NYU Physics I—Problem Set 9

Due Thursday 2017 November 09 at the beginning of lecture.

Problem 1 What is the pressure gradient (units of $\mathrm{Pa\,m^{-1}}$) in air at sea level at STP? What is the pressure gradient in water? What is the pressure gradient in liquid mercury? What would be the pressure gradient in rock, if we treated the rock like a fluid?

Compute the ratio of the pressure gradient in mecury to the pressure gradient in water. How does this ratio compare to the ratio of 33 ft to 760 mm? Why did I ask that?

Problem 2 In a bungy jump, the bungy cord has a rest (unstretched) length of $\ell_0 = 5\,\mathrm{m}$, and has a spring constant k such that it stretches by 1 m for every 200 N of force.

(a) If an adult of mass $M = 80\,\mathrm{kg}$ jumps off of a very tall bridge at time $t = 0$ with this bungy cord attached between her or himself and also the bridge, to what maximum distance h_{max} below the bridge will he or she fall? (You might use energy conservation). Give your answer in terms of the symbols M, ℓ_0, k, and g as well as numerically. That is, we want the numerical and symbolic answers both.

(b) What is the maximum tension T in the bungy? Again, give both answers.

(c) If you stiffen the cord (increase k), but keep everything else fixed, do you increase or decrease the maximum tension?

(d) What acceleration a does the adult feel at the maximum extension of the bungy (that is, at the bottom)? Again, give your answer both symbolically (in terms of the same symbols) and numerically.

Problem 3: Here we consider the construction and tuning of a standard grand piano. Each string of the piano has a mass M, a length L, and a tension T.

(a) Use dimensional analysis to estimate the natural angular frequency ω of a piano string with these properties. That is, what combination has units of frequency?

(b) Look up the natural frequency of a guitar or piano string in its lowest harmonic. You might have to look up "standing wave" or something like that, and you might also have to look up "transverse wave speed" in a string. A string fixed at both ends (like a piano string) is different from an open organ pipe!

(c) Look inside a piano at or near middle C. Roughly what are the diameters of the strings? And what are the lengths of the strings? Use these quantities and the density of steel to estimate the masses of the strings.

(d) Given what you know about the piano—the number of keys, the number of strings per key (which isn't one for most keys), and the range of frequencies and string lengths, estimate *very roughly* what the total stress is

on a piano frame, in Newtons. That is, estimate the total of all the tension forces. Do you understand why piano frames and harps are so heavy?

Problem 4: Everything submerged in the Earth's atmosphere is subject to a buoyant force from the air. In the following, use a sensible (reasonably accurate) measure of the density of air at STP.

(a) When you measure your weight on a standard bathroom scale, you are measuring the *normal force* between yourself and the floor. This normal force opposes the *combination* of gravity and buoyancy. What is the correction to your weight coming from buoyancy, roughly? Express it as a *fraction* of the gravitational force. Is this correction positive or negative—that is, does it increase or decrease the weight measured by the scale?

(b) Look up the "volume" of the Goodyear blimp model GZ-22. Imagine that it is floating in an air atmosphere at STP, and that the gas inside the blimp is *also* at STP. What is the approximate buoyant force on the blimp if it is filled with helium? What about if it were filled with hydrogen (molecular hydrogen)? Compare these numbers with the gross weight and capacity of the blimp.

(c) Will the buoyant force increase, decrease, or stay the same as you decrease the temperature? Assume that the blimp contents are always at the same pressure as the exterior air (and therefore the volume of the blimp must change).

Extra Problem (will not be graded for credit): A child of mass m sits exactly on top of a hemispherical mound of frictionless ice of radius R. If the child is displaced a tiny (ie, small relative to R) horizontal distance x from the top of the mound of ice, what is the x-component F_x of the net force on the child? Write down the differential equation relating the $x(t)$ to its second derivative (with respect to time). Use the small-angle approximation to get rid of trigonometric functions! What functions $x(t)$ solve your equation? Try to be as general as possible. *Hint: Try exponentials!*.

Extra Problem (will not be graded for credit): Look up the Youngs modulus for steel, and estimate the spring potential energy stored in the middle C string, using what you figured out in the piano problem. Now estimate the total mechanical energy stored in the tuned piano!