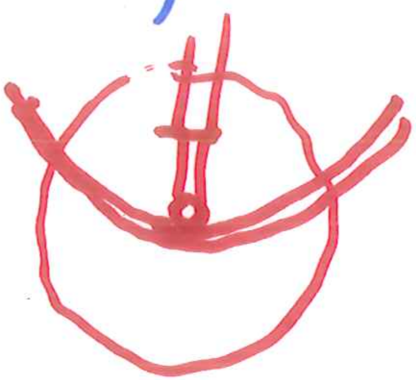
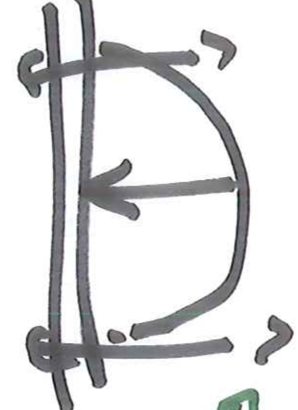
Physics I   2017-10-24.

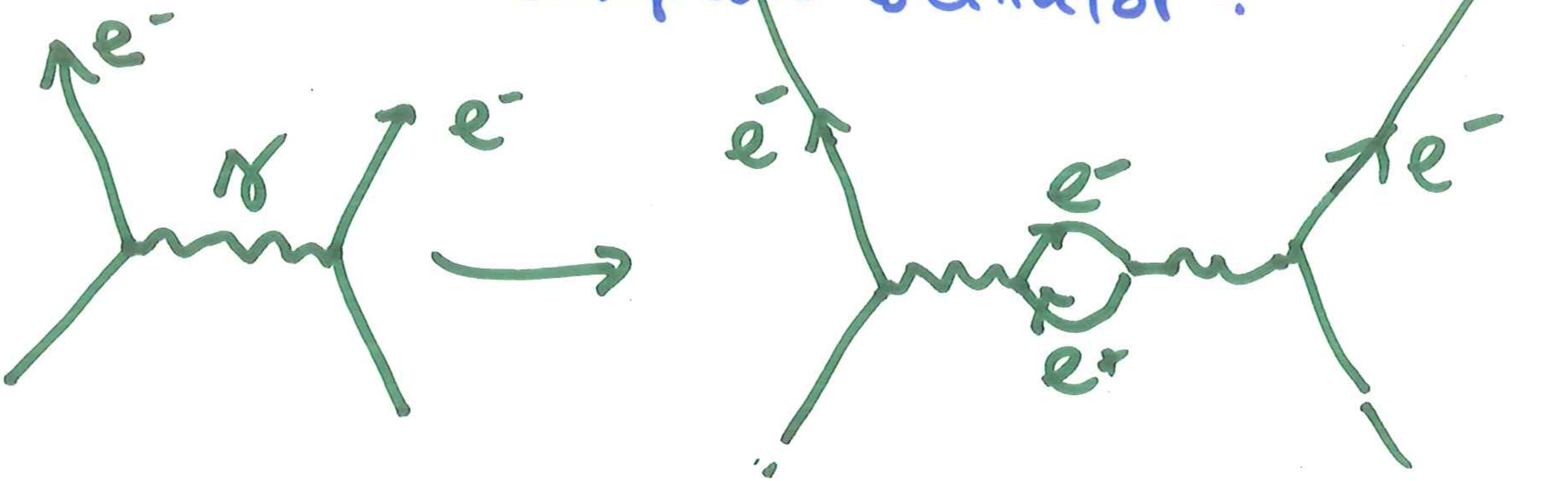
Agenda - Reading < "damped" resonance, Q quality factor

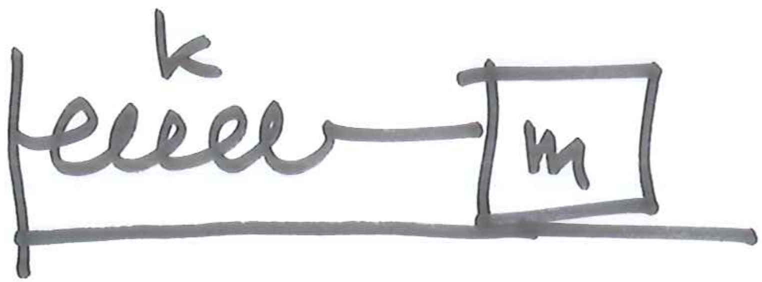
- Questions





- damped oscillator.



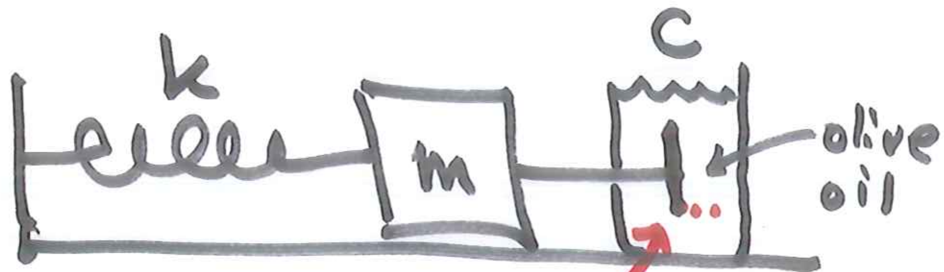


$$m\frac{d^2x}{dt^2} + kx = 0$$

$F = -kx$

$x = A\cos\omega t + B\sin\omega t$

$\omega \equiv \sqrt{\frac{k}{m}}$



→ viscous ← drag.

$F_x = -kx - cv_x$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

out of scope.

guess $x = \overset{A}{\lambda} e^{\alpha t}$

$(e^{\pi/\alpha})$

units:
$A : m$
$\alpha : \frac{1}{s}$

check

$x = Ae^{\alpha t}$ $\quad \frac{dx}{dt} = \alpha A e^{\alpha t}$ $\quad \frac{d^2x}{dt^2} = \alpha^2 A e^{\alpha t}$

$$m\alpha^2 \cancel{Ae^{\alpha t}} + c\alpha \cancel{Ae^{\alpha t}} + k \cancel{Ae^{\alpha t}} = 0$$

$m\alpha^2 + c\alpha + k = 0$ ← equation for α.

$$\alpha = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$ is this okay?

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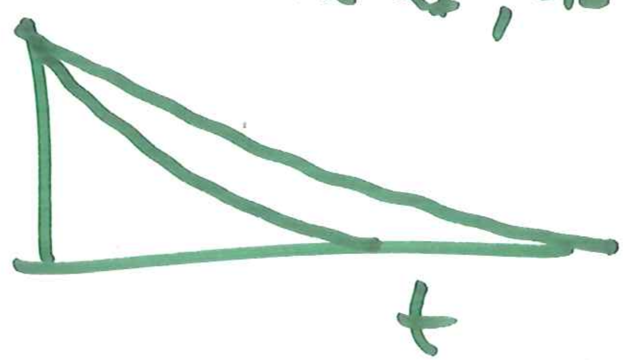
over-damped

if $c^2 - 4mk > 0$ then α is real

both α_+, α_- are negative

Solution is



under-damped case

if $c^2 - 4mk < 0$: then α is <u>complex</u>

$$\alpha = \frac{-c}{2m} \pm i \sqrt{\frac{4mk - c^2}{2m}} \qquad x(t) = Ae^{\alpha t}$$

$$\alpha = -\frac{c}{2m} \pm i \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

$$= -\gamma \pm i \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega_0 \equiv \sqrt{\frac{k}{m}}$$

$$\gamma \equiv \frac{c}{2m}$$

$e^{iqt} = \cos qt + i \sin qt$

$Z e^{iqt} + Z^* e^{-iqt}$

$= \mathrm{Re}(Z) \cos qt$

$+ \mathrm{Im}(Z) \sin qt$.

aaagh ...

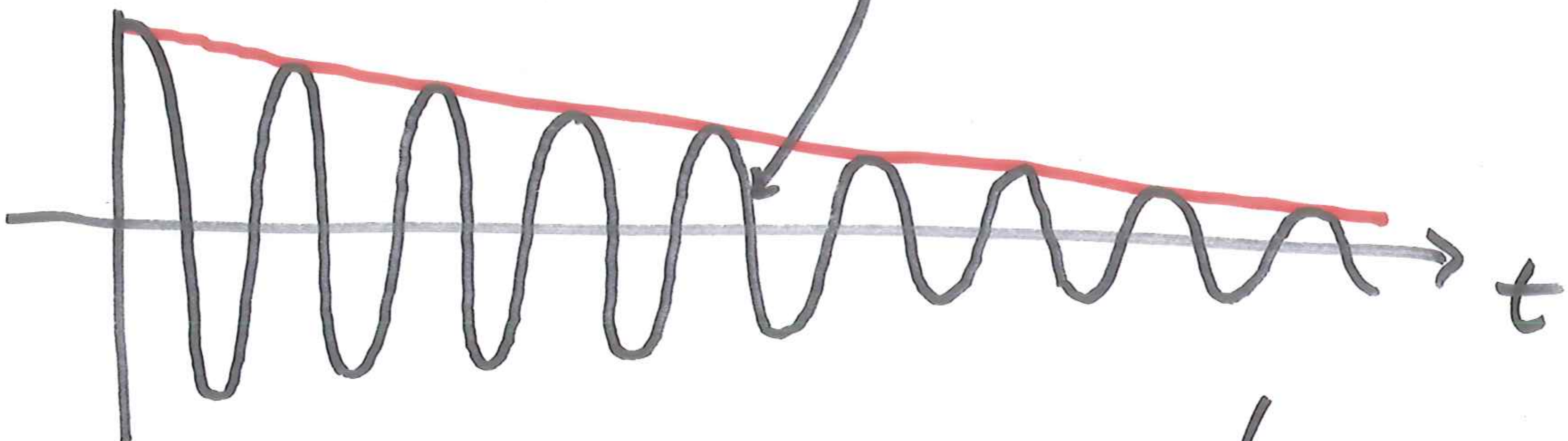
$$x(t) = Ae^{-\gamma t} \cos \omega_1 t + Be^{-\gamma t} \sin \omega_1 t$$

$$\omega_1^2 \equiv \omega_0^2 - \gamma^2$$

... back into scope ...!

under-damped case:

$$x(t) = \underbrace{Ae^{-\gamma t}}\cos\omega t$$



$$\left(T.E. = KE + PE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = kA^2\right)$$

$$T.E = KE + PE = kA^2 e^{-2\gamma t}$$

Q : # of oscillations to decay in energy by 1/e