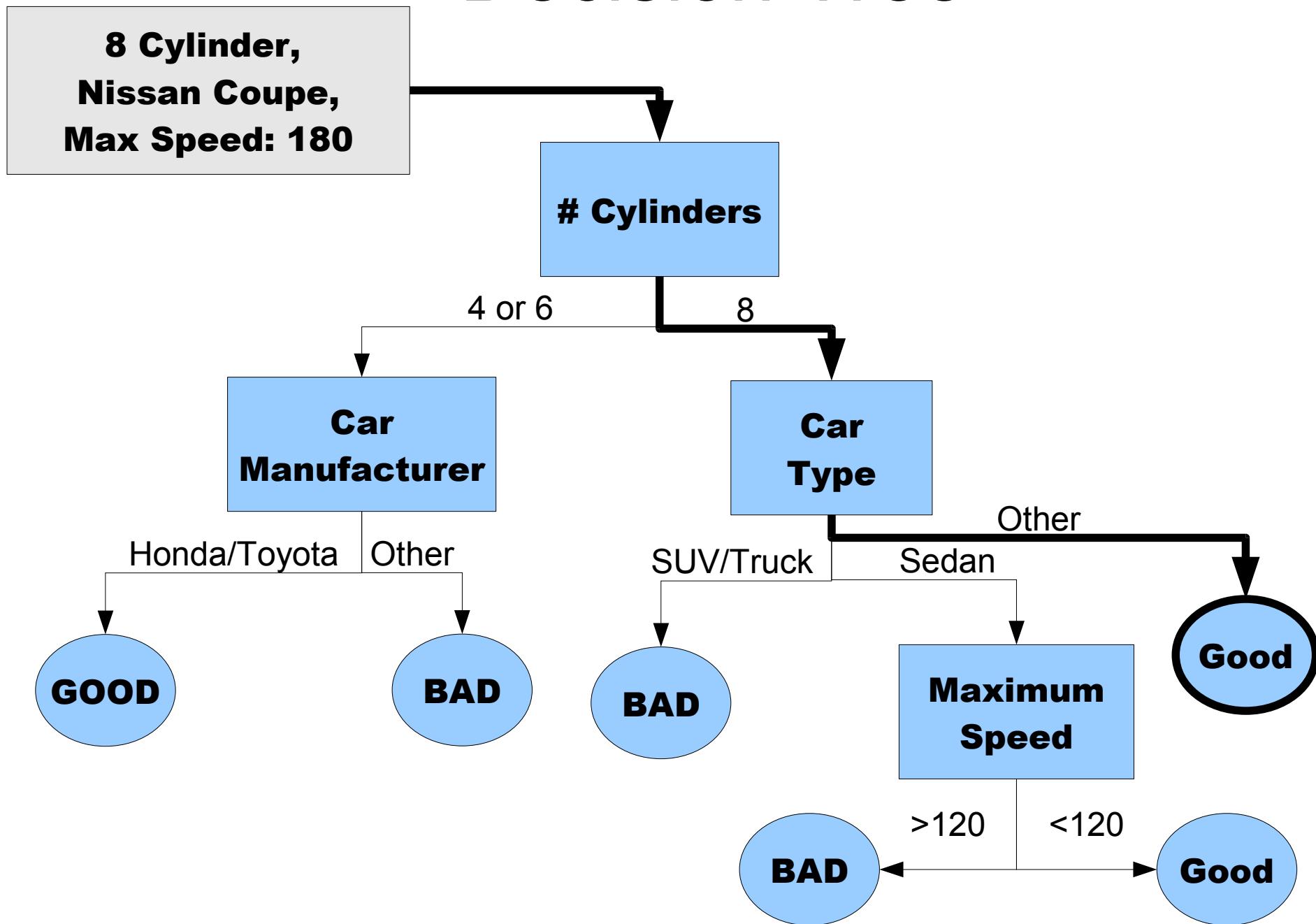
# Decision Tree



# Decision Tree



# Decision Tree



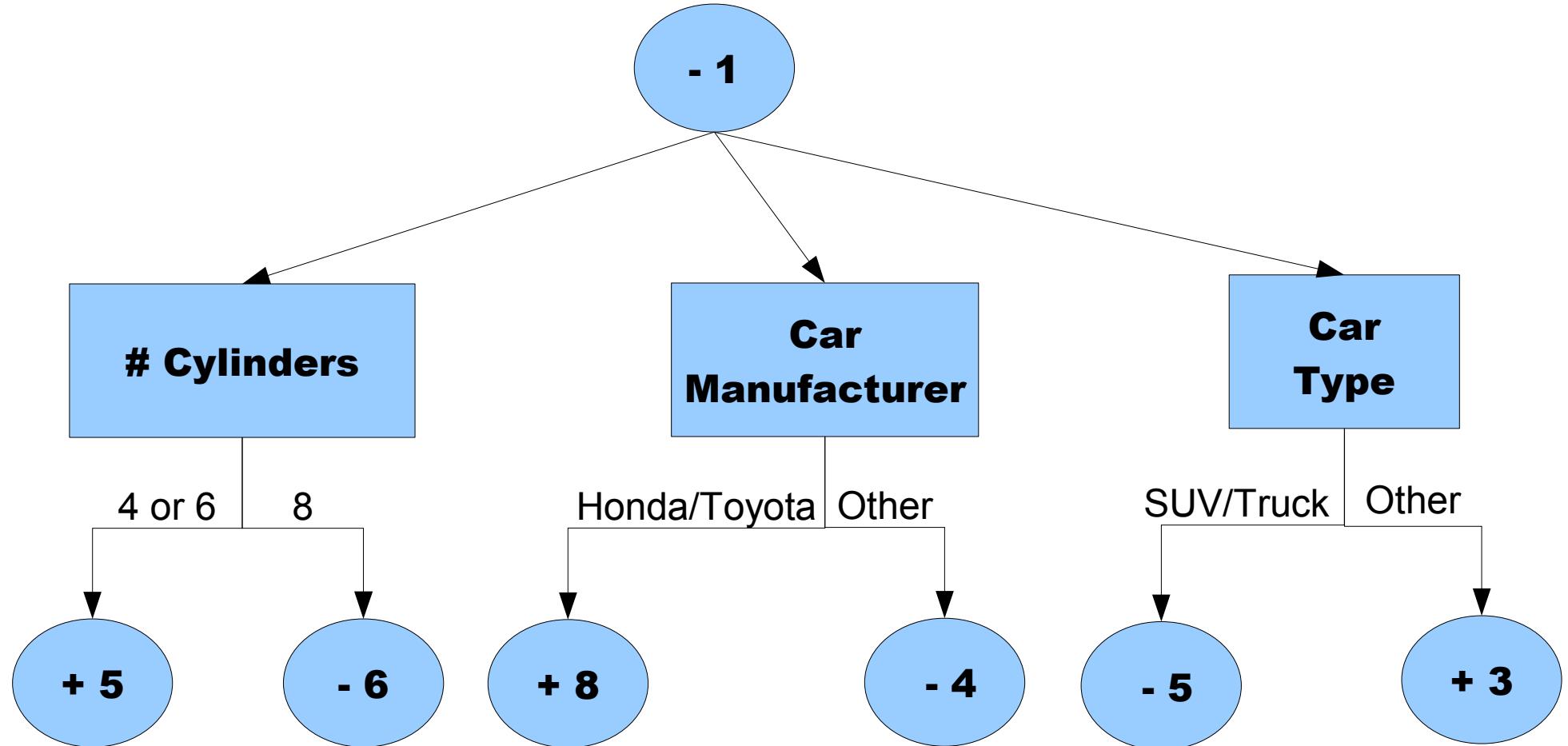
# Decision Trees

- Follow a single path until reach decision
- No confidence levels
- Many criterion for growing decision trees

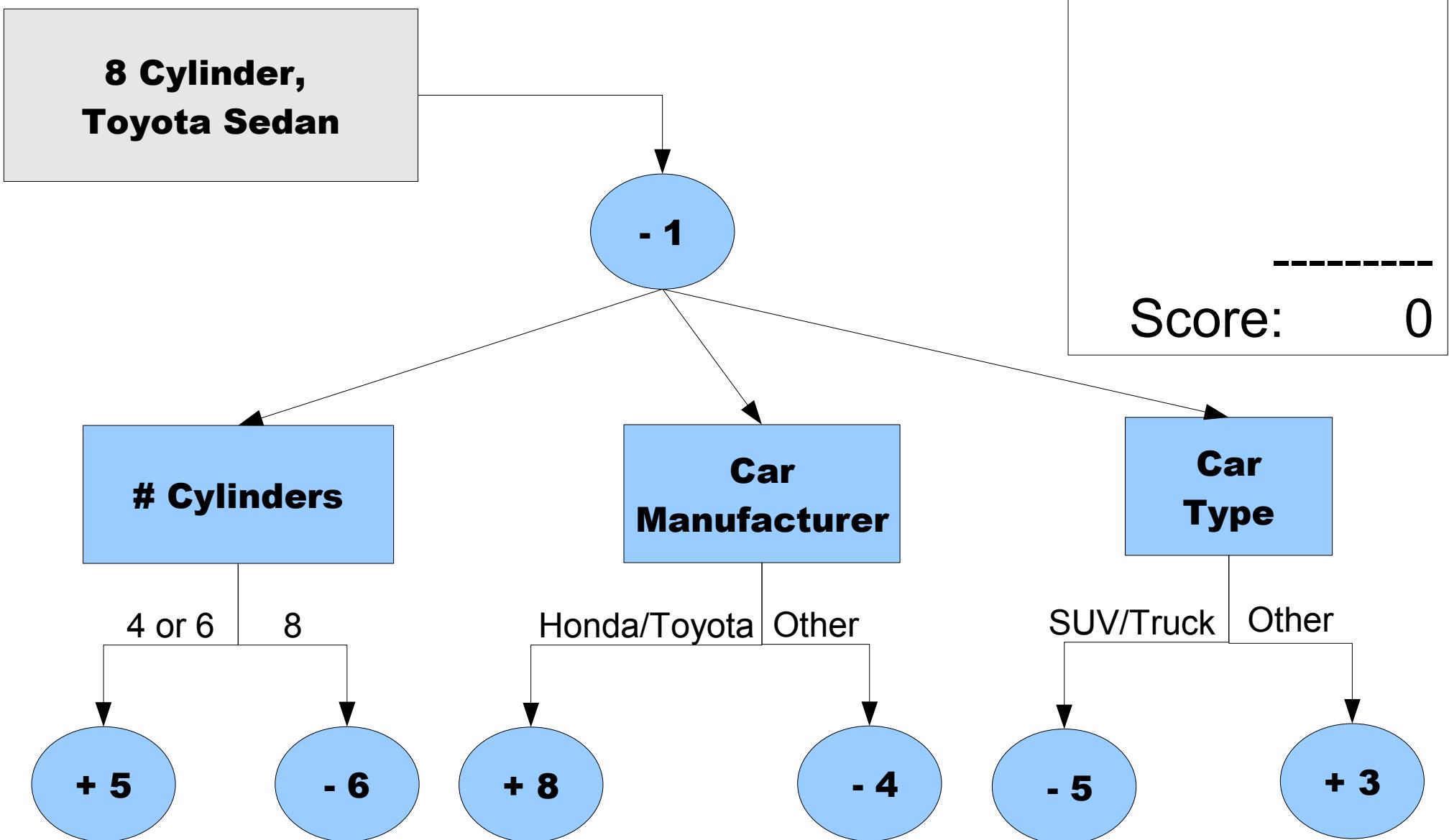
# Final Classifier: Alternating Decision Tree

- Each path in tree is series of weak hypotheses
- Does the car have 4 or 6 cylinders?
  - Yes => +5, No => -6
- Is the car a Toyota or Honda?
  - Yes =>+8, No => -3
- A Honda with 8 cylinders => +2

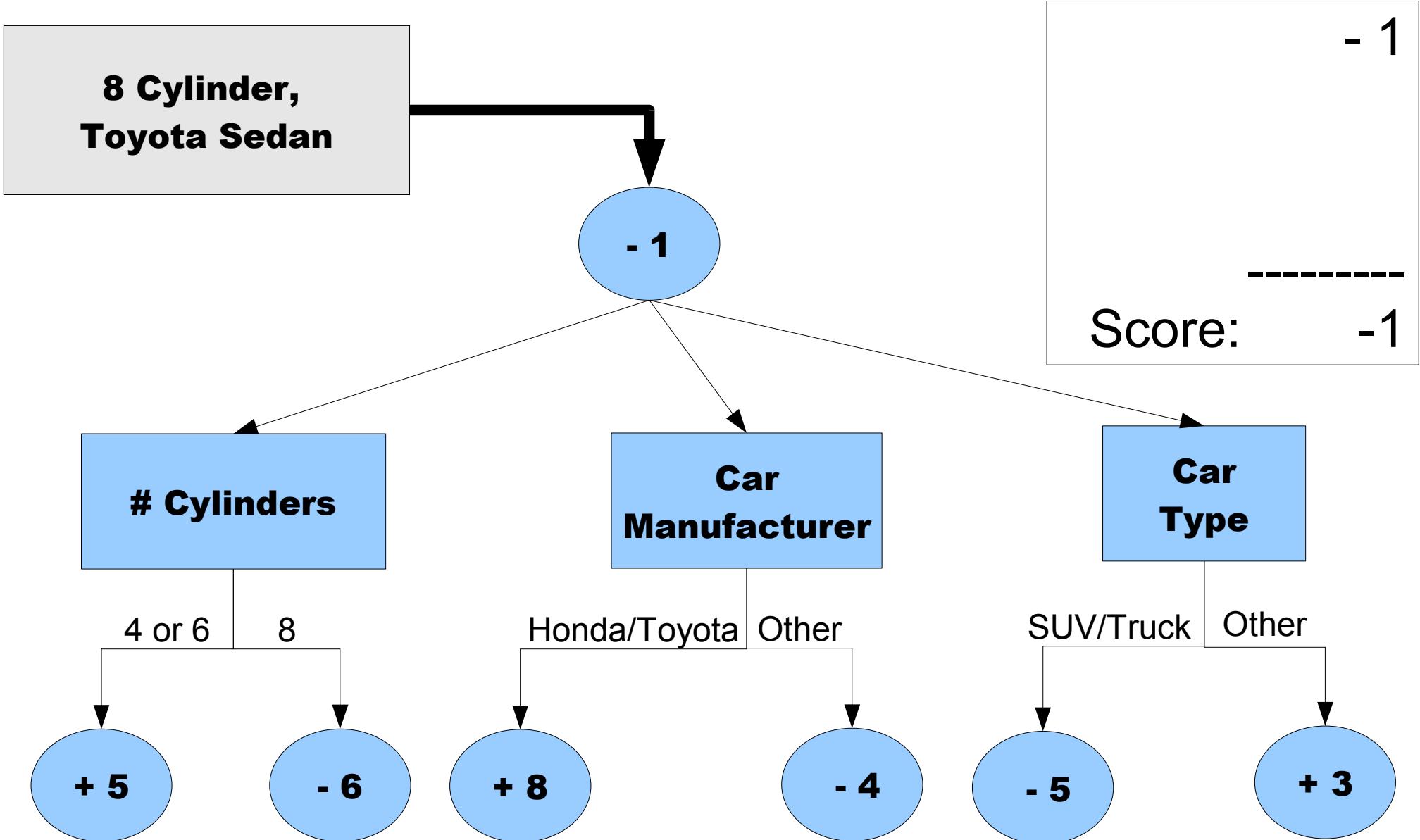
# Alternating Decision Tree



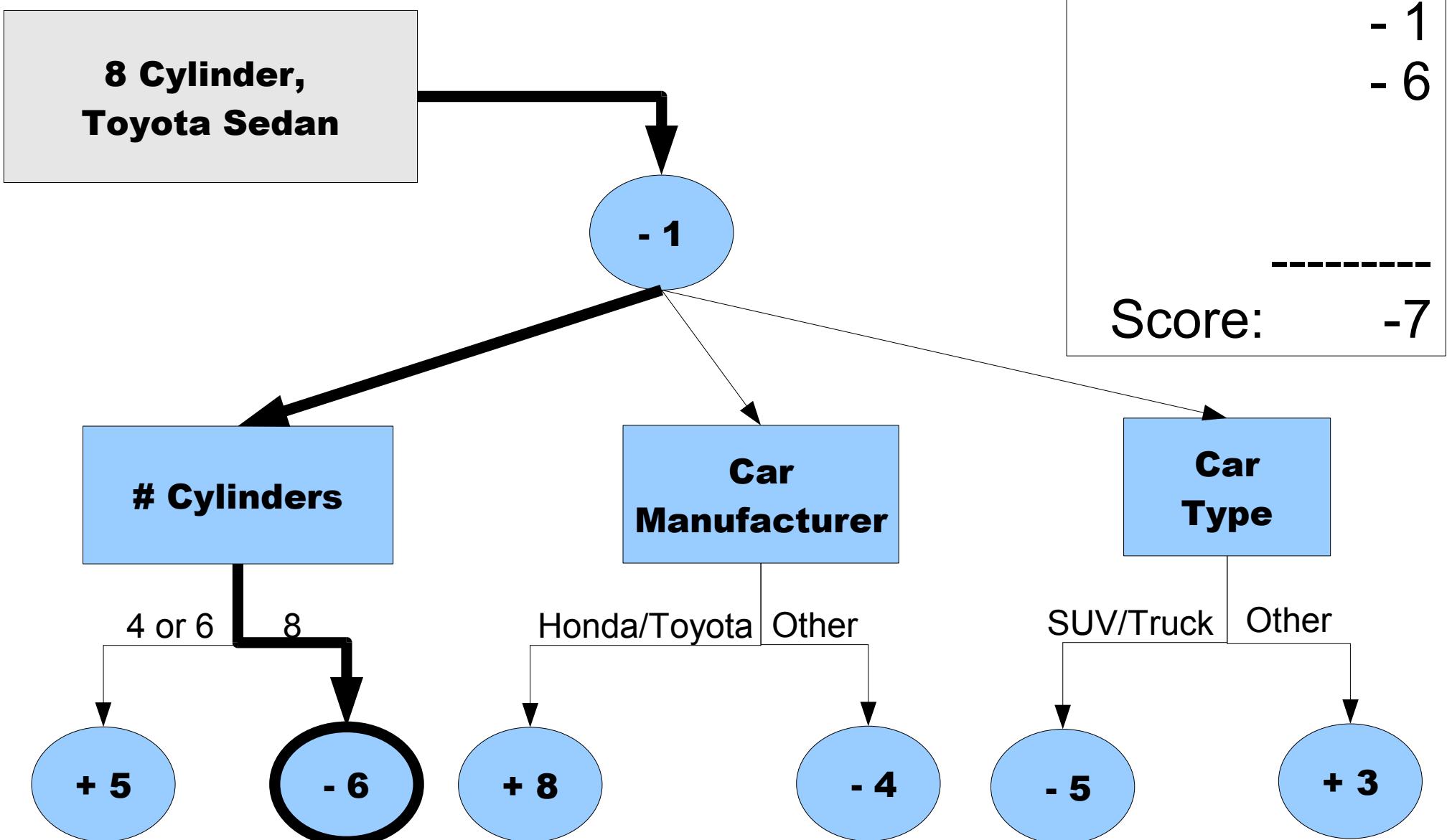
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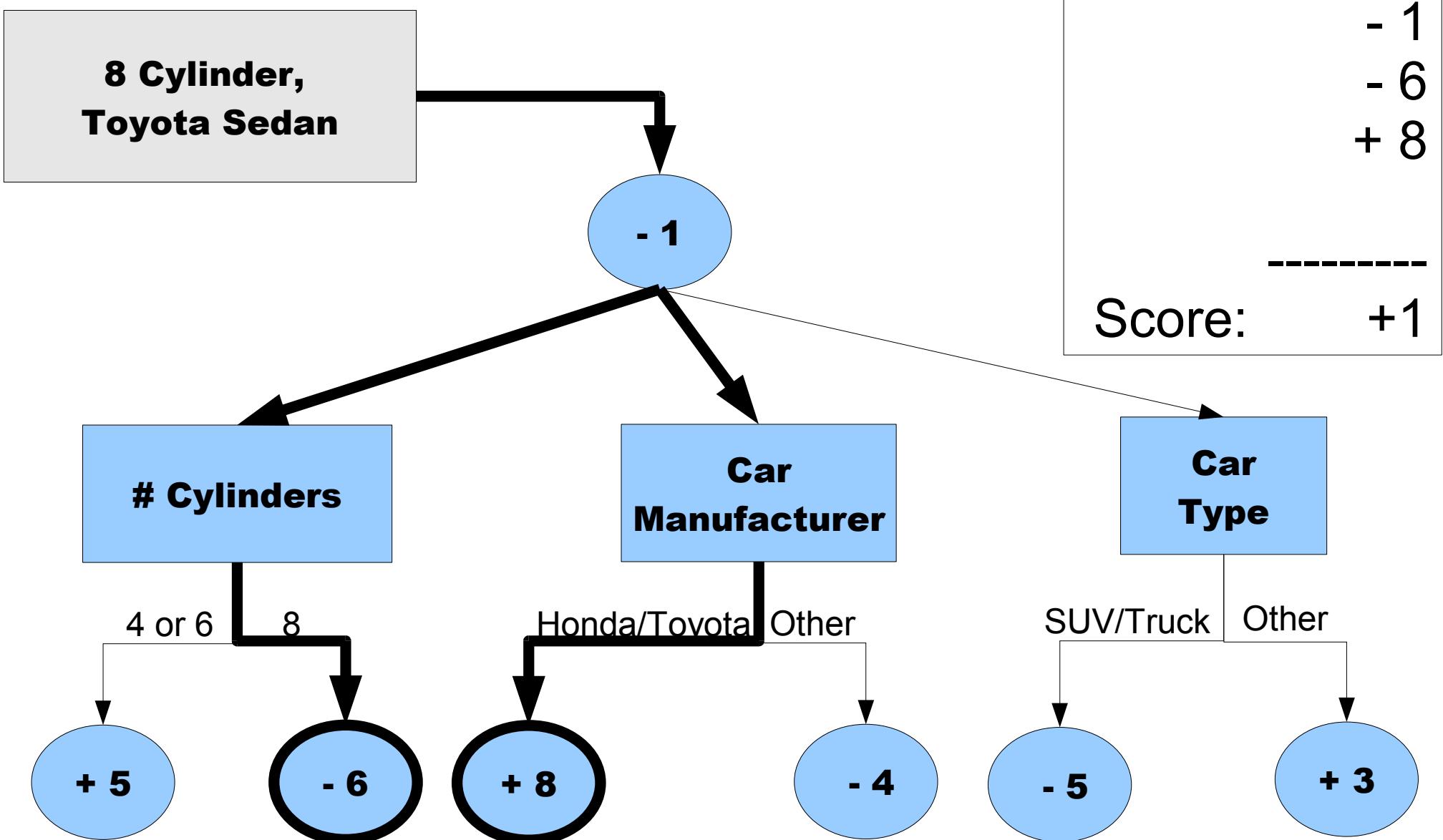
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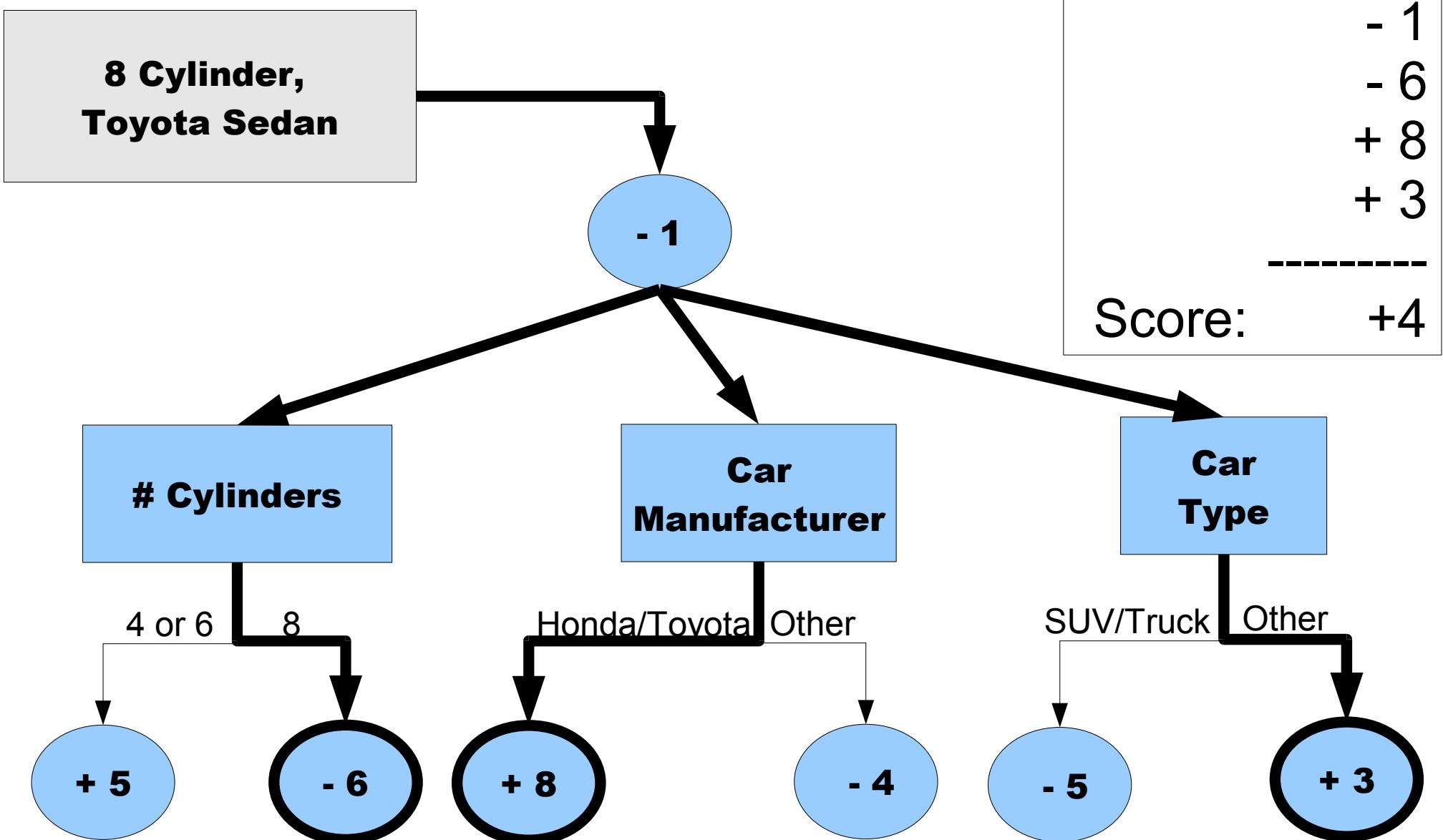
# Alternating Decision Tree



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# Alternating Decision Tree



# Another Example

- Previous example was pretty simple
  - Just a series of decisions with weights
  - A basic additive linear model
- Next example shows a more interesting ATree
  - Has greater depth
  - Some weak hypotheses abstain
- Two inputs are shown

**8 Cylinder,  
Nissan Sedan,  
Max Speed: 180**

- 1

- 1

**# Cylinders**

**Car  
Manufacturer**

**Score:**

- 1

4 or 6      8

Honda/Toyota      Other

+ 5

- 6

+ 8

- 4

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

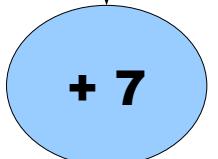
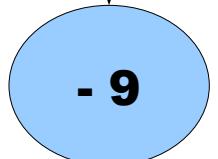
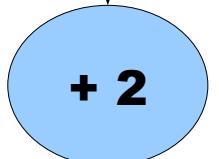
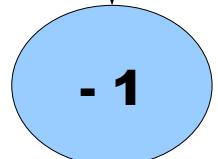
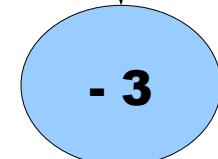
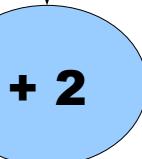
- 3

- 1

+ 2

- 9

+ 7



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Nissan Sedan,  
Max Speed: 180**

- 1  
- 6

- 1

**# Cylinders**

**Car  
Manufacturer**

Score: - 7

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

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**Max Speed**

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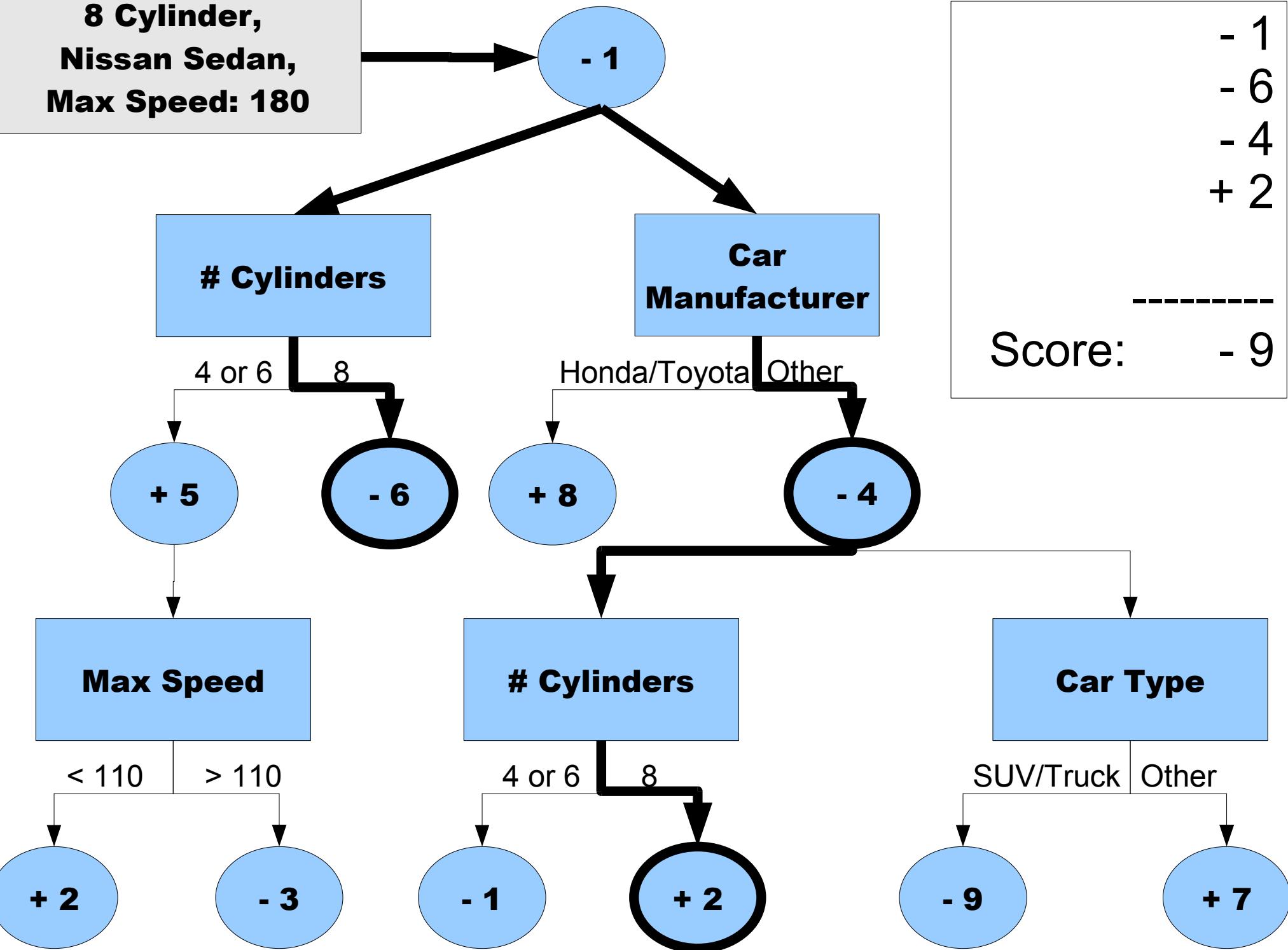
- 3

-

**8 Cylinder,  
Nissan Sedan,  
Max Speed: 180**

- 1  
- 6  
- 4  
+ 2

Score: - 9



# Another Example

**8 Cylinder,  
Honda SUV,  
Max Speed: 90**



- 1

**# Cylinders**

**Car  
Manufacturer**

- 1  
-----  
**Score:** - 1

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

**Max Speed**

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SUV/Truck

Other

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**8 Cylinder,  
Honda SUV,  
Max Speed: 90**



- 1

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-----

**8 Cylinder,  
Honda SUV,  
Max Speed: 90**

- 1

**# Cylinders**

**Car  
Manufacturer**

- 1  
- 6  
+ 8

Score: + 1

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

+ 2

- 3

- 1

+ 2

- 9

+ 7

4 or 6

8

SUV/Truck

Other

# Another Example

**4 Cylinder,  
Honda SUV,  
Max Speed: 90**



- 1

**# Cylinders**

**Car  
Manufacturer**

- 1  
-----  
**Score:** - 1

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**4 Cylinder,  
Honda SUV,  
Max Speed: 90**



- 1

**# Cylinders**

**Car  
Manufacturer**

- 1  
+ 5

-----  
Score: + 4

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

- 1

+ 2

- 9

+ 7

**4 Cylinder,  
Nissan SUV,  
Max Speed: 90**



- 1

**# Cylinders**

**Car  
Manufacturer**

- 1  
+ 5  
+ 2

-----  
Score: + 6

4 or 6

8

Honda/Toyota

Other

+ 5

- 6

+ 8

- 4

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

4 or 6

8

SUV/Truck

Other

+ 2

- 3

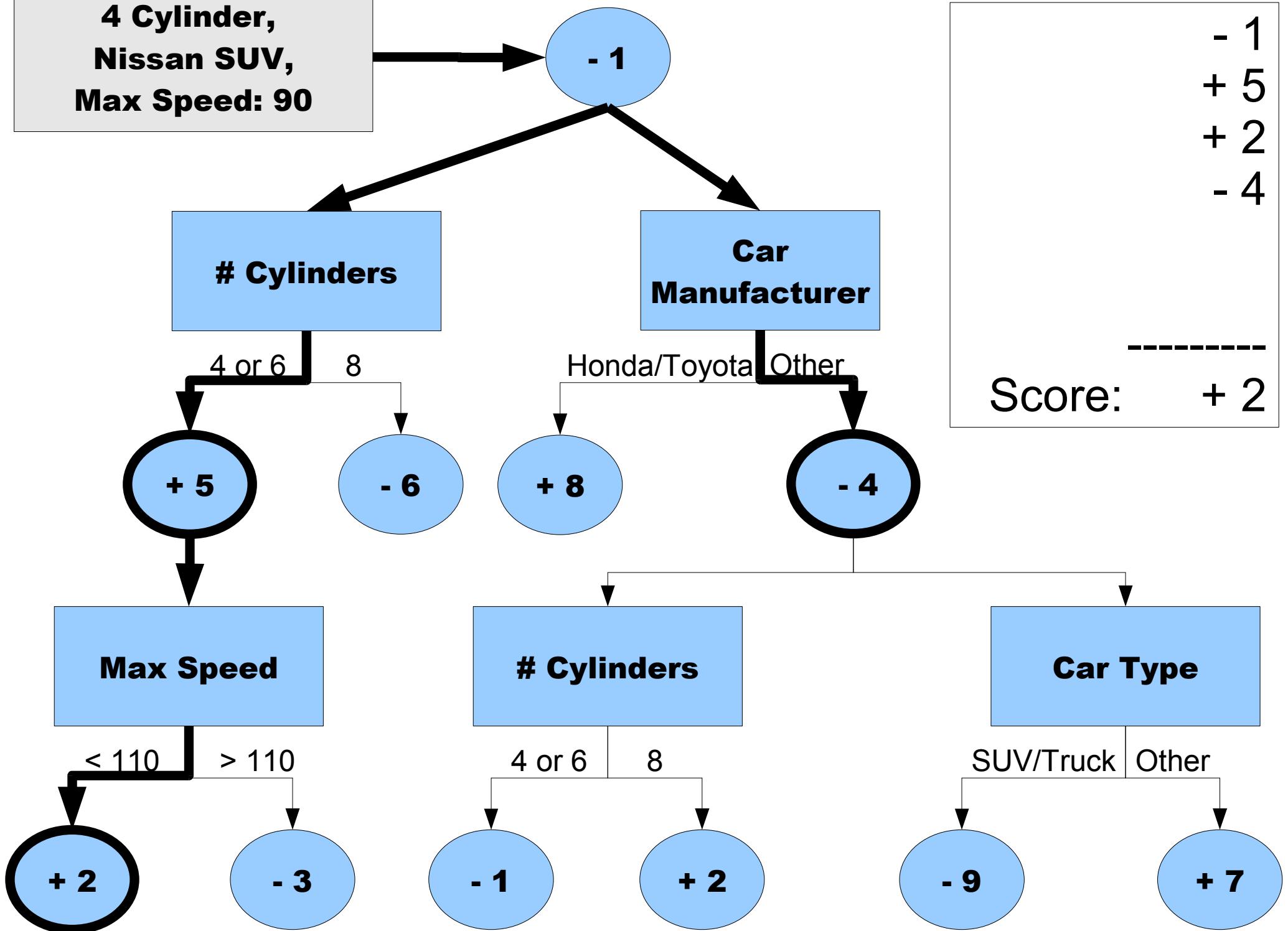
- 1

+ 2

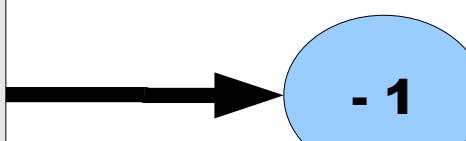
- 9

+ 7

**4 Cylinder,  
Nissan SUV,  
Max Speed: 90**



**4 Cylinder,  
Nissan SUV,  
Max Speed: 90**



- 1  
+ 5  
+ 2  
- 4  
- 1  
+ 7

Score: + 8

**# Cylinders**

**Car  
Manufacturer**

4 or 6      8

Honda/Toyota      Other

**+ 5**

**- 6**

**+ 8**

**- 4**

**Max Speed**

**# Cylinders**

**Car Type**

< 110

> 110

4 or 6      8

SUV/Truck      Other

**+ 2**

**- 3**

**- 1**

**+ 2**

**- 9**

**+ 7**

# ATree Pros and Cons

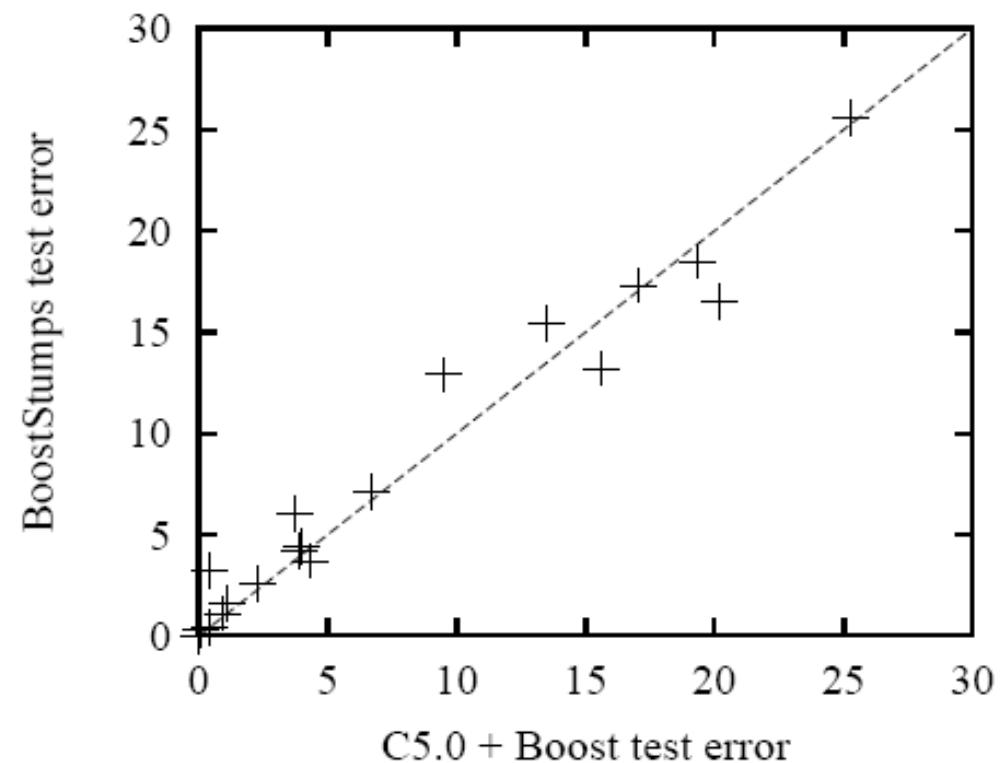
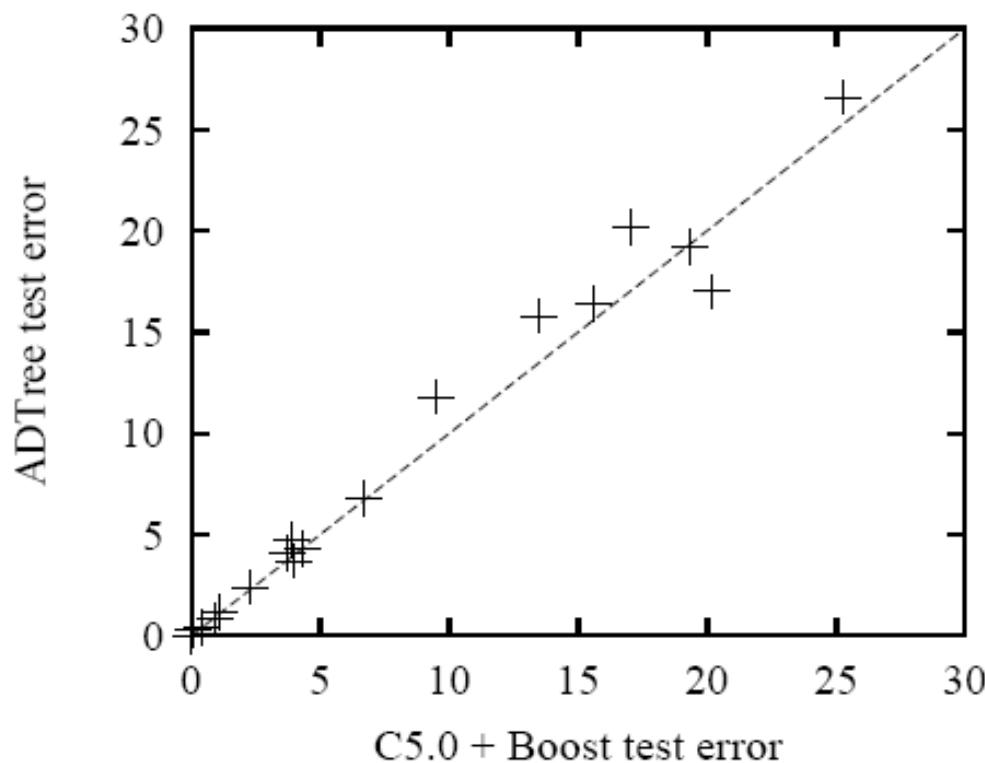
## Pros

- Can focus on specific regions
- Similar test error to other boosting methods
- Requires far fewer iterations
- Easily visualizable

## Cons

- Larger VC-dimension
  - Increased proclivity for overfitting

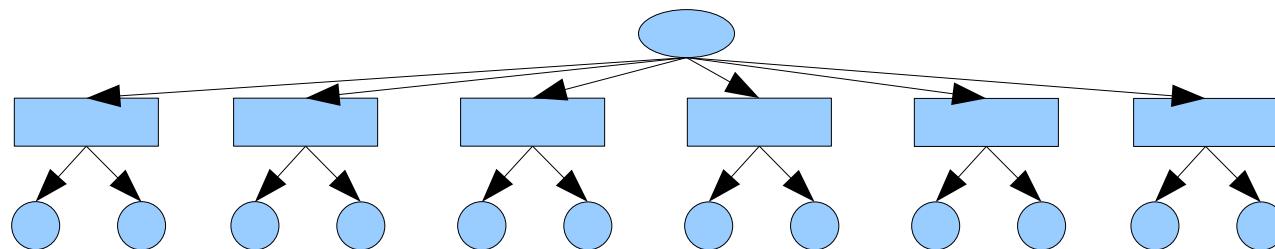
# Error Rates



Taken from Freund & Mason 1997

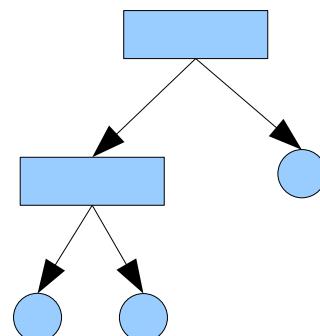
# Some Basic Properties

- ATrees can represent decision trees, boosted decision-stumps, and boosted decision trees
- ATrees for boosted decision stumps:

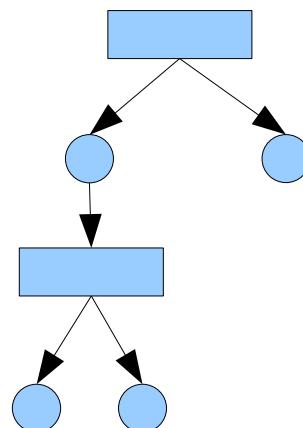


- ATrees for decision trees:

Decision Tree



Alternating Tree



# Resources

- Boosting.org
- JBoost software available at  
<http://www.cs.ucsd.edu/users/aarvey/jboost/>
  - Implementation of several boosting algorithms
  - Uses ATrees as final classifier
- Rob Schapire keeps a fairly complete list  
<http://www.cs.princeton.edu/~schapire/boost.html>
- Wikipedia