Physics I — 2018-10-25

- ULC. — Javan
  nyu.edu/ulc
- HW.
  726 Bwy #1005
- Qs.
- damped oscillator.

- harmonic oscillator.
- damped H.O.

guessed $A \overset{\sin}{\cos}\, \omega t = x(t)$

got $\frac{d^2x}{dt^2} = -\omega^2 A \overset{\sin}{\cos}\, \omega t$

$$\frac{d^2x}{dt^2} + \text{☺}\, x = 0$$

$$-\omega^2 \cancel{A \overset{\sin}{\cos}}\, \omega t + \text{☺}\, \cancel{A \overset{\sin}{\cos}}\, \omega t = 0$$

<u>yes</u>!! iff $-\omega^2 + \text{☺} = 0$

$$\omega = \sqrt{\text{☺}}$$

so: $\omega = \sqrt{\frac{k}{m}}$ or $\omega = \sqrt{\frac{g}{l}}$

($\omega = \frac{2\pi}{T}$ so $T = 2\pi\sqrt{\frac{m}{k}}$ or $T = 2\pi\sqrt{\frac{l}{g}}$)

guessed $x(t) = Ae^{\alpha t}$

sol $\frac{d^2x}{dt^2} = \alpha^2 A e^{\alpha t}$

$$\frac{d^2x}{dt^2} + \text{☺}\, x = 0$$

$$\alpha^2 \cancel{Ae^{\alpha t}} + \text{☺}\, \cancel{Ae^{\alpha t}} = 0$$

yes!! $\alpha^2 + \text{☺} = 0$

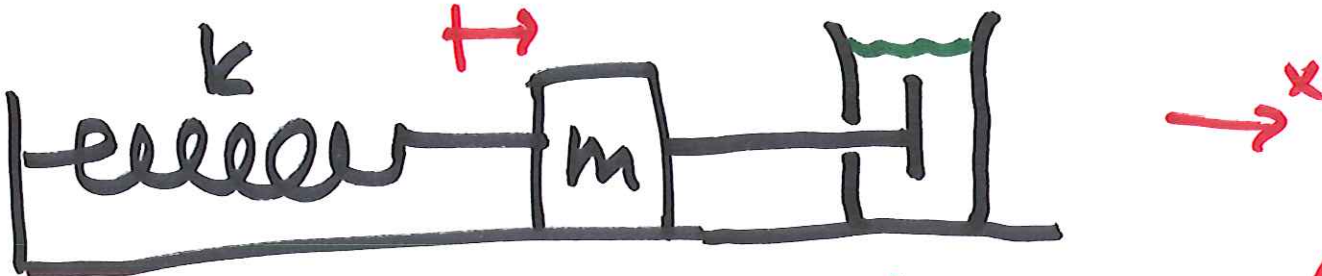
works if $\alpha^2 = -\text{☺}$ $\alpha = \sqrt{\text{☺}}\, i$

imaginary number!

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\pi} = -1$$

$$e^{i\pi} + 1 = 0$$



$-kx$
Hooke's law

$\underset{=}{-bv}$
viscous drag.

$F_d \sim \rho A \underset{=}{v^2} \ldots$
ram pressure

$$m\frac{d^2x}{dt^2} = -kx - bv$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0}$$

guess: $x = Ae^{\alpha t}$

$$\frac{dx}{dt} = \alpha A e^{\alpha t}$$

$$\frac{d^2x}{dt^2} = \alpha^2 A e^{\alpha t}$$

σ.ο.ς.

$$\alpha^2 \cancel{A}\cancel{e^{\alpha t}} + \frac{b}{m}\alpha \cancel{A}\cancel{e^{\alpha t}} + \frac{k}{m}\cancel{A}\cancel{e^{\alpha t}} = 0$$

$$\alpha^2 + \frac{b}{m}\alpha + \frac{k}{m} = 0$$

$$\alpha = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{k}{m}}}{2} = \boxed{-\frac{b}{2m} \pm \sqrt{-\frac{k}{m} + \frac{b^2}{4m^2}}}$$

$$\alpha = -\frac{b}{2m} \pm \sqrt{-\text{☹}}$$

$$x(t) = Ae^{\alpha t} = Ae^{-\frac{bt}{2m}}\, e^{\pm i\sqrt{☹}\, t}$$

b is <u>small</u>, $-☹$ is negative — oscillations

b is <u>large</u>, $-☹$ is positive — exponential

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b small: