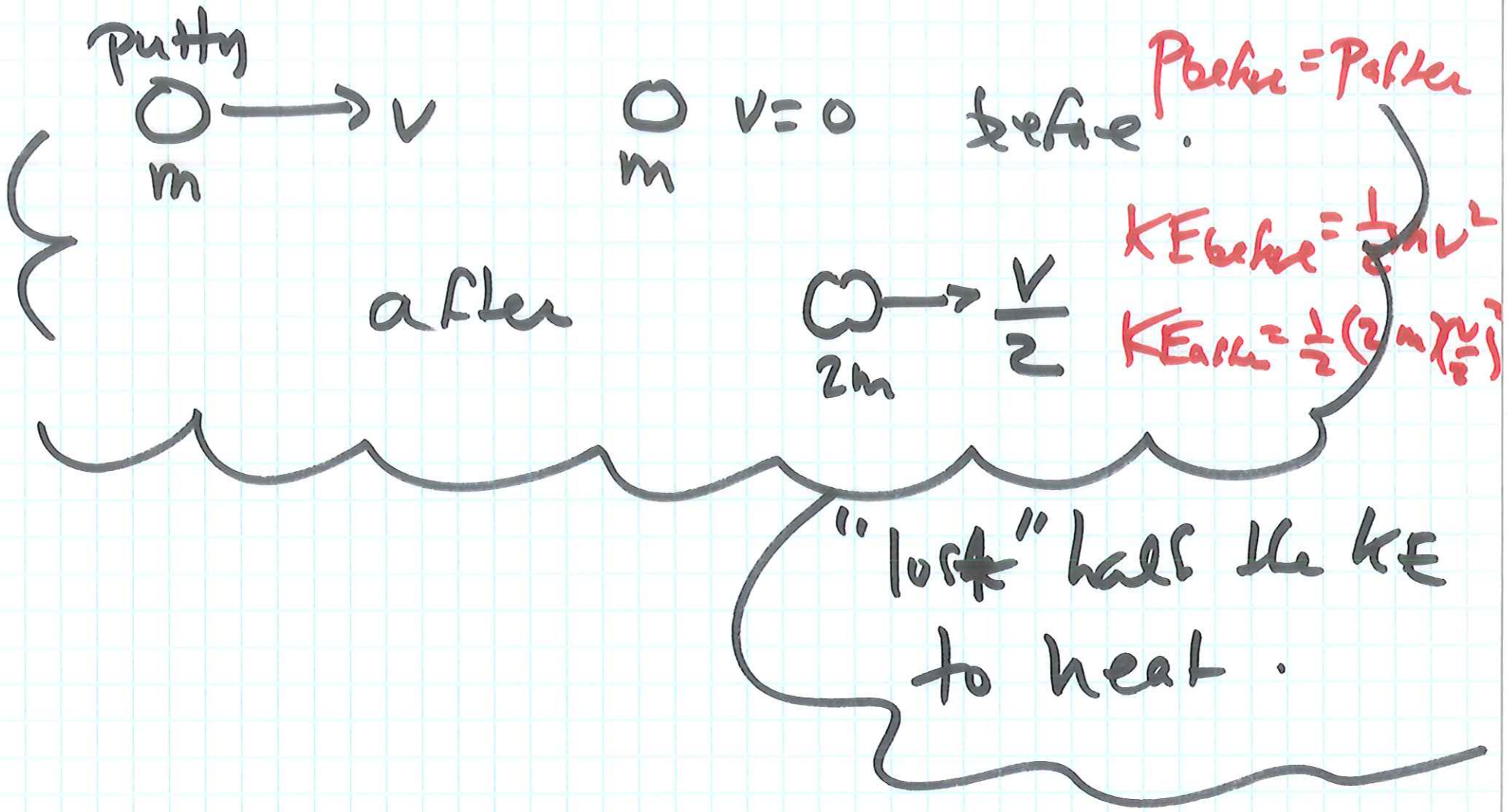
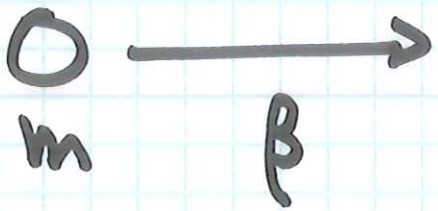
NYU Physics I – 2016-12-15

Agenda:
- Holiday Reading.
- Return Exams & old work.
- Office hours.
- Qs.
- $E = mc^2$ or $E^2 = m^2c^4 + p^2c^2$
- Things I didn't do.


putty
m
v
m
v = 0
before
$P_{before} = P_{after}$
after
2m
$\frac{v}{2}$
$KE_{before} = \frac{1}{2}mv^2$
$KE_{after} = \frac{1}{2}(2m)(\frac{v}{2})^2$
"lost" half the KE
to heat.



before

$$\vec{P}_{before} = (\gamma mc, \gamma mc\beta, 0, 0) + (mc, 0, 0, 0)$$

(4-momentum) $\left(P_t = \frac{E}{c}, P_x, P_y, P_z\right)$

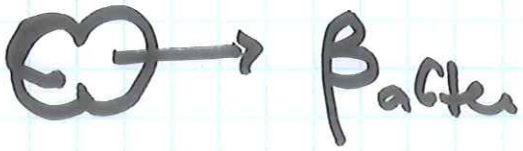
Aside: @ small β

$$\frac{E}{c} = \gamma mc = \frac{mc}{\sqrt{1-\beta^2}} = mc(1-\beta^2)^{-1/2} \approx mc\left(1+\tfrac{1}{2}\beta^2\right) \quad \text{if } \beta \ll 1$$

$$E = mc^2 + \tfrac{1}{2}mv^2$$

mc^2: rest energy

$\tfrac{1}{2}mv^2$: kinetic energy



β_{after}

after

$$p_{after} = (\gamma mc + mc, \gamma mc\beta, 0, 0)$$

$$\equiv \left(\frac{E_{after}}{c}, p_{after}, 0, 0\right)$$

$$\vec{p} = (\gamma mc, \gamma mc\beta, 0, 0)$$

$$|\vec{p}|^2 = p_t^2 - p_x^2 - p_y^2 - p_z^2$$

$$= \gamma^2 m^2 c^2 - \gamma^2 m^2 c^2 \beta^2 = \gamma^2 (1-\beta^2) m^2 c^2$$

$$= m^2 c^2$$

$$\frac{|\vec{p}|^2}{c^2} = m^2$$

$$|\vec{u}|^2 = c^2$$

what is Mass?

$$\frac{|\vec{p}|^2}{c^2} = m_{alk}^2 = \frac{1}{c^2}\left(\left[\gamma mc + mc\right]^2 - \left[\gamma mc\beta\right]^2\right)$$

$\gamma^2 m^2 c^2 (1-\beta^2) \equiv m^2 c^2$

$$= \frac{1}{c^2}\left(\left[\gamma^2 m^2 c^2 + 2\gamma m^2 c^2 + m^2 c^2 - \gamma^2 m^2 c^2 \beta^2\right]\right)$$

$$= \left[m^2 + m^2 + 2\gamma m^2\right]$$

$$= m^2\left[2 + 2\gamma\right] > (2m)^2 \text{ !!!}$$

>4

what if β is small?

fractional increase $\equiv m_{eff}$

$$\gamma = (1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$$

$$\therefore m_{alk}^2 = m^2\left[2 + 2 + \beta^2\right] = (2m)^2\left(1 + \frac{\beta^2}{4}\right)$$

$$\Delta m = 4m\frac{\beta^2}{4} = m\frac{\beta^2}{4}$$

$$m_{after} - m_{before} = 2m\sqrt{1+\frac{\beta^2}{4}} - 2m$$

$$\Delta m \approx m\frac{\beta^2}{4} \quad (\beta \text{ is } \ll 1)$$

Sophomorically say: $E = mc^2$!!

$$\Delta E = mc^2\frac{\beta^2}{4} = \frac{1}{4}mv^2 = \text{half the orig. mol KE.}$$