NYU Physics I—potential energy

For a conservative force, the potential energy is the (negative of the) line integral of the force and the force is the (negative of the) derivative of the potential. This is related to the fundamental theorem of calculus but it is also very fundamental in physics. Here we do a non-trivial example.

1 Consider a particle subject to a potential energy of the form

$$U = \frac{B}{x^2} - \frac{A}{x} \quad , \tag{1}$$

where A and B are positive constants. If x is measured in m, what are the units of A and B?

Consider another particle subject to a potential energy of the form

$$U = \frac{1}{2}\,k\,(x - x_0)^2 \tag{2}$$

where k is a constant. If x is measured in m, what are the units of k?

2 Sketch both potentials U vs x.

3 Take derivatives for each potential, and use these to get the "force law" for each potential. Plot the x component of the force as a function of position x. Be careful about the direction (sign) of the force.

4 Find the equilibrium position (or positions, if there are more than one) x_{eq} for each of the potentials. That is, compute the x positions where there is no force and the potential is at a minimum.

5 Take a second derivative for each potential at the (or an) equilibrium point. What are the units of this second derivative? Interpret it physically in terms of force and distance. What does this have to do with the Taylor series? Discuss.

6 In the second potential (2), you know what the dynamics are: what happens to the system as a function of time if displaced from equilibrium? How does this relate to what happens in the first potential (1)? There is a limit in which these look *very similar*; what is it?

7 Of what kind of system might potential (1) be a model?