NYU Physics I

- QS.
- Final Exam
- $E = mc^2$

2018-12-13.

Chs. 5, 6.

rest mass

4-momentum: $\mathbb{P} = (\gamma m c, \gamma m \vec{v})$

3-vector $\vec{v}$

γ corresponding to $\vec{v}$

$$\mathbb{P}^2 = \gamma^2 m^2 c^2 - \gamma^2 m^2 \vec{v}^2$$

$$\cancel{= m^2 c^2 (\gamma^2} = \gamma^2 m^2 c^2 (1 - \beta^2)$$

$$= m^2 c^2$$

(1) $$\frac{\mathbb{P}^2}{c^2} = m^2$$

"magnitude" of $\mathbb{P}$ is rest mass.

$$\mathbb{P} = \left(\frac{E}{c}, \vec{p}\right)$$

$$m^2c^2 = \mathbb{P}^2 = \frac{E^2}{c^2} - \vec{p}^{\,2}$$

$$\boxed{E^2 = m^2c^4 + \vec{p}^{\,2}c^2}$$

$\lim_{\vec{p}\to 0}$: $E = mc^2$ ← Hollywood

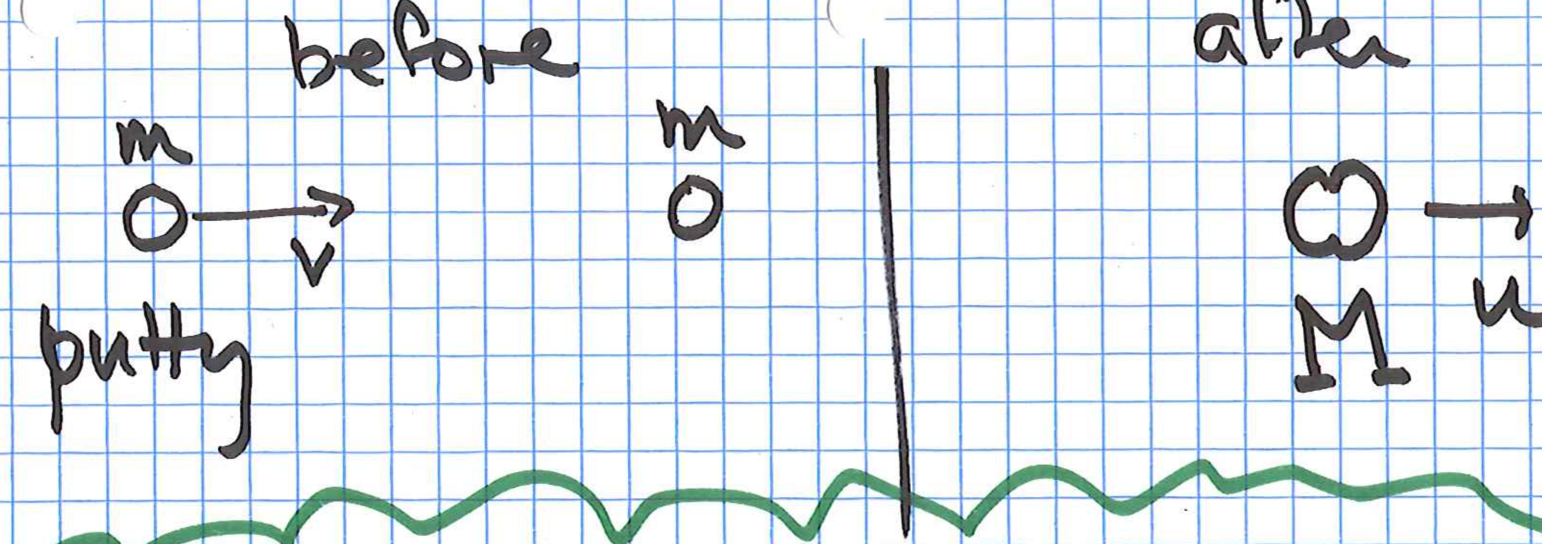
$\lim_{m\to 0}$ $E = |\vec{p}|c$ ← photons.

$$\frac{E}{c} = \gamma mc \qquad \vec{p} = \gamma m\vec{v}$$

$$E = \gamma mc^2$$

$$= \underbrace{mc^2}_{\text{rest energy}} + \underbrace{(\gamma-1)mc^2}_{\text{kinetic energy}}$$

$\sim \frac{1}{2}mv^2$ @ low v.



non-rel. $M = 2m$ $\quad u = \frac{v}{2}$

$KE_{before} = \frac{1}{2} m v^2$ $\quad KE_{after} = \frac{1}{2} M u^2 = \frac{1}{4} m v^2$

$\mathbb{P} = (\gamma mc, \gamma m\vec{v}) + (mc, \vec{0})$ $\quad \mathbb{P}_f = (\gamma_f Mc, \gamma_f M\vec{u})$

$\mathbb{P}_f = (\gamma mc + mc, \gamma m\vec{v}) = (\gamma_f Mc, \gamma_f M\vec{u})$

$$M^2c^2 = (\gamma mc + mc)^2 - (\gamma m\vec{v})^2$$

$$= (\gamma^2 + 2\gamma + 1)m^2c^2 - \gamma^2 m^2 \vec{v}^2$$

m^2c^2

$$= m^2c^2(1 + 2\gamma + 1) = m^2c^2(2\gamma + 2)$$

$$M = m\sqrt{2\gamma + 2}$$

$$\lim_{\gamma \to 1} M = 2m \ !!$$

$$\beta_f = \frac{\gamma m\vec{v}}{\gamma mc + mc} = \frac{\gamma}{\gamma + 1}\frac{\vec{v}}{c}$$

$$\vec{u} = \frac{\gamma}{\gamma + 1} \cdot \vec{v}$$

$$\lim_{\gamma \to 1} \vec{u} = \frac{\vec{v}}{2}$$

@ low v: $\gamma = 1 + \frac{1}{2}\beta^2$]-

$$M = m\sqrt{2\gamma + 2} = m\sqrt{2 + \beta^2 + 2} = m\sqrt{4 + \beta^2}$$

$$M = 2m\sqrt{1 + \frac{\beta^2}{4}} = 2m + m\frac{\beta^2}{4}$$

$(1+\epsilon)^{1/2} = 1 + \frac{1}{2}\epsilon + \ldots$

leading order correction to $M = 2m$

The energy "lost to heat" is $\underline{\text{not}}$ $\underline{\text{lost}}$ — it is converted into mass.