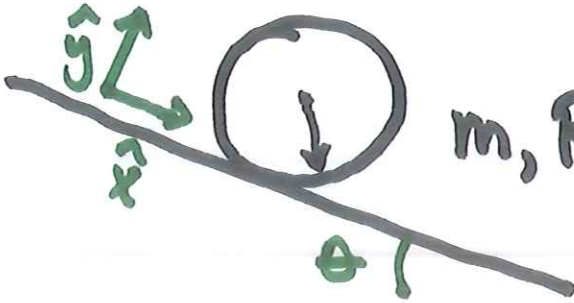
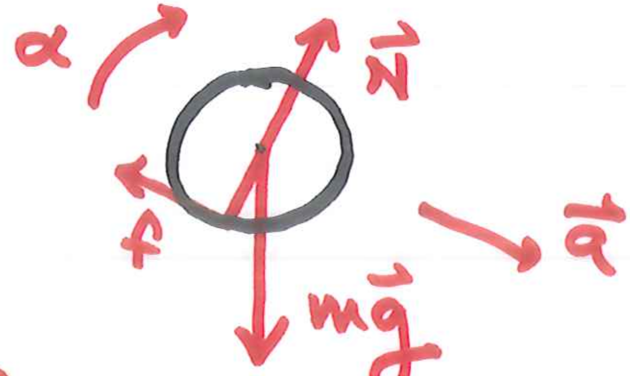
NYU Physics I — 2016-11-10

Agenda — Reading —
— finish rolling.
— collisions of extended objects.



m, R, I



① $\hat{y}: |\vec{N}| - m|\vec{g}|\cos\theta = 0$ kinematic beliefs!

② $\hat{x}: m|\vec{g}|\sin\theta - f = m|\vec{a}|$

③ angular : $fR = I\alpha$

④ rolling without slipping : $|\vec{a}| = \alpha R$

③: $f = \dfrac{I\alpha}{R}$ ④: $f = \dfrac{I|\vec{a}|}{R^2}$

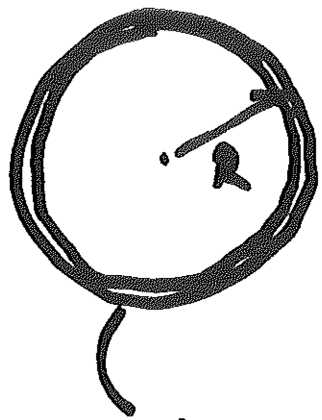
②: $mg\sin\theta - \dfrac{Ia}{R^2} = ma$

$$mg\sin\theta = m\left[a + \frac{Ia}{mR^2}\right] = ma\left[1 + \frac{I}{mR^2}\right]$$

Answer:

$$\boxed{a = \frac{g\sin\theta}{\left[1 + \frac{I}{mR^2}\right]}}$$

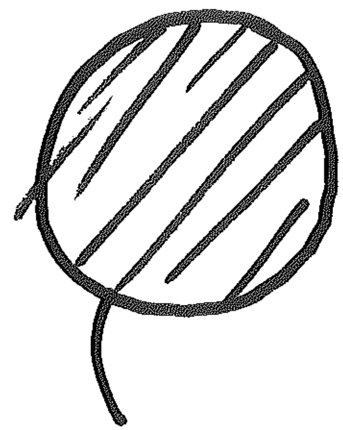
Moments of inertia:



thin hoop

$I = mR^2$

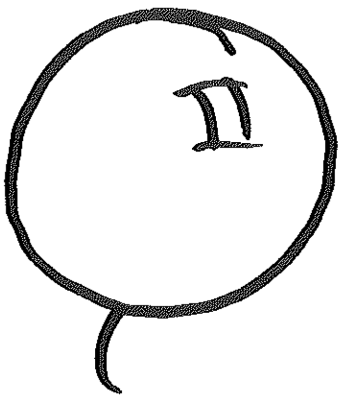
~ empty can



solid disk

$I = \frac{1}{2} mR^2$

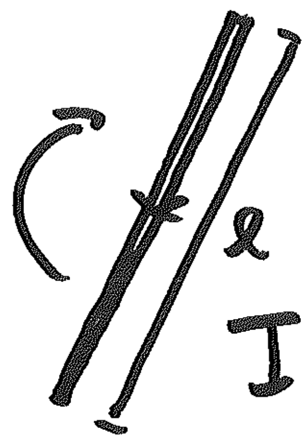
~ Smuckers (tm) Jam.



solid sphere

$I = \frac{2}{5} mR^2$?

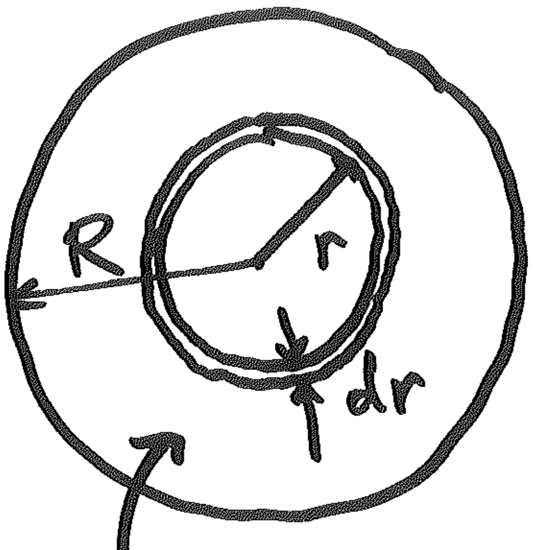
~ cueball



thin rod

$I = \frac{1}{12} m l^2$

Solid, uniform cylinder (disk)



area πR^2
mass M

$$dA = \cancel{2\pi R\, dR} \qquad \cancel{\frac{d\,\pi R^2}{dR} \cdot dR}$$

$$2\pi r\, dr \qquad \left(\frac{d}{dr}\pi r^2\right) dr$$

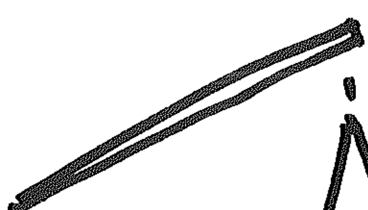
$$dM = M \cdot \frac{2\pi r}{\pi R^2}\, dr$$

$$dI = dm\, r^2 = M \cdot \frac{2\cancel{\pi} r^3}{\cancel{\pi} R^2}\, dr$$

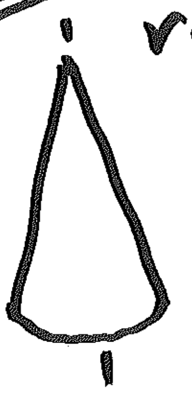
$$I = \int_{\text{everything}} dI = \int_0^R M \cdot \frac{2r^3}{R^2}\, dr$$

$$= \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R = \frac{1}{2} M R^2$$

next up:



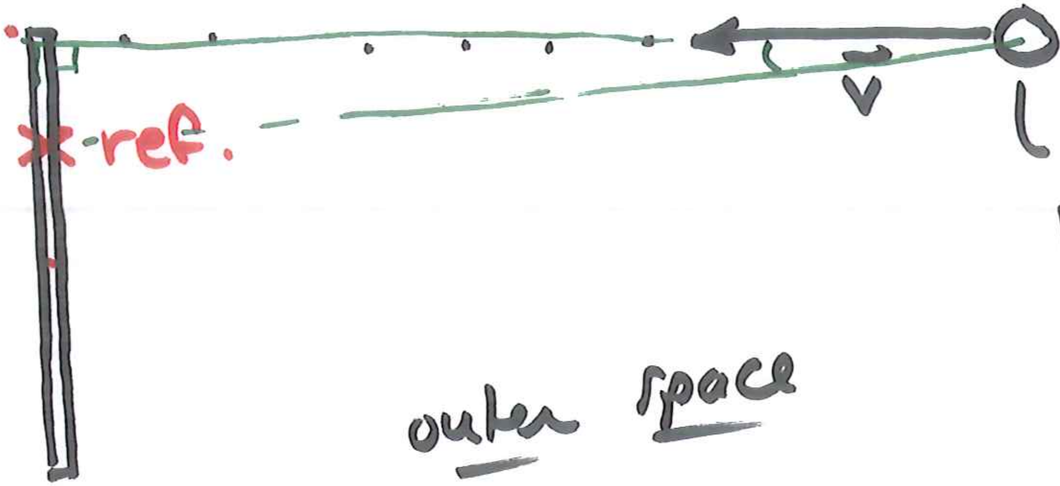
thin rod



spherical shell

solid sphere

– rectangular parallelepiped



blob of putty of mass $M = m$ compact.

outer space

rod of length ℓ, mass m

before

after

Before:

$\vec{p} = m\vec{v} + 0$

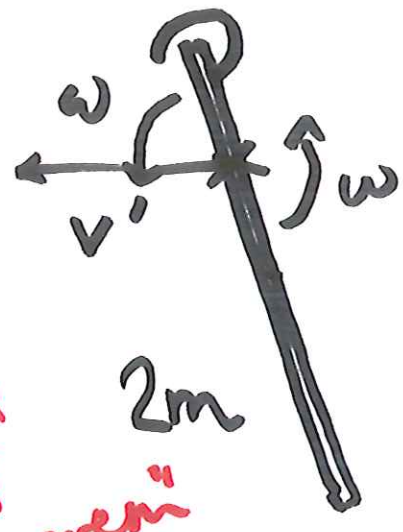
$K = \frac{1}{2}mv^2 + 0$

$L = |\vec{r} \times \vec{p}| = \frac{\ell}{4}mv$

$$I = \underbrace{\frac{1}{12}m\ell^2}_{\text{rod}} + \underbrace{m\left(\frac{\ell}{4}\right)^2}_{\text{shift}} + \underbrace{m\left(\frac{\ell}{4}\right)^2}_{\text{putty}}$$

$$\boxed{I = \frac{5}{24}m\ell^2}$$

read: "parallel axis theorem"



After: $\vec{p} = 2m\vec{v}'$

stick + putty $= \frac{5}{24} m\ell^2$

$$K = \frac{1}{2}(2m)v'^2 + \frac{1}{2} I\omega^2$$

$$L = I\omega = \frac{5}{24} m\ell^2 \omega$$

---

no external force

$\vec{P}_{after} = \vec{P}_{before}$: $m\vec{v} = 2m\vec{v}'$ $\boxed{\vec{v}' = \frac{\vec{v}}{2}}$

no external torque.

$L_{after} = L_{before}$: $\frac{\ell}{4} \cancel{m} v = \frac{5}{24} \cancel{m} \ell^2 \omega$

$$\boxed{\omega = \frac{6}{5} \frac{v}{\ell}}$$