NYU Physics I—Problem Set 8

Due Thursday 2016 November 03 at the beginning of lecture.

Problem 1: A very thin ladder of length L and mass M leans against a vertical wall, on a horizontal floor, making an angle of θ with respect to the wall. Imagine that there is a large coefficient of friction μ at the floor so that the ladder is in static equilibrium, but assume that the wall is effectively frictionless.

(a) Draw a free-body diagram for the ladder, showing all forces acting.

(b) Using the bottom of the ladder as the axis of rotation or origin, compute all the forces and torques on the ladder such that it is in equilibrium.

(c) Why did I make the wall "effectively frictionless"?

(d) Re-solve the problem using the *top* of the ladder as the axis of rotation or origin. What is different in the end?

(e) At what angles θ would the ladder start to slip? If $\mu = 0.8$ (not unreasonable for a ladder with hard rubber feet on a wood floor), what is the maximum angle at which you could lean the ladder?

Problem 2: A long, thin rod of length L and cross-sectional area A and elastic (Young's) modulus E has mass M.

(a) Think of the rod as being like a Hooke's Law spring; it can be stretched by applying a force. What is the spring constant k for this spring?

(b) By dimensional analysis, can you combine L, A, E, and M into a frequency ω? Do you have more than one choice? If so, which of the choices makes most sense? That is, think about how your answer should scale with changes to the problem.

(c) Look up the properties of a femur bone and compute this the constant k and the frequency ω for the femur bone, and then for a steel rod with the same properties, and then for a (hypothetical) diamond (!) rod.

Problem 3: In lecture, you saw something like the damped harmonic oscillator differential equation

$$m\,\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + c\,\frac{\mathrm{d}x}{\mathrm{d}t} + k\,x = 0 \quad , \tag{1}$$

where m is the mass, c is a damping coefficient, and k is a restoring constant (a spring constant). Here we are going to show that

$$x(t) = A\,\mathrm{e}^{-\frac{\gamma}{2}\,t}\,\cos(\omega\,t + \phi) \tag{2}$$

can be a solution to the differential equation.

(a) What are the units of c, A, γ, and ϕ?

(b) Take a derivative of $x(t)$ to get $v(t)$. Take another to get $a(t)$.

(c) If your derivatives are correct, what is the potential energy and the kinetic energy as a function of time? Recall that potential energy for a spring is $(1/2)\,k\,x^2$. What is the total mechanical energy of the system as a function

of time? *Hint:* The sines and cosines should disappear from this total energy expression!

(d) Now plug your derivatives into the differential equation, and see if there is a setting of the parameters γ and ω such that the differential equation can be satisfied? *Hint:* Group sine and cosine terms separately; both sets of terms must sum to zero for the differential equation to be satisfied. This is related to the concept of *detailed balance*.

(e) Did you have to assume things about m, c, k to make your answer work? What things?

Extra Problem (will not be graded for credit): Re-do the previous problem using complex exponentials. That is, assume

$$x(t) = Z\,\mathrm{e}^{\alpha\,t} \tag{3}$$

where Z and α are complex numbers. What's different, and what's the same?

Extra Problem (will not be graded for credit): Show that these two descriptions of a simple harmonic oscillator

$$x(t) = A\,\cos(\omega_0\,t) + B\,\sin(\omega_0\,t) \tag{4}$$

$$x(t) = X\,\cos(\omega_0\,t + \phi) \tag{5}$$

are completely equivalent by finding the relationship between A, B and X, ϕ that makes them identical.