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UNIT-II   Advanced Differential calculus

- Maxima & minima of function of two or more variable

working rule –

i, write a given func^n & find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ & $\frac{\partial u}{\partial z}$

ii, put $\frac{\partial u}{\partial x} = 0$ — i, & $\frac{\partial u}{\partial y} = 0$ — ii, solving eq. i, & eq. ii, suppos $x = a$ & $y = b$ is sol^n of eq^n these point (a, b) is critical point.

iii, Find $r = \frac{\partial^2 u}{\partial x^2}$, $s = \frac{\partial^2 u}{\partial x . \partial y}$, $t = \frac{\partial^2 u}{\partial^2 y}$ at point (a, b)

iv, calculate $rt - s^2$ & examine following cases –

(a) $rt - s^2 > 0$, $r > 0$ then point (a, b) is minima & $f(a, b)$ is minimum value of f^n.

(b) $rt - s^2 > 0$, $r < 0$ then point (a, b) is maxima & $f(a, b)$ is maximam value of f^n

(c) $rt - s^2 < 0$, then point (a, b) is neither maxima nor minima called saddle point.

d) $rt - s^2 = 0$, then further investigation is required

v, Put the value on the given func^n then find u_{max} & u_{min}

Q Discuss the max & min value of the function $x^3y^2(1-x-y)$

Sol $u = x^3y^2(1-x-y) = x^3y^2 - x^4y^2 - x^3y^3$

now, $\frac{\partial u}{\partial x} = 3x^2y^2 - 4x^3y^2 - 3x^2y^3$

$\frac{\partial u}{\partial y} = 2x^3y - 2x^4y - 3x^3y^2$

Put $\frac{\partial u}{\partial x} = 0$ & $\frac{\partial u}{\partial y} = 0$

$x^2y^2(3 - 4x - 3y) = 0$ & $x^3y(2 - 2x - 3y) = 0$

$4x + 3y - 3 = 0$ — i, & $2x + 3y - 2 = 0$ — ii,

now solving eq i, & ii,

$$4x + \cancel{3y} - 3 = 0$$
$$2x + \cancel{3y} - 2 = 0$$
$$2x - 1 = 0$$
$$x = 1/2$$

put eq ii,

$$2 \times \frac{1}{2} + 3y - 2 = 0$$
$$y = \frac{1}{3}$$

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The critical point is $\left(\frac{1}{2}, \frac{1}{3}\right)$, then.

$$r = \frac{\partial^2 u}{\partial x^2} = 6xy^2 - 12x^2y^2 - 6xy^3.$$

$$s = \frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = 6x^2y - 8x^3y - 9x^2y^2.$$

$$t = \frac{\partial^2 u}{\partial y^2} = 2x^3 - 2x^4 - 6yx^3$$

at poin $(1/2, 1/3)$ $r = -\frac{1}{9}$, $s = \frac{-1}{12}$, $t = -\frac{1}{8}$

now, $rt - s^2 = \left(-\frac{1}{9}\right)\cdot\left(-\frac{1}{8}\right) - \left(\frac{-1}{12}\right)^2 = \frac{1}{72} - \frac{1}{144} = +ve.$

& $r < 0$

Then, Umax at $\left(\frac{1}{2}, \frac{1}{3}\right) = \frac{1}{8} \times \frac{1}{9}\left(1 - \frac{1}{2} - \frac{1}{3}\right)$

$$= \frac{1}{72} \times \frac{1}{6} \Rightarrow \frac{1}{432} \text{ Ans}$$

- Partial differential.

Q if $u = (x^2+y^2+z^2)^{-1/2}$ then prove that.

i, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$ & ii, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u$

Sol $u = (x^2+y^2+z^2)^{-1/2}$

on taking power (−2) in both side.

$u^{-2} = x^2+y^2+z^2$ — i,

Partially diff w r t x

$$-\cancel{2}u^{-3}\cdot\frac{\partial u}{\partial x} = \cancel{2}x$$

$$\frac{\partial u}{\partial x} = -xu^3.$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\left[u^3 + 3u^2x\cdot\frac{\partial u}{\partial x}\right]$$

$$= -\left[u^3 + 3u^2x\cdot(-xu^3)\right]$$

$$= 3x^2u^5 - u^3$$

- Partial diff eq-i, w r t y.

$$-2u^{-3}\frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial u}{\partial y} = -yu^3$$

similarly, $\frac{\partial^2 u}{\partial y^2} = 3y^2u^5 - u^3$

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Similarly $\dfrac{\partial u}{\partial z} = -zu^3$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2u^5 - u^3$$

i, $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

$= 3x^2u^5 - u^3 + 3y^2u^5 - u^3 + 3z^2u^5 - u^3$

$= 3u^5(x^2+y^2+z^2) - 3u^3$

$= 3u^5(u^{-2}) - 3u^3$

$= 0.$ H.P

ii, $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$

$x(-xu^3) + y(-yu^3) + z(-zu^3)$

$= -u^3[x^2+y^2+z^2]$

$= -u^3[u^{-2}]$

$= -u$ H.P.

Q if $u = f\left(\dfrac{y}{x}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

Sol $u = f\left(\dfrac{y}{x}\right)$

$\Rightarrow \dfrac{\partial u}{\partial x} = f'\left(\dfrac{y}{x}\right)\cdot\left(\dfrac{-y}{x^2}\right)$

$\Rightarrow \dfrac{\partial u}{\partial y} = f'\left(\dfrac{y}{x}\right)\cdot\left(\dfrac{1}{x}\right)$

now, $x f'\left(\dfrac{y}{x}\right)\cdot\left(\dfrac{-y}{x^2}\right) + y f'\left(\dfrac{y}{x}\right)\cdot\left(\dfrac{1}{x}\right)$

$= 0$ H.P.

Q if $u = \log(\tan x + \tan y + \tan z)$ then, show that

$$\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$$

Sol. $\left\{\dfrac{\partial u}{\partial x} = \dfrac{1}{\tan x + \tan y + \tan z}\cdot(\sec^2 x + \sec^2 y + \sec^2 z)\right\}$

$u = \log(\tan x + \tan y + \tan z)$

$e^u = e^{\log(\tan x + \tan y + \tan z)}$

$e^u = \tan x + \tan y + \tan z$

now, $e^u\dfrac{\partial u}{\partial x} = \sec^2 x$ ~~$+ \sec^2 y + \sec^2 z$~~

$e^u\dfrac{\partial u}{\partial y} = \sec^2 y$

$e^u\dfrac{\partial u}{\partial z} = \sec^2 z$

now, $\sin 2x\cdot\dfrac{\sec^2 x}{e^u} + \sin 2y\dfrac{\sec^2 y}{e^u} + \sin 2z\dfrac{\sec^2 z}{e^u}$

$\Rightarrow \dfrac{2\sin x \cos x}{\cos^2 x\cdot e^u} + \dfrac{2\sin y\cos y}{\cos^2 y\, e^u} + \dfrac{2\sin z\cos z}{\cos^2 z\, e^u}$

$\Rightarrow \dfrac{2\tan x}{e^u} + \dfrac{2\tan y}{e^u} + \dfrac{2\tan z}{e^u}$

$\Rightarrow 2\left(\dfrac{e^u}{e^u}\right) = 2$ Ans

Euler's theorem :- Applied when given fuc^n is homogenous.

State & prove Euler's theorem for homogenous function of degree 'n'.

Statement : if u is a homogenous function of degree 'n' in two variable then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

Proof : u is homogenous function of degree 'n' in two variable (x, y) then

$$u = x^n f\left(\frac{y}{x}\right) \quad \text{— i)}$$

∴ Partially diff w r t x

$$\frac{\partial u}{\partial x} = nx^{n-1} \cdot f\left(\frac{y}{x}\right) + x^n \cdot f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)$$

multiply by x.

$$x\frac{\partial u}{\partial x} = nx^n \cdot f\left(\frac{y}{x}\right) - x^{n-1} y \cdot f'\left(\frac{y}{x}\right) \quad \text{— ii)}$$

∴ Partially diff w r t y.

$$\frac{\partial u}{\partial y} = x^n f'\left(\frac{1}{x}\right)$$

multiply by y

$$y\frac{\partial u}{\partial y} = x^n y f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) \quad \text{— iii)}$$

now adding eq ii) & iii) , then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nx^n f\left(\frac{y}{x}\right) - \cancel{x^{n-1} y f'\left(\frac{y}{x}\right)}$$

$$+ \cancel{x^{n-1} y f'\left(\frac{y}{x}\right)}$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nx^n f\left(\frac{y}{x}\right)$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$$ Hence proved.

Note : if u is a homogenous function then $\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}$

Relation b/w 2nd order derivative of homogenous function where n is degree of homogenous func.

i. $x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial x \cdot \partial y} = (n-1) \frac{\partial u}{\partial x}$

ii. $y \frac{\partial^2 u}{\partial y^2} + x \frac{\partial^2 u}{\partial x \cdot \partial y} = (n-1) \frac{\partial u}{\partial y}$

iii. $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \cdot \partial y} = n(n-1) u$.

⇒ if u is not a homogenous func. but $f(u)$ is homogenous fun' of degree n

i. $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{f(u)}{f'(u)}$

ii. $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \cdot \partial y} = g(u)[g'(u) - 1]$ where, $g(u) = n \frac{f(u)}{f'(u)}$

Q if $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.

Sol. Here, u is non-homogenous function, then

$$\sin u = \frac{x^2+y^2}{x+y}$$

$$\sin u = \frac{x^2(1+(y/x)^2)}{x(1+y/x)} = x f\left(\frac{y}{x}\right)$$

i.e $\sin u$ is homogenous func of degree '1'.

By euler's theorem

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n u$$

$$x \frac{\partial \sin u}{\partial x} + y \frac{\partial \sin u}{\partial y} = \sin u$$

$$x \cos u \cdot \frac{\partial u}{\partial x} + y \cos u \cdot \frac{\partial u}{\partial y} = \sin u$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u \quad \text{H.P.}$$

Q if $u = \sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that.

i. $x\frac{\partial u}{\partial x} + y\frac{\partial v}{\partial y} = \frac{1}{2}\tan u$

ii. $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\sin u \cos 2u}{4\cos^3 u}$

Sol. $\sin u = \frac{x-y}{\sqrt{x}\;\sqrt{y}} = \frac{\overset{\sqrt{x}}{x}(1-y/x)}{\sqrt{x}(1-\sqrt{y/x})} = x^{1/2} f\left(\frac{y}{x}\right)$

Here, $f(u) = \sin u$ which is homogenous of x & y in degree $\frac{1}{2}$

By euler's theorem,

$$x\frac{\partial \sin u}{\partial x} + y\frac{\partial \sin u}{\partial y} = \frac{1}{2}\sin u$$

$$x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial x} = \frac{1}{2}\sin u.$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{1}{2}\frac{\sin u}{\cos u} = \frac{1}{2}\tan u \quad \text{H.P.}$$

now, $g(x) = n\, f(u)/f'(u) = \frac{1}{2}\frac{\sin u}{\cos u} = \frac{1}{2}\tan u$ $\quad g'(x) = \frac{1}{2}\sec^2 u$

we know that

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \cdot \partial y} = g(x)[g'(x)-1]$$

$$= \frac{1}{4}\tan u[\sec^2 u - 1 - 1] = \frac{1}{2}\tan u\left[\frac{1}{2}\sec^2 u - 1\right]$$

$$= \frac{1}{4}\tan u[\tan^2 u - 1]$$

$\sec^2 x = 1 + \tan^2 x.$

$$= \frac{1}{4}\frac{\sin^3 u}{\cos^3 u} - \frac{\sin u}{\cos u} \qquad = \frac{1}{4}\frac{\sin u}{\cos u}\times\frac{1}{\cos^2 u} - \frac{1}{2}\frac{\sin u}{\cos u}$$

$$= \frac{1}{4}\left(\frac{\sin^3 u - \sin u\cos^2 u}{\cos^3 u}\right) \qquad = \frac{1}{4}\frac{\sin u - 2\sin u\cos^2 u}{\cos^3 u}$$

$$= \frac{1}{4}\frac{\sin u(\sin^2 u - \cos^2 u)}{\cos^3 u} = \frac{1}{4}\frac{\sin u\cos 2u}{\cos^3 u} \quad \text{Ans}$$

Jacobian :-

Let u, v are func of variables x & y that is $u = u(x,y)$, $v = v(x,y)$ than the jacobian of u & v with respect to x & y is denotated by $\frac{\partial(u,v)}{\partial(x,y)}$ or $J(u,v)$ is defined as

$$J(u,v) = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \partial u/\partial x & \partial v/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix}$$

Similarly, $u = u(x,y,z)$ then
$v = v(x,y,z)$
$w = w(x,y,z)$

$$j(u,v,w) = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \partial u/\partial x & \partial v/\partial x & \partial w/\partial x \\ \partial u/\partial y & \partial v/\partial y & \partial w/\partial y \\ \partial u/\partial z & \partial v/\partial z & \partial w/\partial z \end{vmatrix}$$

and

$$J'(u,v) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix}.$$

Q if $x = u(1+v)$, $y = v(1+u)$ find the jacobian of (x,y) with u & v.

Sol $J(x,y) = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1+v & u \\ v & 1+u \end{vmatrix} = (1+v)(1+u) - vu$

Properties :-

i. $JJ' = 1 \Rightarrow \frac{\partial(u\,v)}{\partial(x\,y)} \times \frac{\partial(x\,y)}{\partial(u\,v)} = 1$

ii. if u & v are function of r and s where r and s are function of x & y, then

$$\frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(x,y)}$$

iii. The variable x y u v are connected by implicit func that is $f_1(x,y,u,v) = 0$; where, u & v are implicit func
$f_2(x,y,u,v) = 0$ of x & y then

$$\frac{\partial(u\,v)}{\partial(x,y)} = (-1)^n \frac{\frac{\partial(f_1, f_2)}{\partial(x,y)}}{\frac{\partial(f_1, f_2)}{\partial(u,v)}}$$

where, n is no of independent variable

Q if $u = \frac{2yz}{x}$, $v = \frac{3zx}{y}$ & $w = \frac{4xy}{z}$ calculate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

Sol $J(u,v,w) = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} -\frac{2yz}{x^2} & \frac{2z}{x} & \frac{2y}{x} \\ \frac{3z}{y} & -\frac{3zx}{y^2} & \frac{3x}{y} \\ \frac{4y}{z} & \frac{4x}{z} & -\frac{4xy}{z^2} \end{vmatrix} = \frac{2\times3\times4}{x^2y^2z^2} \begin{vmatrix} -yz & zx & yx \\ zy & -zx & xy \\ yz & xz & -xy \end{vmatrix}$

$$\cancel{= -\frac{2yz}{x^2}\left[\left(\frac{-3zx}{y^2}\right)\left(\frac{-4xy}{z^2}\right) - \left(\frac{4x}{z}\right)\left(\frac{3x}{y}\right)\right] - \frac{2z}{x}\left[\left(\frac{3z}{y}\right)\left(\frac{-4xy}{z^2}\right) - \left(\frac{4y}{z}\right)\left(\frac{3x}{y}\right)\right] + \frac{2y}{x}\left[\left(\frac{3z}{y}\right)\left(\frac{4x}{z}\right) + \left(\frac{3zx}{y^2}\right)\left(\frac{4y}{z}\right)\right]}$$

$$= \frac{24}{x^2y^2z^2}(x^2y^2z^2) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 24(4) = 96$$

Thus $J \cdot J' = 1$

$96 J' = 1 \Rightarrow J' = \frac{1}{96}$ Ans.

Q Find the value of the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$ where $u = x^2 - y^2$, $v = 2xy$

$x = r\cos\theta$, $y = r\sin\theta$.

Sol. $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2x & -2y \\ 2y & 2x \end{vmatrix} = 4x^2 + 4y^2 = 4u$

$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$

now, $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(r,\theta)}$

$4u \times r = \frac{\partial(u,v)}{\partial(r,\theta)}$ Ans.

Q if $f_1 = x^2 + y^2 + u^2 - v^2$ & $f_2 = uv + xy$ then show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2 - y^2}{u^2 + v^2}$

Sol

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{\frac{\partial(f_1,f_2)}{\partial(x,y)}}{\frac{\partial(f_1,f_2)}{\partial(u,v)}} = \frac{\begin{vmatrix} 2x & 2y \\ y & x \end{vmatrix}}{\begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix}}$$

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$$= \frac{2x^2 - 2y^2}{2u^2 + 2v^2} = \frac{x^2 - y^2}{u^2 + v^2} \quad \text{H.P.}$$

function depandance :-

if $J(u, v) \neq 0 \Rightarrow$ Independent

if $J(u, v) = 0 \Rightarrow$ dependent

Q if $u = x^2 + y^2 + z^2$, $v = x + y + z$ & $w = xy + yz + zx$ $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. ~~is~~ vanishes identicaly also find the relationship b/w $(u, w \& v)$

Sol
$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = \begin{vmatrix} 2x & 2y & 2z \\ 1 & 1 & 1 \\ y & z & x \end{vmatrix}$$

$$= 2x(x - z) - 2y(x - y) + 2z(z - y)$$

$$= 2x^2 - 2xz + 2y^2 - 2xy + 2z^2 - 2zy$$

$$= 2(x^2 + y^2 + z^2) - 2(xy + yz + zx)$$

Lagrange's method of undetermined multiplier.

working rule : i) let $f(x,y,z)$ be the fun^n & $\phi(x,y,z)=0$ is any condition.

if $\begin{cases} f(x,y,z,\lambda) = f(x+y+z) + \lambda\,\phi(x,y,z) \\ f(x,y,z,\lambda_1,\lambda_2) = f(x,y,z) + \lambda_1\phi_1(x,y,z) + \lambda_2\phi_2(x,y,z) \end{cases}$

where, λ is Lagrang's multiplier.

eq^n is know as lagrange's auxilary eq^n.

ii) find $\frac{\partial f}{\partial x}=0$, $\frac{\partial f}{\partial y}=0$ & $\frac{\partial f}{\partial z}=0$ and solving them for find x,y,z in terms of λ

iii) By condition $\phi(x,y,z)=0$ put x,y,z & find λ with the help of λ find critical point (x,y,z)

iv) Examine the value of $f(x,y,z)$ at critical point. maxima / minima ??

Q find the maximum & minimum distance of a point $(3,4,12)$ from the sphere $x^2+y^2+z^2=4$.

Sol Let $P(x,y,z)$ be a any point in the sphere $(x^2+y^2+z^2=4)$ and point $A(3,4,12)$

& $\phi(x^2+y^2+z^2-4)=0$ —— i)

now, The distance b/w point P & A

Let $f = AP^2 = (x-3)^2 + (y-4)^2 + (z-12)^2$ —— ii)

~~now~~, $\cancel{\frac{\partial f}{\partial x} = 2(x-3) = 0}$ making Lagrang's auxilary eq^n

$\cancel{x = \frac{3}{2}}$ $f = (x-3)^2 + (y-4)^2 + (z-12)^2 + \lambda(x^2+y^2+z^2-4)$

now, $\frac{\partial f}{\partial x}=0$	$\frac{\partial f}{\partial y}=0$	$\frac{\partial f}{\partial z}=0$
$2(x-3)+2\lambda x=0$	$2(y-4)+2\lambda y=0$	$2(z-12)+2\lambda z=0$
$2x-6+2\lambda x=0$	$2y-8+2\lambda y=0$	$2z-24+2\lambda z=0$
$\cancel{2}x(1+\lambda)=\cancel{6}\,3$	$\cancel{2}y(1+\lambda)=\cancel{8}\,4$	$z(1+\lambda)=\cancel{24}\,12$
$x=\frac{3}{1+\lambda}$ —— iii)	$y=\frac{4}{1+\lambda}$ —— iv)	$z=\frac{12}{1+\lambda}$ —— v)

put value of x, y & z in eq i), then

$$\left(\frac{3}{1+\lambda}\right)^2 + \left(\frac{4}{1+\lambda}\right)^2 + \left(\frac{12}{1+\lambda}\right)^2 = 4$$

$$\frac{9+16+144}{(1+\lambda)^2} = 4$$

$169 = 4(1+\lambda)^2$

$1+\lambda = \pm\frac{13}{2}$

(+ve) $\lambda = 11/2$

(-ve) $\lambda = -15/2$

now, put value of λ in eq iii, iv, v

+ve $(x, y, z) = \frac{6}{13}, \frac{8}{13}, \frac{24}{13}$

-ve $(x, y, z) = -\frac{6}{13}, \frac{-8}{13}, \frac{-24}{13}$

∵ There are two critical point $\left(\frac{6}{13}, \frac{8}{13}, \frac{24}{13}\right)$ & $\left(\frac{-6}{13}, \frac{-8}{13}, \frac{-24}{13}\right)$

$$f\left(\frac{6}{13}, \frac{8}{13}, \frac{24}{13}\right) = \left(\frac{6}{13}-3\right)^2 + \left(\frac{8}{13}-4\right)^2 + \left(\frac{24}{13}-12\right)^2$$

$AP^2 = 121$

$AP = 11$

and $f\left(\frac{-6}{13}, \frac{-8}{13}, \frac{-24}{13}\right) = \left(-\frac{6}{13}-3\right)^2 + \left(-\frac{8}{13}-4\right)^2 + \left(\frac{-24}{13}-12\right)^2$

$AP^2 = 225$

$AP = 15$

Thus, the maximum distance is 15 & minimum is 11. Ans