Bernoulli Distribution:

This distribution best describes all situations where a "trial" is made resulting in either "success" or "failure," such as when tossing a coin, or when modeling the success or failure of a surgical procedure. The Bernoulli distribution is defined as:

$$f(x) = p^x (1-p)^{1-x}, \quad \text{for } x = 0, 1,$$

where

p is the probability that a particular event (e.g., success) will occur.

Binomial Distribution

Suppose we repeat a Bernouilli p experiment n times and count the number X of successes, the distribution of X is called the Binomial $B(n,p)$ random variable.

Probability mass function:

$$P(X = k) = \binom{n}{k} p^k q^{(n-k)},$$

where $q = 1 - p, \quad k=0, 1, 2, ..., n.$

$$E(X) = np$$

$$Var(X) = np(1-p)$$

Odds Ratios and Mode: The odds of k successes relative to (k-1) successes are:

$$\frac{P(X = k)}{P(X = k - 1)} = \frac{n - k + 1}{k}\frac{p}{q}$$

This is very useful for computing by recursion the probability mass of the binomial.

Property:

For X a B(n,p) random variable with probability of success p neither 0 or 1, then as k

varies from 0 to n, $P(X=k)$ first increases monotonically and then decreases monotonically, (it is unimodal) reaching its highest value when k is the largest integer less or equal to $(n+1)p$.

Proof:

$$P(X=k) \geq P(X=k-1)$$

is equivalent to

\begin{displaymath}(n-k+1)p\geq k(1-

The value where the the probability mass function takes on its maximum is called the mode.

Examples:

1. A manufacturer of nails claims that only 3% of its nails are defective. a random sample of 24 nails is selected, and it is found that two of them are defective. Is it fair to reject the manufacturer's claim based on this observation.

2. A certain rare blood type can be found in only 0.05% of people. If the population of a randomly selected group is 3000, what is the probability that at least two persons in the group have this rare blood type.