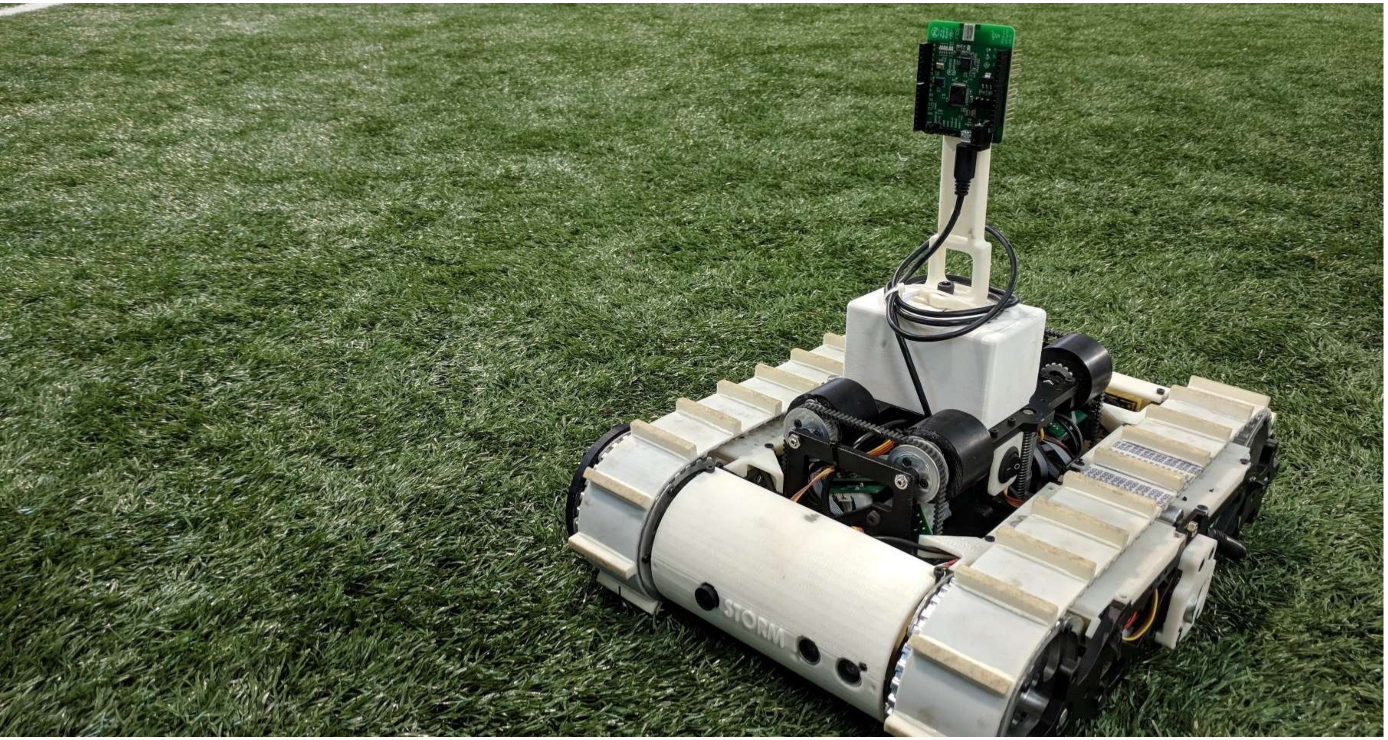


ED 5215
INTRODUCTION TO MOTION PLANNING

Instructor: Bijo Sebastian

MOTIVATION

Information about the goal state can be leveraged to find an optimal path while expanding lesser nodes in the process (efficient search)

Informed search methods:

- Heuristic
- A*
- Details on heuristic functions

HEURISTIC

Heuristic: Educated guess (rule of thumb)

- In this case, how close or far away we are from goal
- Function of the current state, $H(current\ state) = h$
- Specific to the application

From the perspective of a search problem:

Strategy:

For a set of nodes in fringe:

- Calculate the heuristic for that node
- Calculate sum of cumulative cost, g and heuristic, h
- Expand the node with the lowest $g + \ h$ value

→ A* (pronounced as A-star search)

A* SEARCH

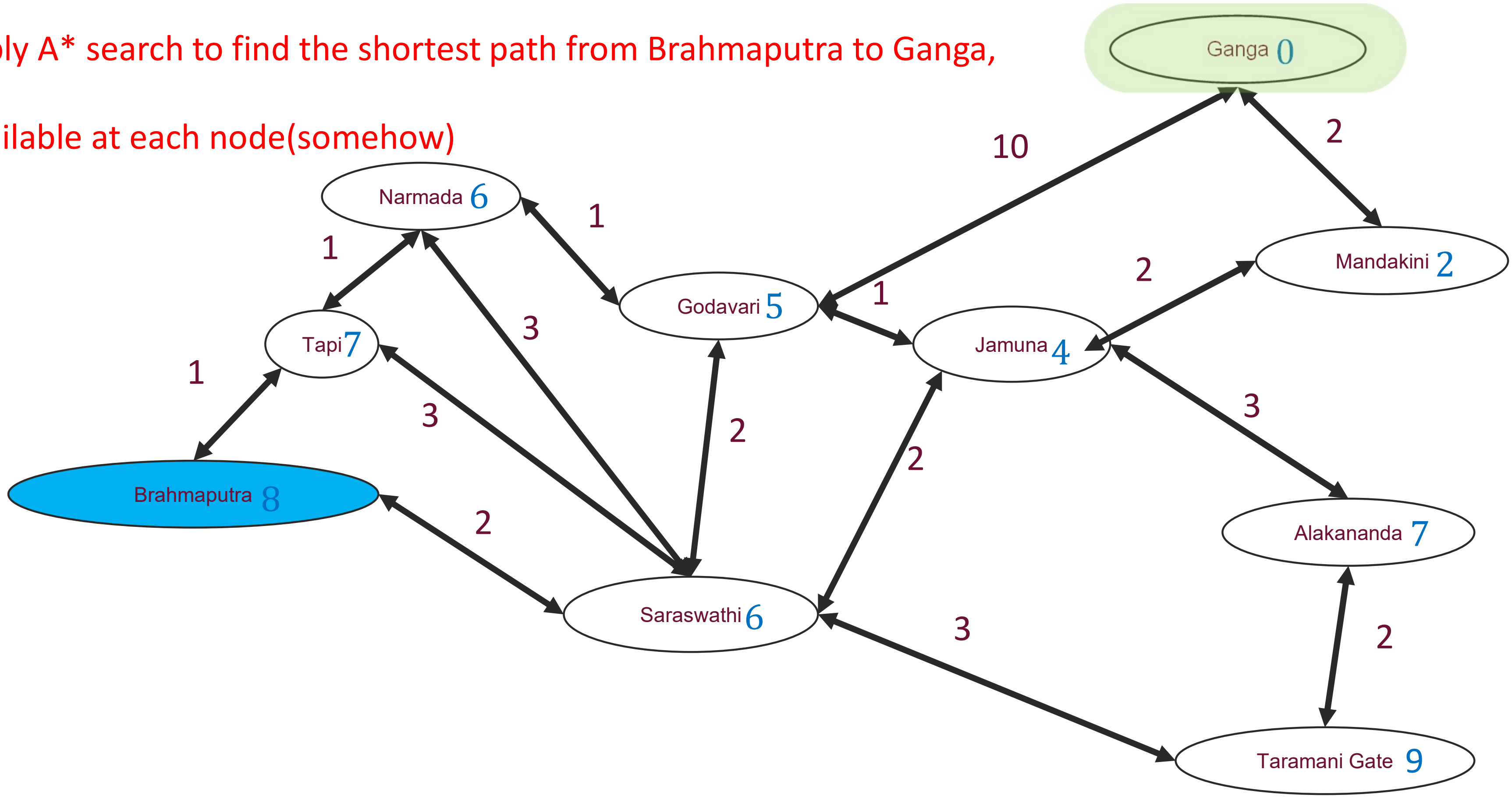
Pseudo code:

```
function A*search ( problem) returns a solution, or failure
        create empty fringe and closed set
        compute heuristic,h for start node
        add the initial state of problem to fringe with a cost, g of 0
        loop do
                if there are no candidates for expansion in the fringe then return failure
                pop node with least g + h from fringe
                if the popped node is goal state, then return the corresponding solution
                else
                add the popped node to closed set
                expand the popped node by performing all possible actions
                for each resulting child node:
                        compute cost, g = popped node cost + cost of action to get to child
                        compute heuristic, h for the child state
                        add the child node to the fringe
                        except:
                                if child node exists in closed set
                                if duplicate child node exists in fringe:
                                        store the one with lower cost
```

A* SEARCH

Consider the graph below apply A* search to find the shortest path from Brahmaputra to Ganga,

assume distance to goal is available at each node(somehow)



A* SEARCH

<u>Strategy</u>: Expand the cheapest (cumulative cost) node first

Fringe (B8(0+8)): pop B8

Fringe (T8(1+7),S8(2+6)): pop T8(1+7)

Fringe (S8(2+6), N8(2+6)): pop S8(2+6)

Fringe (N8(2+6), J(4+4), G(4+5), TG(5+9)): pop N8(2+6)

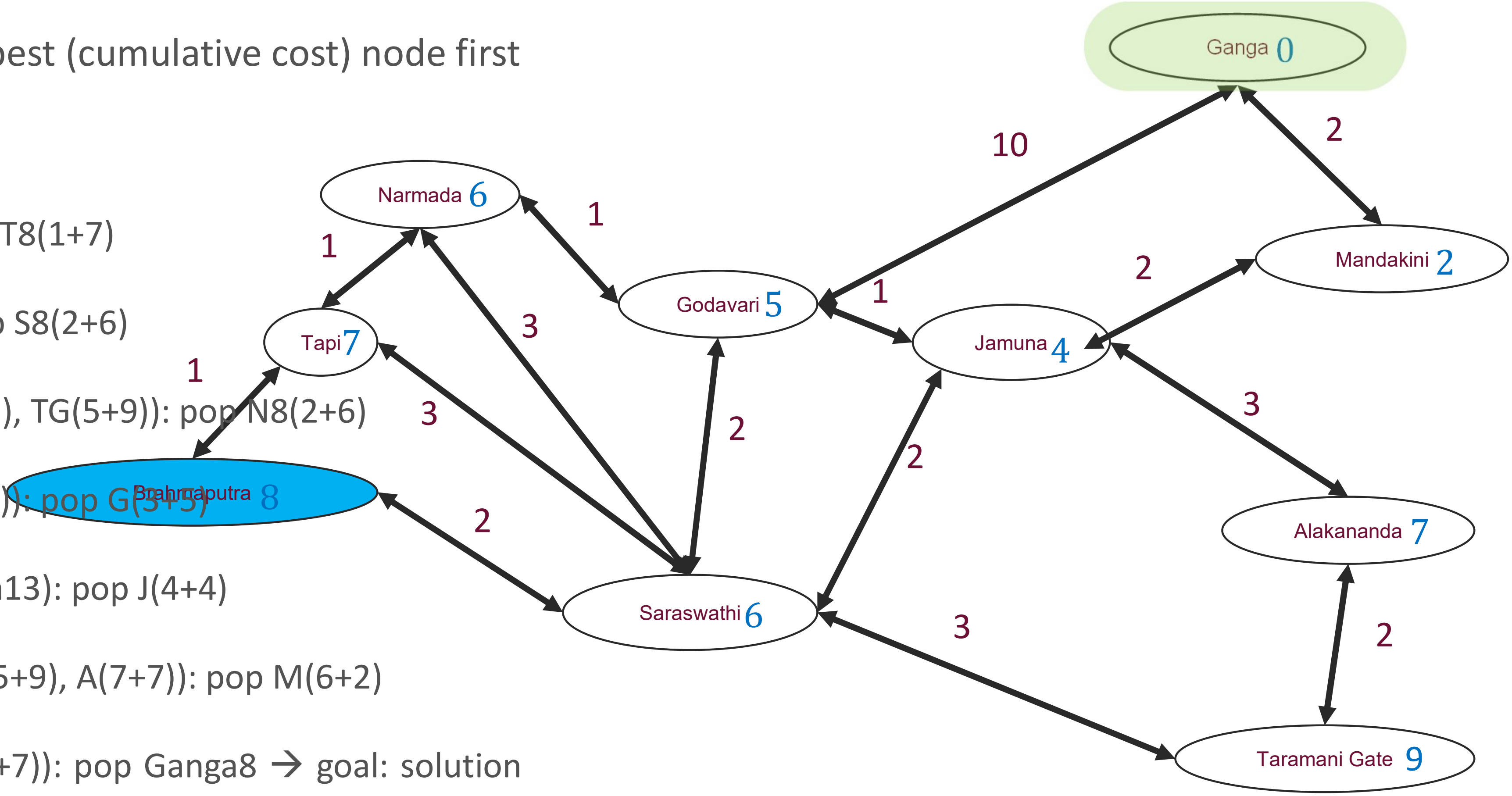
Fringe (G(3+5), J(4+4), TG(5+9)): pop G(3+5)

Fringe (J(4+4), TG(5+9), Ganga13): pop J(4+4)

Fringe (M(6+2), Ganga13 ,TG(5+9), A(7+7)): pop M(6+2)

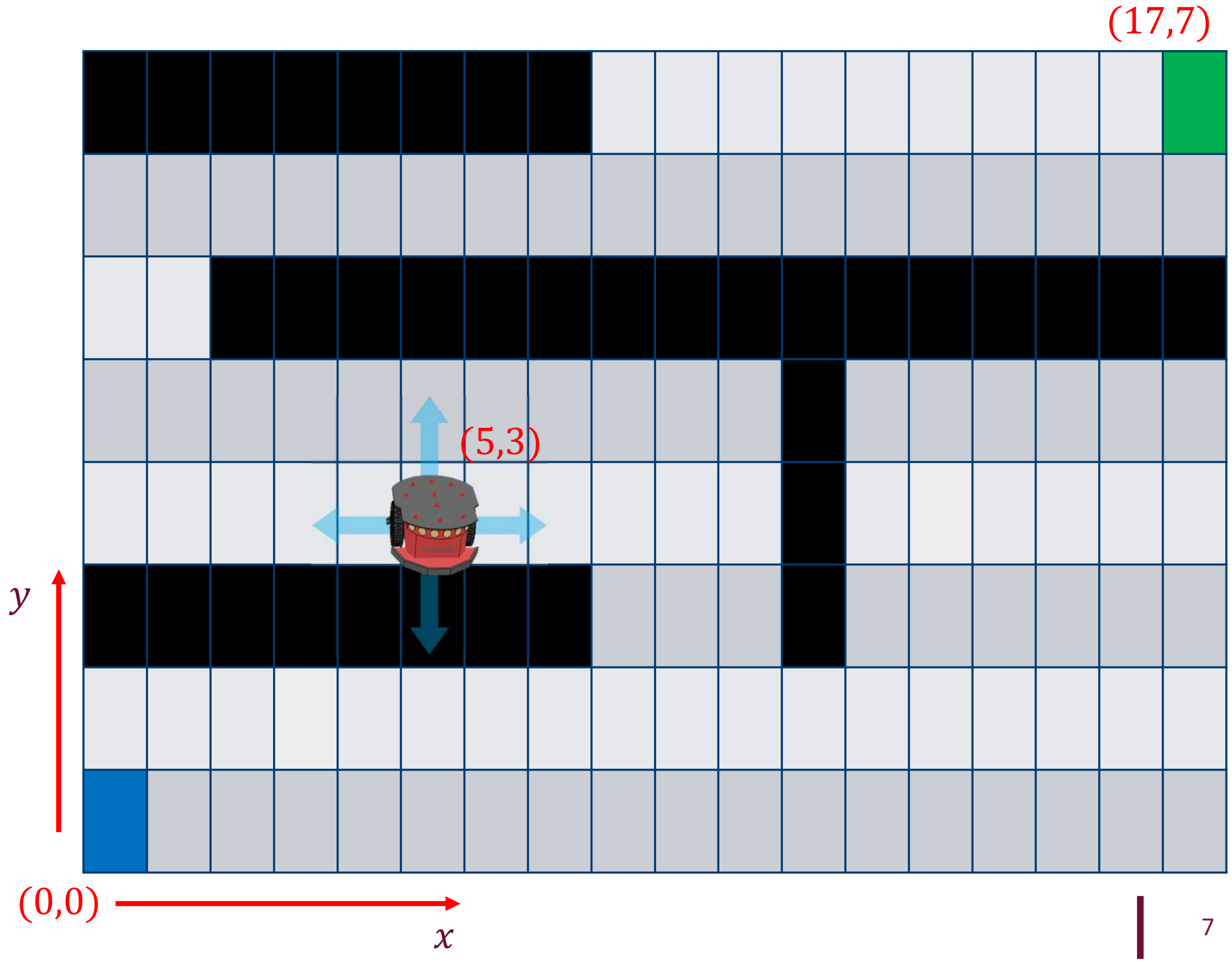
Fringe (Ganga8, TG(5+9), A(7+7)): pop Ganga8 → goal: solution

<u>Brahmaputra, Saraswathi, Jamuna, Mandakini, Ganga</u>



HEURISTIC

Question: Consider point robot in a grid world, trying to get to goal: what would be a good heuristic?



HEURISTIC

Question: Consider point robot in a grid world, trying to get to goal: what would be a good heuristic?

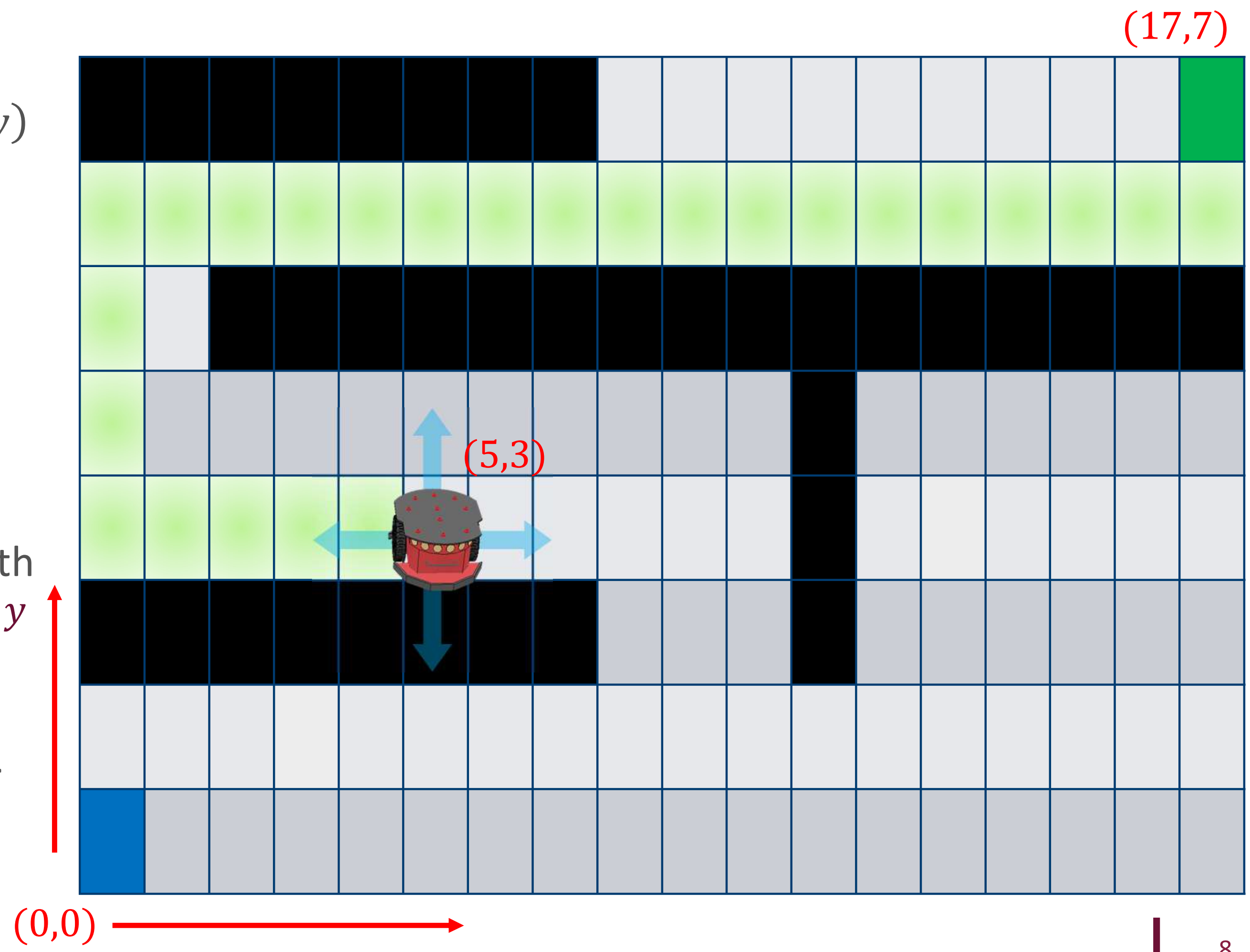
- Manhattan distance = $abs(\Delta x) + abs(\Delta y)$
  = (17-5) + (7-3) = 16

- Euclidean distance = $\sqrt{\Delta x^2 + \Delta y^2}$
  = 12.64

Note that shortest (optimal) path length = 26

→ Heuristic does not have to be the exact path length to goal, an educated estimate is sufficient

Question: Which among the above is a better heuristic?



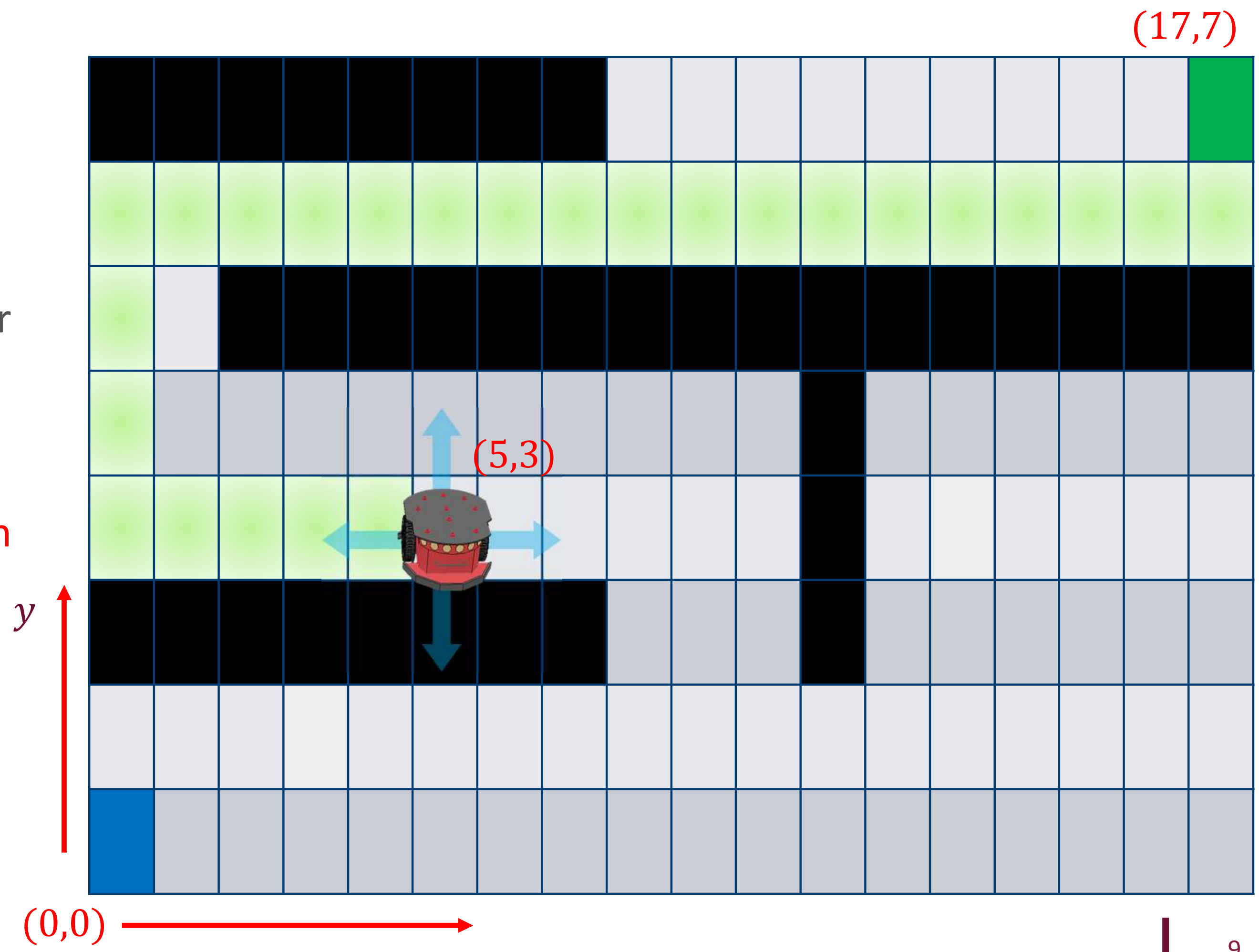
HEURISTIC

Question: Consider point robot in a grid world, trying to get to goal: what would be a good heuristic?

- Manhattan distance
- Euclidean distance

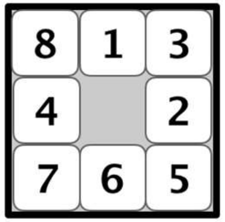
Question: Which among the above is a better heuristic?

The closer the heuristic is towards the actual path length the better it is, in terms of search efficiency → Manhattan distance



HEURISTIC - EIGHT PUZZLE

Question: Consider eight puzzle, what would be a good heuristic?



board

goal

HEURISTIC - EIGHT PUZZLE

Question: Consider eight puzzle, what would be a good heuristic?

Heuristic: To measure how close a board is to the goal board.

Options:

- The Hamming distance between a board and the goal board is the number of tiles in the wrong position

  Implementation: count the number of wrong tiles

HEURISTIC - EIGHT PUZZLE

Question: Consider eight puzzle, what would be a good heuristic?

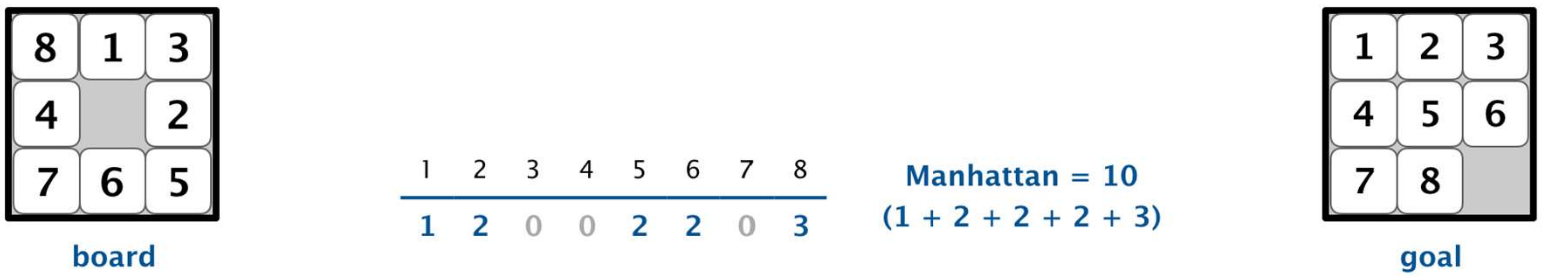
Heuristic: To measure how close a board is to the goal board.

Options:

- The Manhattan distance between a board and the goal board is the sum of the Manhattan distances (sum of the vertical and horizontal distance) from the tiles to their goal positions

  Implementation: For each tile in wrong positon,

$$\Delta x = x_{tile} - x_{ideal}, \Delta y = y_{tile} - y_{ideal}$$
$$h = abs(\Delta x) + abs(\Delta y)$$

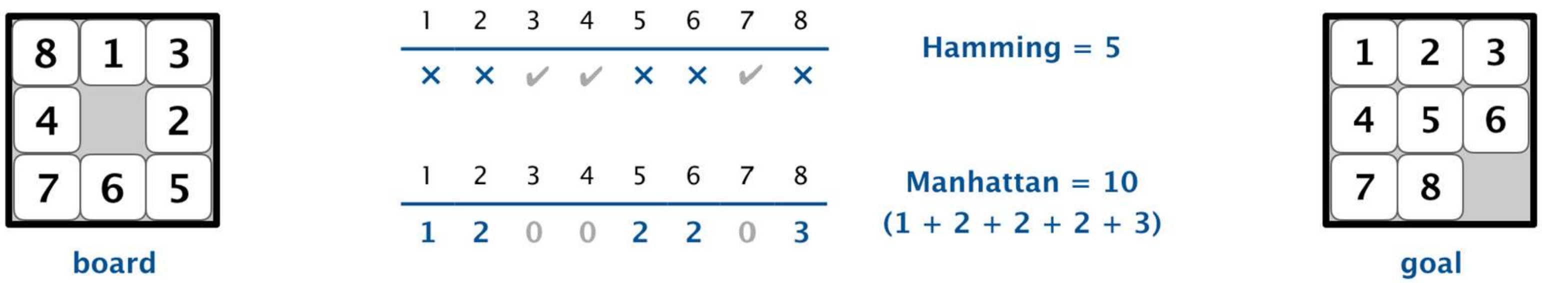


HEURISTIC - EIGHT PUZZLE

Heuristic: To measure how close a board is to the goal board.

Options:

- The Hamming distance
- The Manhattan distance



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HEURISTIC - EIGHT PUZZLE

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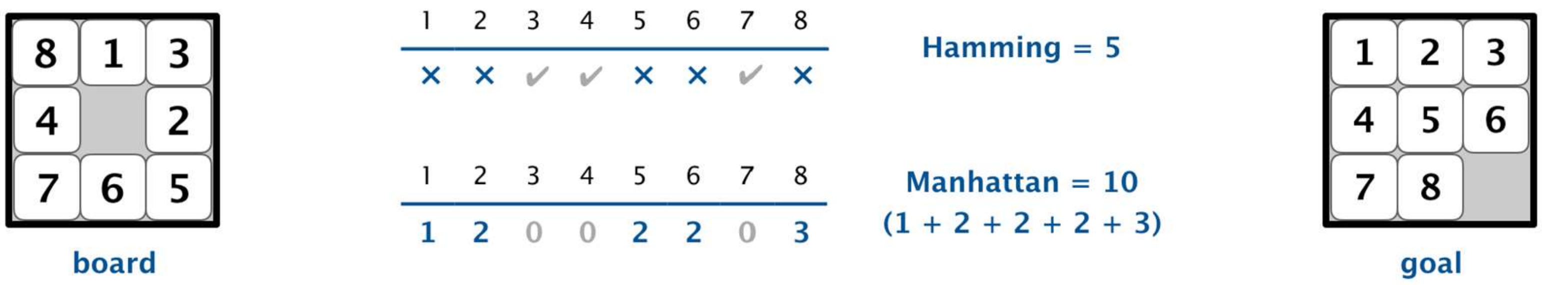


Question: Which among the above is a better heuristic?

The closer the heuristic is towards the actual path length the better it is, in terms of search efficiency → Manhattan distance

HEURISTIC

So now we have seen a couple of cases:

Question: How would A* perform in each of the following cases?

- Zero heuristic → ??
- Heuristic that underestimates distance to goal → ??
- Heuristic that gives exact length to goal → ??
- Heuristic that overestimates the length to goal → ??

HEURISTIC

So now we have seen a couple of cases:

Question: How would A* perform in each of the following cases?

- Zero heuristic → No difference as compared to Uniform Cost Search
- Heuristic that underestimates distance to goal, $h(n)$ → Better than UCS, can be quite easily formulated
- Heuristic that gives exact length to goal, $h^*(n)$ → Best performance in terms of search efficiency, but requires solving the search problem itself
- Heuristic that overestimates the length to goal → A* No longer gives optimal solution

→ Inadmissible heuristic

HEURISTIC

Admissible heuristic

A heuristic h is admissible (optimistic) if: $0 \leq h(n) \leq h^*(n)$

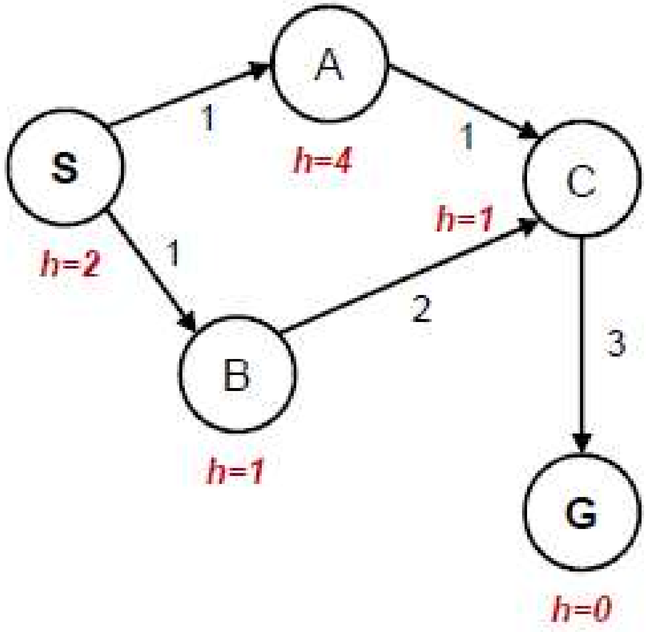
where $h^*(n)$ is the true cost to goal

In this sense, an admissible heuristic can be considered as an optimistic heuristic

Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

HEURISTIC

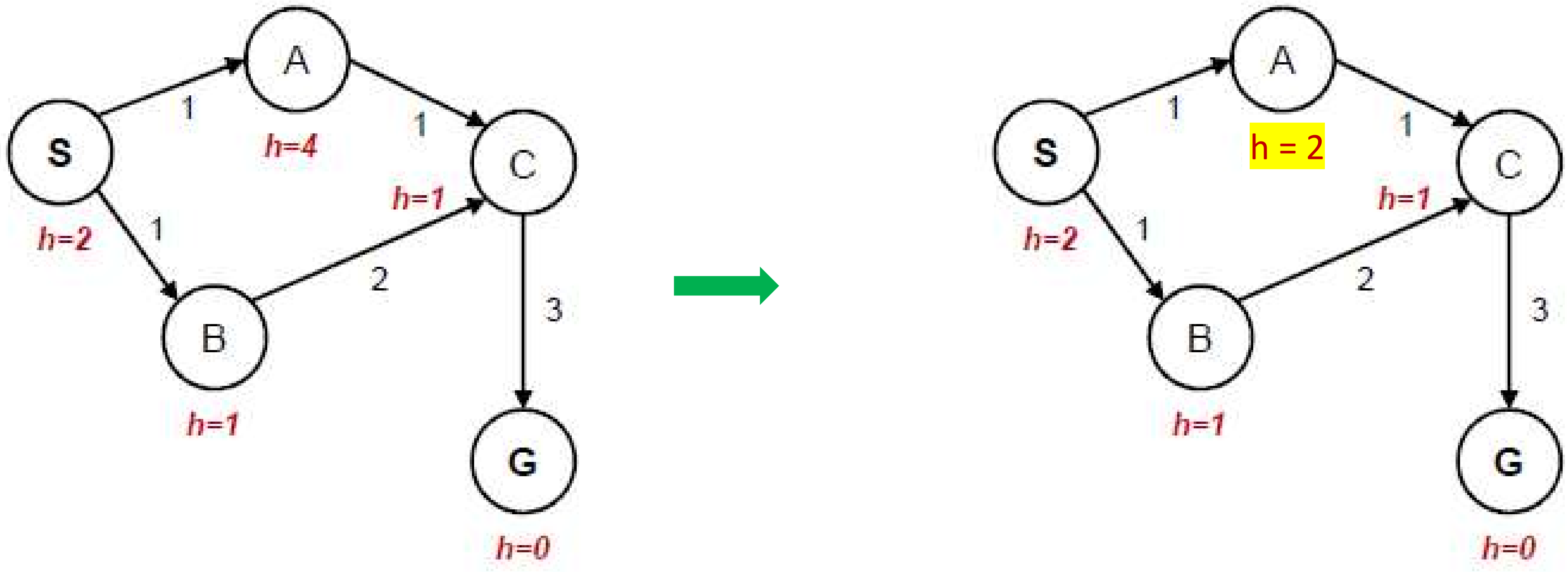
Question: Is admissibility enough to ensure optimality ?



HEURISTIC

Question: Is admissibility enough to ensure optimality ?

No, it also has to be consistent

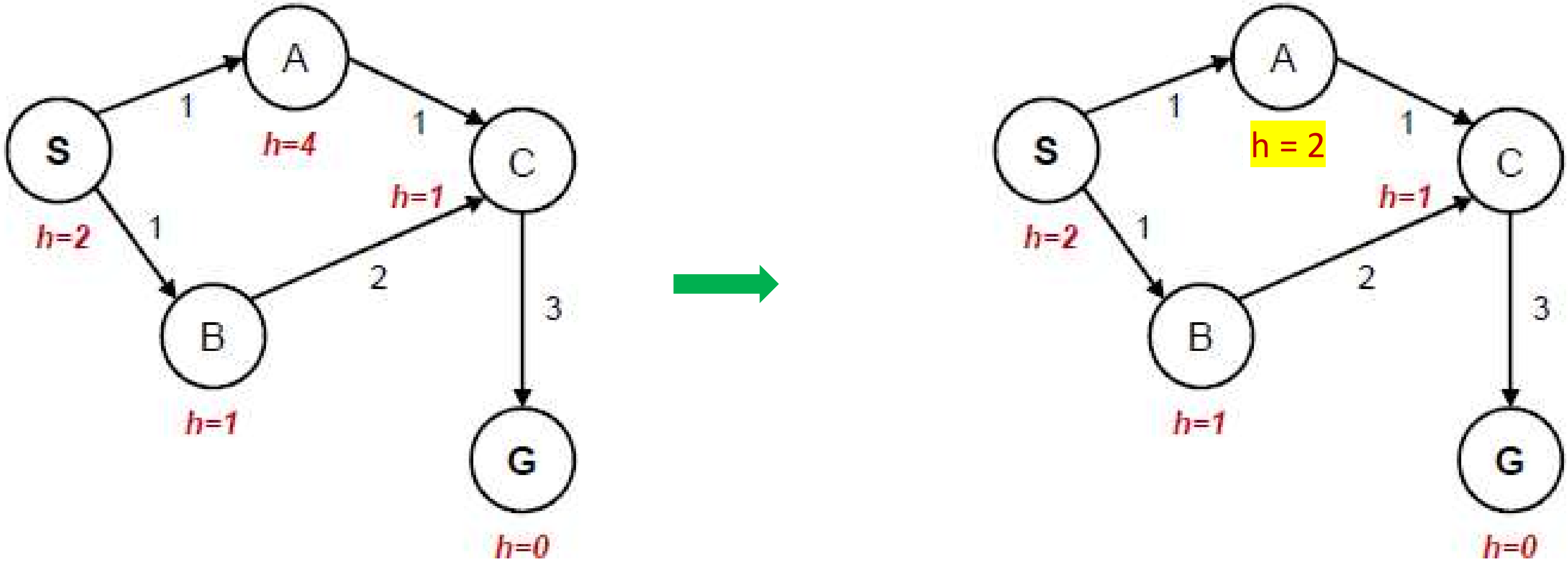


Issue is inconsistent heuristic

HEURISTIC

Consistent heuristic

A heuristic h is consistent if heuristic cost ≤ actual cost for each arc



h(A) – h(C) ≤ cost(A to C)

Consistency implies admissibility

ADDITIONAL READING

Question: What can we do when the search problem changes with time, due to say changes in environmental conditions? Should we replan always?

Additional reading:

- Chapter 2 of “Planning Algorithms” by S. M. LaValle
- Chapter 3 of “Principles of Robot Motion, Theory, Algorithms, and Implementation” by Howie Choset, Kevin Lynch, Seth Hutchinson, George Kantor, Wolfram Burgard, Lydia Kavraki, and Sebastian Thrun