NYU Physics I—Problem Set 7

Due Thursday 2017 October 26 at the beginning of lecture.

Problem 1: What is the length of a classical pendulum (light string, heavy point mass at the end) which oscillates in small oscillations with a roughly 2-s period? use $g = 9.8\,\mathrm{m\,s^{-2}}$, and give your answer to three digits of accuracy. This is called a "seconds pendulum" (because it ticks off seconds). Apparently it isn't a coincidence that the answer to this is close to $1\,\mathrm{m}$!

Problem 2: A typical adult is holding her or his left arm at a right angle, so the upper arm is pointing straight down, and the forearm is pointing horizontally forwards. The hand is oriented palm-up. The arm is holding a $10\,\mathrm{kg}$ grocery bag by its handle in the hand. Look up the point of attachment of the relevant tendon and make sensible estimates (or look them up) for all lengths and masses. In what follows, treat the "hand plus forearm" to be one monolithic object; that is, we primarily want to understand the forces at or near the elbow.

(a) Draw a free-body diagram for the hand-plus-forearm system, identifying all significant forces acting on it (including from the bag handle, and don't forget the elbow joint—the contact force from the upper arm bones).

(b) Compute the magnitudes and directions of all forces, and the magnitudes and directions of all torques, taking the elbow to be the axis of rotation (that is, the origin or reference point). For simplicity, take the tendon direction and joint contact force both to be precisely vertical. That is, treat all angles as being right angles. This is not a bad approximation.

(c) Look up the definition of "mechanical advantage" and compute the mechanical advantage the grocery bag has over the tendon. Why would evolution (such a brilliant designer) decide to put tendons under this kind of stress?

Problem 3: Consider a mass M attached to a spring of natural (equilibrium) length ℓ, with spring constant k, and hanging from the ceiling. The system is subject to gravity with gravitational acceleration g.

(a) Because gravity is acting, the equilibrium position of the mass will not be at the equilibrium length of the string, but instead stretched. Compute the amount the spring is stretched at equilibrium.

(b) Carefully choose the following coordinate system: $\hat{y}$ points upwards (opposite to gravity), and the position $y = 0$ is where the spring has length ℓ. That is, the zero of the coordinate system is where the spring would be unstretched in the absence of gravity. Make $y = 0$ also the zero of the gravitational potential energy. In this coordinate system, plot the potential energy as a function of y position. You need to take into account both the gravitational potential energy and the spring potential energy. Show that the equilibrium position you computed in the previous part is the minimum of this potential energy curve.

(c) Compare the second derivative of the potential energy at the minimum to the same for the same spring *not* subject to a gravitational force. What are the implications of this calculation?

(d) Write down the differential equation relating y to its second derivative, and show that this differential equation is solved by sinusoidal motion. What are the angular frequency and period of the oscillating solutions?

Problem 4: Determine the value of π by integration!

(a) Make a spreadsheet with a dimensionless column marked "t" that goes from 0.0 to 20.0 in steps of 0.05. Now make columns marked "c" and "s". Integrate the functions $c(t)$ and $s(t)$ with the properties that:

$$
\begin{aligned}
c(0) &= 1.0 && (1)\\
s(0) &= 0.0 && (2)\\
\Delta c &= -s\,\Delta t && (3)\\
\Delta s &= c\,\Delta t \quad . && (4)
\end{aligned}
$$

That is, you are integrating functions that are each other's derivatives:

$$
\begin{aligned}
\frac{\mathrm{d}c(t)}{\mathrm{d}t} &= -s(t) && (5)\\
\frac{\mathrm{d}s(t)}{\mathrm{d}t} &= +c(t) \quad . && (6)
\end{aligned}
$$

Make a graph of $c(t)$ vs t and $s(t)$ vs t. Remind you of anything?

(b) Make an expanded graph in the region $1.5 < t < 1.6$ and look where one of the curves crosses zero. Multiply your answer by 2 and you have an estimate of π! Why does that work?

(c) There are many estimates of π you can make, given your spreadsheet. What do you think will be the most accurate one and why? Check.