NYU General Physics 1—Problem set 9

Problem 1: A long, thin rod of length L and cross-sectional area A and elastic (Young's) modulus E has mass M.

(a) Think of the rod as being like a Hooke's Law spring; it can be stretched by applying a force. What is the spring constant k for this spring?

(b) By dimensional analysis, can you combine L, A, E, and M into a frequency? Do you have more than one choice? If so, which of the choices makes most sense?

(c) Look up the properties of a femur bone and compute this frequency for the femur bone.

(d) Repeat part *(c)* but replacing the mass M of the femur bone with the mass of a typical college-age human. Hold everything else constant. That is, think of this problem as being a human mass on a femur-bone spring.

(e) Compare the frequency you got in part *(c)* to the dimensional analysis frequency you can obtain by combining the acceleration due to gravity g with L and M. What does this frequency represent? Is it higher or lower? Does that jive with your intuition?

Problem 2: Consider a mass M on a spring of spring constant k, released from rest but from a distance X (in the x-direction, which is parallel to the spring) away from the equilibrium position for the mass. Subsequently, the mass on the spring oscillates without any loss of energy. Plot the position x as a function of time, the velocity v as a function of time, the acceleration a, the kinetic energy K, the spring potential energy U, and the total energy E. Make your plots in a time-aligned stack so that you can compare them.

Problem 3: Consider a function

$$x(t) = A\,\sin(\omega\,t + \phi) \quad ,$$

where A, ω, and ϕ are constants. Now consider the "differential equation"

$$m\,\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -k\,x \quad .$$

Take derivatives of $x(t)$ and plug them into the left-hand side of the differential equation. Under what conditions will the given $x(t)$ satisfy the differential equation? That is, what needs to be true about A, ω, and ϕ? How is the story different if you have cos instead of sin?