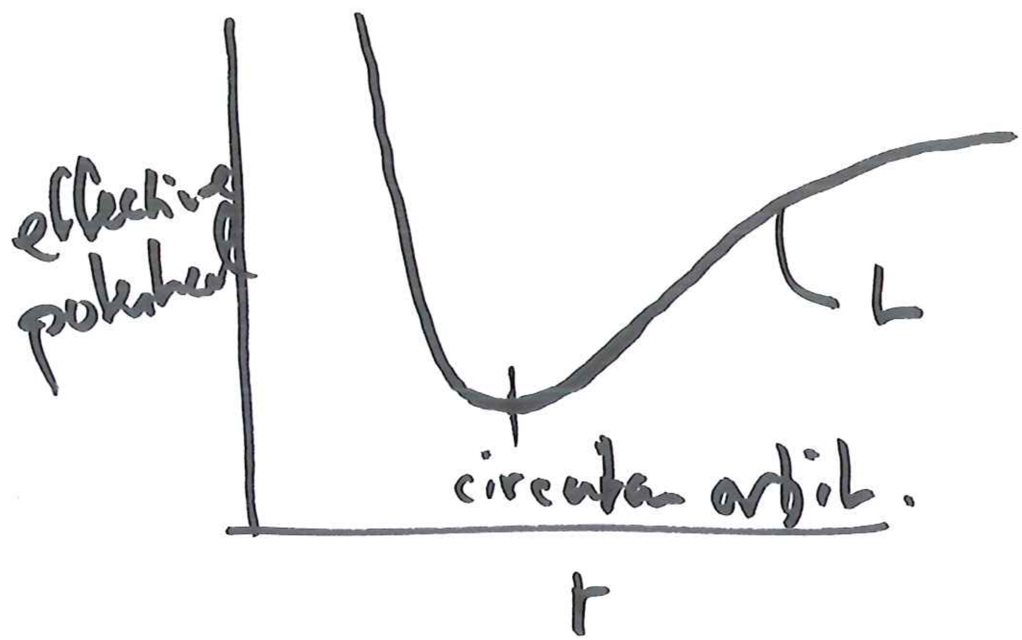
NYU Physics I — 2016-11-17

Agenda:
- reading: orbits.
- registration.
- eccentricity.
- Exam 5.

$$E = \underbrace{\frac{1}{2} m v_r^2}_{\text{radial kinetic energy}} + \underbrace{\underbrace{\frac{L^2}{2 m r^2}}_{\text{transverse K.E.}} - \underbrace{\frac{GMm}{r}}_{\text{grav. potential}}}_{\text{effective potential}}$$



circular orbit: $v_r = 0$ $\quad r =$ constant.

$$E = \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

$$\frac{v^2}{r} = \frac{GM}{r^2}$$ v is tangential.

$$\underbrace{\left|m\frac{v^2}{r}\right|}_{ma} = |F| = \left|\frac{GMm}{r^2}\right|$$

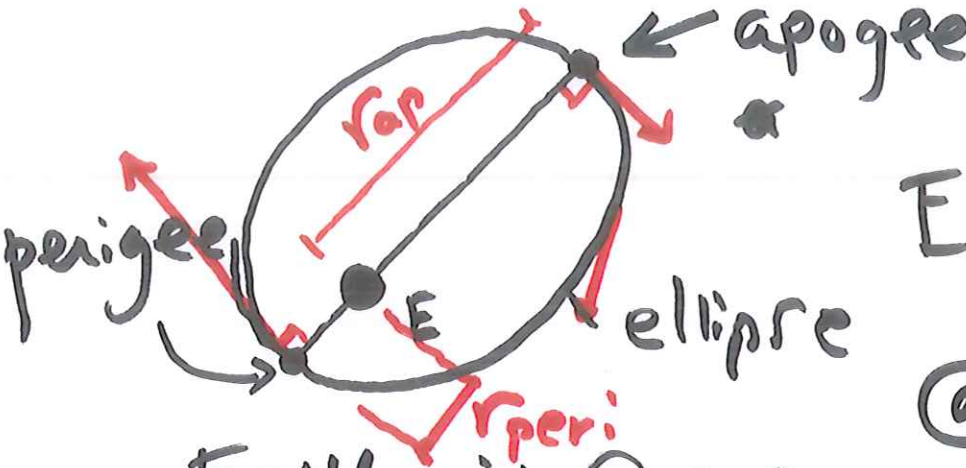
for a circular traj.

$$v = \sqrt{\frac{GM}{r}}$$

$$L = mvr = \sqrt{GMr}\, m$$

$$\frac{L^2}{2mr^2} = \frac{GM\cancel{r}m^{\cancel{2}}}{2\cancel{m}r^{\cancel{2}}} = \frac{GMm}{2r}$$

$$\boxed{E = -\frac{GMm}{2r}}$$ circular orbit.



Earth is @ one focus.

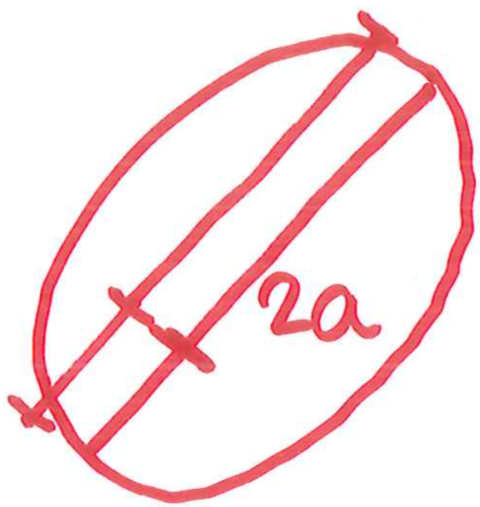
$$E = \frac{1}{2} m v_r^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r}$$

$$@\ r_{ap},\quad v_r = 0$$

$$E = 0 + \frac{L^2}{2m r_{ap}^2} - \frac{GMm}{r_{ap}}$$

$$r_{ap}^2 E + GMm\, r_{ap} - \frac{L^2}{2m} = 0$$

$$r_{ap} = -\frac{GMm}{2E} \pm \frac{\sqrt{(GMm)^2 + 2\frac{EL^2}{m}}}{2E}$$



$$r_{ap} = -\frac{GMm}{2E} \pm \sqrt{\left(\frac{GMm}{2E}\right)^2 + \frac{L^2}{2Em}}$$

$$2a = r_{ap} + r_{peri}$$

$$a = -\frac{GMm}{2E}$$

$$\boxed{E = -\frac{GMm}{2a}}$$ elliptical orbit.

$$e \equiv \frac{r_{ap} - r_{peri}}{r_{ap} + r_{peri}}$$ "asymmetry"

eccentricity