NYU Physics I—numerical integration

The question here is: How long does it take for an object to fall 10 m near the surface of the Earth in the absence of air resistance? You will need to perform numerical integrations throughout this course, so we begin by solving the problem that way.

1 Choose a partner (if you wish), obtain a sheet of notebook paper and put your names on it. Start a table with 21 rows and four columns for four variables t, a, v, and x.

2 In the t column, put in times 0.0 s, 0.1 s, 0.2 s, ..., 2.0 s. In the a column, put $-9.8 \,\mathrm{m\,s^{-2}}$ in all 21 rows.

3 In the first entry of the v column, put $0.0 \,\mathrm{m\,s^{-1}}$. In each subsequent row $i + 1$ put in for the v entry v_{i+1}:

$$v_{i+1} = v_i + a_i \,\Delta t = v_i + a_i \left[t_{i+1} - t_i\right] \quad . \tag{1}$$

Do that for all 20 subsequent rows. Keep "sufficient" accuracy as you go. Use a phone or calculator; no need to do this calculating by hand!

4 In the first entry of the x column, put 10.0 m. In each subsequent row $i + 1$ put in for the x entry x_{i+1}:

$$x_{i+1} = x_i + v_i \,\Delta t = x_i + v_i \left[t_{i+1} - t_i\right] \quad . \tag{2}$$

Do that for all 20 subsequent rows. Keep "sufficient" accuracy as you go.

5 (Answer this and the following parts on your sheet of paper, below the table or on the other side.) Does the object get to $x = 0$ m within the time span in your table? If it does, make an estimate of the time t at which the object gets to $x = 0$ m.

6 Do you expect your answer to be an overestimate or underestimate of the true time obtained by an analytic calculation? Give a written explanation.

7 Do you recall this equation?

$$x = x_0 + v_0\, t + \frac{1}{2}\, a\, t^2 \tag{3}$$

Use this to compute the true time it takes the object to fall $10\,\mathrm{m}$. Were you correct in your expectation? How far off was your numerical answer?

8 Name two things you could have done to make your integration more accurate.