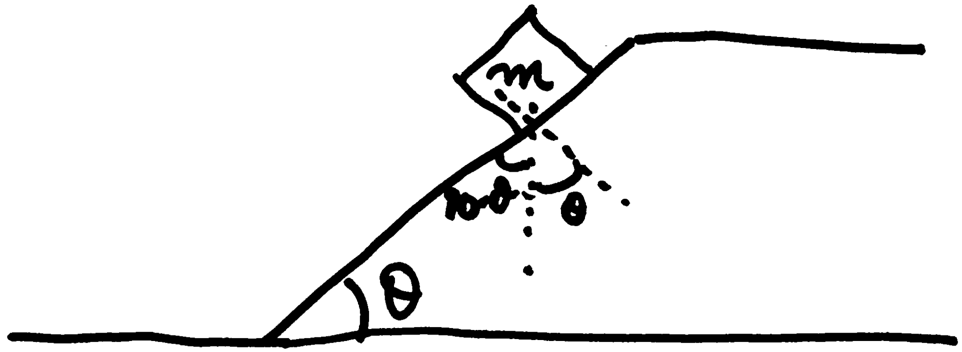
2013-09-19

Last time.....



released from rest.

Slides down a frictionless plane inclined at an angle θ w.r.t. the ground

"with respect to"

Q: 1) What are the forces acting on the block?

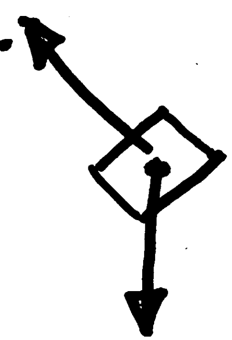
2) what's the acceleration of the block?

NB: accel is a vector! ($\longrightarrow$)

All answers in terms of m, θ, and g

Gabe's sol'n :

(1) Draw a FBD on the obj. whose motion concerns us (or obj's)



$F_{plane,m}$ (pseudonym: F_N)

"Force that acts normally in this problem"

mg → just a non-neg. # (the mag. of the Force vector I just drew)

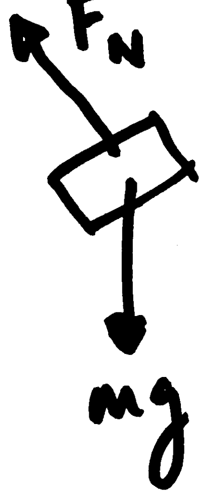
normal = perpendicular (tech math jargon) syn: orthogonal

Notation :

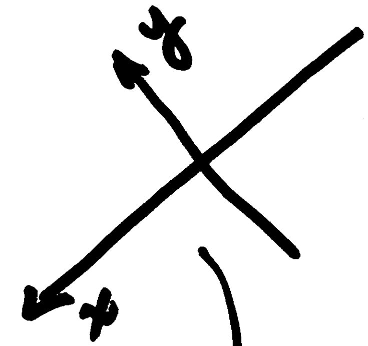
$\vec{F}_{A,B}$ ≡ $\underbrace{\text{Force A applies to B}}$

$F_{A,B}$ ≡ magnitude of "

Cleanup



as of now unknown mag.



(2) Set up a convenient coord system (make this choice EXPLICIT)

Make this choice so that it simplifies subsequent calculation...

<u>GUIDELINES</u>: • if you <u>know</u> that accel in some dir is 0, choose that dir to be an axis (makes $a_y = 0$ in this case)

Another <u>GUIDELINE</u>:

- ~~Try~~ Try to choose your axes so that as many $\vec{F}$'s as possible in your FBD have zero-valued components along one of your axes (in this problem, either axes would leave us w/ 1 force that doesn't point along a coord. direction. Guideline #1 breaks the tie)

(3) Break any $\vec{F}$'s that ~~don't~~ ~~a~~ aren't already point along one of your axes into components ("EXPLODING the FBD")

~~shown in red~~ ~~as~~

$\cos\theta = \frac{adj}{hyp}$

$\boxed{hyp \cdot \cos\theta = adj}$

(4) Apply $\vec{F}_{net} = m\vec{a}$ componentwise to the obj's whose motion you care about

$$y: \quad F_N - mg\cos\theta = m\underbrace{0}_{\vec{a}_y}$$

$$\Big[\ \vec{F}_{N_y} + \underbrace{(\overrightarrow{mg\cos\theta})_y}_{\text{negative b/c it points in } -y\text{-direction}}\ \Big]$$

NB: $\vec{a}_y = 0$ b/c we chose our axes at the outset so that this would be true

not especially informative in this problem... but will be once we add friction

$$\boxed{F_N = mg\cos\theta}$$

$$x: \quad mg\sin\theta = m \cdot \underset{\sim}{a}$$

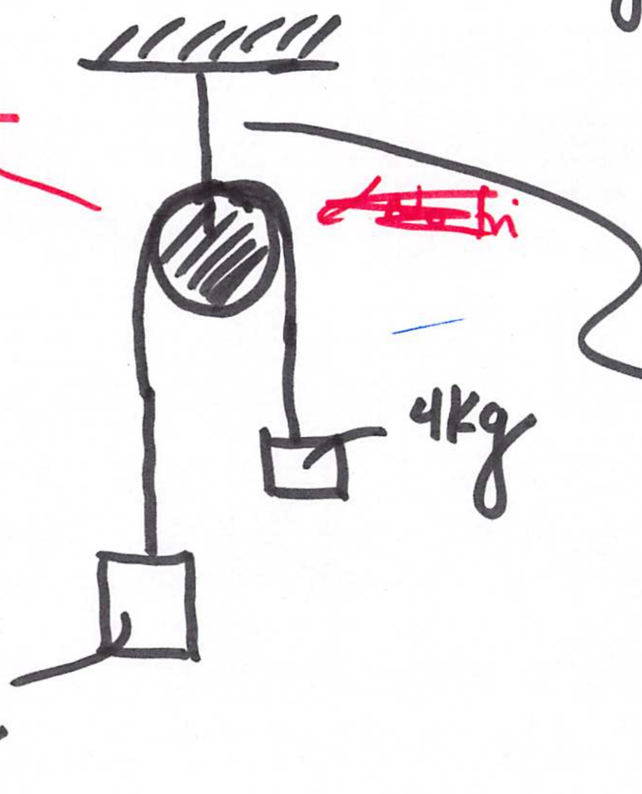
really $\vec{a}_x$, but that's equal to $|\vec{a}|$ b/c $\vec{a}$ points entirely in the $+x$-dir.

$$\cancel{m}g\sin\theta = \cancel{m}a$$

$$\boxed{g\sin\theta = a}$$

(5) Do algebra

New problem:



Q's:

1. what happens?

2. Tension in rope

   5% >

   40% <   combined weight of the blocks?

   55% =

   class votes on the answer

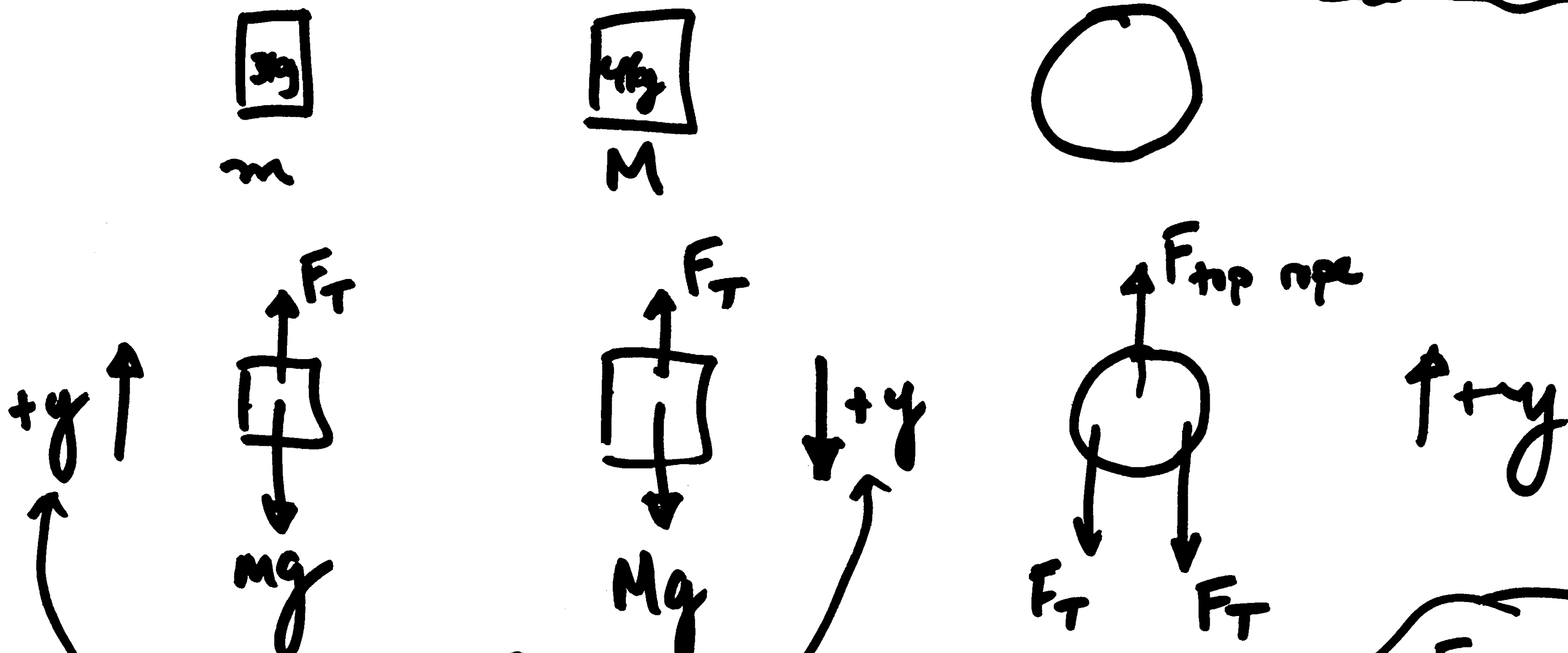
Rope assumptions:

- massless → uniform tension in rope
- inextensible → lets me keep all masses in lockstep ⇒ only one acceleration magnitude to track

SOL'N (same 5 steps as before)

(1) FBD's

NB: Below, step (3) is unnecessary b/c all $\vec{F}$'s already point along our axes



(3) $F_{top\ rope} = 2F_T$

(2) coord sys.

ok to choose diff. axes for every obj.

(4)

m: $F_T - mg = m \cdot a$ (1)

M: $Mg - F_T = M \cdot a$ (2)

same b/c rope doesn't stretch

Now need to solve the sys of eqns...

Step (5) → The algebra (this was not actually covered in the lecture)

We have 3 eq'ns:

$$\left.\begin{array}{ll} (1) & F_T - mg = ma \\ (2) & Mg - F_T = Ma \end{array}\right\} \text{ let's solve these for } F_T\text{, then use (3) to find } F_{\text{top rope}}\ldots$$

(3) $F_{\text{top rope}} = 2F_T$

We could find "a" first and then backsubstitute into (1) or (2) to find F_T, but here's a way to find F_T directly: ELIMINATE "a" from the eq'ns

(→)

$M \times ①: \quad MF_T - Mmg = Mma$

$m \times ②: \quad mMg - mF_T = mMa$

Subtract these eq'ns $\Longrightarrow$ $\underbrace{MF_T + mF_T}_{F_T(M+m)} \underbrace{- Mmg - mMg}_{-2Mmg} = \cancelto{0}{Mma - mMa}$

$$\Longrightarrow \quad F_T(M+m) = 2Mmg$$

$$\boxed{F_T = \frac{2Mm}{(M+m)} g}$$

Sub into ③:

$$F_{\text{top rope}} = 2F_T = \frac{4Mm}{(M+m)} g$$

$m = 3\,kg$, $M = 4\,kg$

Insert #'s:

$$F_{\text{top rope}} = \frac{4(4\,kg)(3\,kg)}{7\,kg} g = \frac{48}{7} kg \cdot g < 7\,kg \cdot g$$

So the magnitude of the tension in the top rope is

$$F_{\text{top rope}} = \frac{48}{7}\,\text{kg} \cdot (10\,\text{m/s}^2) < 7\,\text{kg} \cdot 10\,\text{m/s}^2$$

It's <u>LESS THAN</u> the combined weight of the blocks. How can that be?

Ask Hogg next time....