Partial Differential Equations-

Those equations which contains partial derivatives, independent & dependent variables.

Independent variable will be denoted by 'x' and 'y' and the dependent variable by 'z'

The partial coefficients are denoted as follows:

$$\frac{\partial z}{\partial x} = p \quad , \quad \frac{\partial z}{\partial y} = q \rightarrow \text{First order, First degree.}$$

(Not r, s

$$\downarrow$$

$$\frac{\partial^2 z}{\partial x^2} = r \quad , \quad \frac{\partial^2 z}{\partial x \partial y} = s, \quad \frac{\partial^2 z}{\partial y^2} = t$$

and p, q are higher order DE.

Order and Degree of a partial DE.

It is same as the order and degree of an ordinary DE.

Methods of forming Partial DE.

(1) By Eliminating arbitary constants:

(Que) Form the Partial DE. by Eliminating arbitary const 'a' & 'b'.

$$2x + 0 + 2(z-a)\frac{\partial (z-a)}{\partial x} = 0$$

$$2x + 2(z-a)\left(\frac{\partial z}{\partial x} - \frac{\partial a}{\partial x}\right) = 0$$

$$2x + 2(z-a)p = 0$$

$$(z-a) = \frac{-x}{p} \rightarrow (a)$$

Diff both side in Eq (1) parially wrt 'y'

$$\frac{\partial}{\partial y}\left(x^2+y^2+(z-a)^2\right) = \frac{\partial (b^2)}{\partial y}$$

$$0 + 2y + 2(z-a)\left(\frac{\partial z}{\partial y} - \frac{\partial a}{\partial y}\right) = 0$$

$$2y + 2(z-a)q = 0$$

$$z-a = \frac{-y}{q} \rightarrow (b)$$

From (a) & (b)

$$\frac{-x}{p} = \frac{-y}{q}$$

$\boxed{qx - py = 0}$ required DE.

NOTE: If number of arbitrary

(2) By Eliminating Arbitary functions

(Que) Form partial DE -

$z = f(x^2 - y^2)$ ← arbitary function

Given that,

$$z = f(x^2 - y^2) \quad \rightarrow \text{Eq } ①$$

Diff both side partially wrt 'x'

So we get,

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(f(x^2 - y^2)\right)$$

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2)\,\frac{\partial}{\partial x}(x^2 - y^2)$$

$$\frac{\partial z}{\partial x} = f'(x^2 - y^2)\, 2x.$$

$$f'(x^2 - y^2) = \frac{\partial z}{\partial x}\,\frac{1}{2x} \quad \rightarrow ⓐ$$

$$\text{—''—} = \frac{p}{2x}$$

Now,

Diff Eq ① both side partially wrt 'y'

$$\frac{\partial z}{\partial y} = f'(x^2 - y^2)\,\frac{\partial}{\partial y}(x^2 - y^2)$$

$$\frac{\partial z}{\partial y} = -f'(x^2 - y^2)\cdot 2y$$

$$f'(x^2 - y^2) = -\frac{q}{2y} \quad \rightarrow ⓑ$$

from ⓐ & ⓑ

$$-\frac{q}{2y} = \frac{p}{2x}$$

$$\boxed{qx + py = 0} \quad \text{Req. Patial DE.}$$

1 funct → 1st order 1st degree partial DE.

more than 1 arbitary funct → Higher order 2 degree partial DE

Solution of Partial DE

→ By Direct Integration

(Que) Solve : $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$

We have, $\hookrightarrow$ Eq ①

Int. Eq ① wrt. 'x'

So we get,

$$\frac{\partial^2 z}{\partial x \partial y} = \left[\int \cos(2x+3y)\, \partial x\right] + \phi_1(y)$$

arbitary const in terms of arbitary function

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{2}\sin(2x+3y) + \phi_1(y).$$

$\hookrightarrow$ Eq ②

Now,

Int. Eq ② wrt 'x'.

$$\frac{\partial z}{\partial y} = \left[\int \left[\frac{1}{2}\sin(2x+3y) + \phi_1(y)\right] \partial x\right] + \phi_2(y).$$

$$\frac{\partial z}{\partial y} = \frac{-1}{4}\cos(2x+3y) + \phi_1(y).\int \partial x + \phi_2(y).$$

$$\frac{\partial z}{\partial y} = \frac{-1}{4}\cos(2x+3y) + \phi_1(y)x + \phi_2(y)$$

$\hookrightarrow$ Eq ③

Now, Int Eq ③ wrt 'y'

So, we get,

$$z = \int \frac{-1}{4}\cos(2x+3y)\, dy + \int \phi_1(y)x\, dy + \int \phi_2(y)\, dy + \phi_3(x).$$

$$z = \frac{-1}{12}\sin(2x+3y) + x\underbrace{F_1(y)}_{\int \phi_1(y)\,dy} + \underbrace{F_2(y)}_{\int \phi_2(y)\,dy} + \phi_3(x).$$

Lag Linear Partial DE

An Eq^n of the type.

$$Pp + Qq = R$$

where, P, Q, R are functions of x, y, z

and $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ is called

Lagrenge's linear Partial DE.

And its sol^n is given by

$$\left.\begin{array}{l} f(u, v) = 0 \text{ (OR)} \\ u = f(v) \\ \text{(OR)} \\ v = f(u) \end{array}\right] u, v \text{ functions of } x, y \text{ and } z.$$

Working Rule to solve -

$$Pp + Qq = R$$

① Auxillary Eq^n $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

② Solve the above auxillary Eq^n (A E)
Let the two sol^n be -

$$u = c_1 \quad \text{and} \quad v = c_2$$

③ Then $\left.\begin{array}{l} f(u, v) = 0 \\ u = f(v) \\ v = f(u) \end{array}\right]$ is the required solution -

Grouping Method (By Que).

Que: Solve : $yq - xp = z$ (small p or q → Lag-)

Given that ↳ Eq ①

compare partial DE ① with

$Pp + Qq = R$.

So, $P = -x$

$Q = y$

$R = z$

So, AE will be -

We know that,

for $Pp + Qq = R$

is given by -

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

So, AE for partial DE ① will be

$$\frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$$

For 1st sol^n, we are taking 1st two fraction

i.e. $\frac{dx}{-x} = \frac{dy}{y} \Rightarrow \frac{dx}{x} + \frac{dy}{y} = 0$

which on int gives -

$\ln x + \ln y = \ln C_1$

$\ln (xy) = \ln (C_1)$

After taking anti Log-

$xy = C_1$ (constant).

Now for 2nd sol^n - we are taking last 2 frac

i.e. $\frac{dy}{y} = \frac{dz}{z} \Rightarrow \frac{dy}{dy} - \frac{dz}{z} = 0$

After int

$\ln y - \ln z = \ln C_2$

After taking anti ln

$\frac{y}{z} = C_2$

So, Required sol^n is -

$f[(x, y), (y/z)] = 0$

$xy = f(y/z)$

OR

$y/z = f(xy)$

Que: Solve: $y^2 p - xyq = x(z-2y)$

given that

Eq (1)

Comparing eq (1) with $Pp + Qq = R$

we get,

$P = y^2$

$Q = -xy$

$R = x(z-2y)$

We know that,

Integrating by both sides

$$\boxed{x^2 + y^2 = c_1}$$

Now, For 2nd soln taking last 2n fractions

$$\frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

$$\frac{(z-2y)}{y} dy + dz = 0$$

$$z dy - 2y dy + y dz = 0$$

$$zy - 2y^2 + zy = c_2$$

$$\boxed{2(zy - y^2) = c_2}$$

Method of multipliers -

Let the AE be -

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Let l, m, n may be constants or functions of x, y, z.

Then we have -

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R} = \frac{l dx + m dy + n dz}{lP + mQ + nR}$$

→ so l, m, n are selected in such a way that $\boxed{lP + mQ + nR = 0}$

So, $l dx + m dy + n dz = 0$

which after integration gives 1st set of soln

i.e. $u = c_1$

Illy we select 2nd set of multiplier l_1, m_1, n_1 for the 2nd soln -

i.e. $v = c_2$

$\therefore$ $f(u, v) = 0$ (OR) $u = f(v)$ (OR) $v = f(u)$

is the req. soln.

Solve 1: $x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = z.(x^2-y^2)$

given that, → Eq ①

Compare with $Pp + Qq = R$. with eq ②

$x(y^2+z)\,p - y(x^2+z)\,q = z(x^2-y^2)$

$P = x(y^2+z)$

$Q = -y(x^2+z)$

$R = z(x^2-y^2)$

Now, AE for Eq ①

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$$

(Grouping Not Applicable as one variable can not be cancelled completely in any 2 fraction).

Suppose, it's 1st set of multiplier-
$\boxed{(x, y, -1)}$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} = \frac{x\,dx + y\,dy - 1\,dz}{x^2y^2 + x^2z - x^2y^2 - y^2z - zx^2 + zy^2}$$

Each fraction $= \frac{x\,dx + y\,dy - dz}{0}$

$x\,dx + y\,dy - dz = 0$

After integrating,

$\boxed{x^2 + y^2 - 2z = c_1}$

Now, taking 2nd set of multiplier-

$(\frac{1}{x}, \frac{1}{y}, \frac{1}{z})$

Each fraction $= \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

Int. by both side

$$\ln x + \ln y + \ln z = \ln C_2$$

So, $\boxed{xyz = C_2}$

Que 2: Solve: $x(z^2 - y^2)\frac{dz}{dx} + y(x^2 - z^2)\frac{dz}{dy} = z(y^2 - x^2)$ ↳ Eq ①

It can be written as-

$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ ↳ Eq ②

Comparing it with

$Pp + Qq = R$

we get,

$P = x(z^2 - y^2)$

$Q = y(x^2 - z^2)$

$R = z(y^2 - x^2)$

Now, AE for Eq ① will be.

$$\frac{dx}{x(z^2 - y^2)} = \frac{dy}{y(x^2 - z^2)} = \frac{dz}{z(y^2 - x^2)}$$

Suppose, 1st set of multiplier (x, y, z)

2nd $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$

$(xy^2 + xz^2) - (yx^2 + yz) - (zx^2 - zy^2)$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$xyz = C_2$$

Linear Homogeneous PDE of n^{th} order with const coefficient -

An equation of the type -

$$a_0\frac{\partial^n z}{\partial x^n} + \frac{a_1\partial^n z}{\partial x^{n-1}\partial y} + \frac{a_2\partial^n z}{\partial x^{n-2}\partial y^2} + \cdots + \frac{a_n\partial^n z}{\partial y^n} = \phi(x,y)$$

is known as the linear homogeneous PDE of n^{th} order with cont coeff. → Eq ①

Put $\frac{\partial}{\partial x} = D$ and $\frac{\partial}{\partial y} = D'$

Then, It will be -

$$a_0 D^n + a_1 D^{n-1}D' + a_2 D^{n-2}D'^2 + \cdots + a_n D'^n = \phi(x,y)$$

$$\Rightarrow F(D,D')z = \phi(x,y) \rightarrow \text{Eq } ②$$

$$CS\ (\text{of } 1 \ \& \ 2) = CF + PI \rightarrow \text{Eq } ③$$

Rules for finding the CF -

In the process of finding CF first we find AE by replacing RHS by 0, $D \rightarrow m$, $D' \rightarrow 1$ and $z = 1$ in Eq ②

$\therefore$ AE $\Rightarrow F(m,1) = 0$

ie. $a_0 m^n + a_1 m^{n-1} + a_2 m^{n-2} + \cdots a_n = 0 \rightarrow$ Eq ④

1. Roots of AE are distinct.

$$m = m_1, m_2, m_3, \cdots, m_n$$

$$CF = F_1(y+m_1x) + F_2(y+m_2x) + \cdots + F_m(y+m_ox)$$

2. Roots of AE are equal.

$$m = m_1 = m_2 = \cdots = m_n$$

$$CF = F_1(y+mx) + xF_2(y+mx) + x^2F_3(y+mx) + \cdots + x^nF_n(y+mx).$$

Methods of finding PI

$$F(D, D')z = \phi(x,y)$$

$$PI = \frac{1}{f(D,D')} \phi(x,y)$$

(1) $\phi(x,y) = e^{ax+by}$

$$PI = \frac{1}{f(D,D')} e^{ax+by}$$

$D \to a$; $D' \to b$

$\therefore$ $PI = \frac{1}{f(a,b)} e^{ax+by}$ $[f(a,b) \neq 0]$

If $f(a,b) = 0$ then

$PI = x \frac{1}{\frac{\partial}{\partial D}[f(D,D')]} e^{ax+by}$

D' will be constant here.

$\therefore$ $PI = \frac{x}{f'(a,b)} e^{ax+by}$ $[f'(a,b) \neq 0]$

(2) $\phi(x,y) = \sin(ax+by)$ or $\cos(ax+by)$

$PI = \frac{1}{F(D^2, DD', D'^2)} \sin(ax+by)$ or $\cos(ax+by)$

$D^2 \to -a^2$; $DD' = -ab$, $D'^2 = -b^2$

$\therefore$ $PI = \frac{1}{F(-a^2, -ab, -b^2)} \sin(ax+by)$ or $\cos(ax+by)$

$[F(-a^2, -ab, -b^2) \neq 0]$

If zero, then partially differentiate wrt D.

(3) $\phi(x,y) = x^m y^n$

$PI = \frac{1}{F(D,D')} x^m y^n$

(i) If $m > n$, then $\frac{1}{F(D,D')}$ is expanded in powers $\frac{D'}{D}$.

(ii) If $m < n$, then $\frac{1}{F(D,D')}$ is expanded in powers $\frac{D}{D'}$.

(iii) If $m = n$, then we'll have choice.

(4) $\phi(x,y)$ = Any function in (x,y)

$$PI = \frac{1}{f(D,D')}\phi(x,y)$$

Factorize

$$PI = \frac{1}{(D-m_1D')(D-m_2D')\dots(D-m_nD')}\phi(x,y)$$

$$\frac{1}{(D-m_1D')}\phi(x,y) = \int \phi(x, c+mx)\,dx$$ [$y = c+mx$, c is a constant]

After integrating, we replace c by $y-mx$. We repeat the same process for the remaining factors.

Questions:

(1) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{3\partial^2 z}{\partial x \partial y} + \frac{2\partial^2 z}{\partial y^2} = e^{2x-y} + e^{x+y} + \cos(x+2y)$

Given PDE can be written in the form of differential operation, i.e., $\frac{\partial}{\partial x} = D$, $\frac{\partial}{\partial y} = D'$ as follows:

$z(D^2 - 3DD' + 2D'^2) = e^{2x-y} + e^{x+y} + \cos(x+2y) \rightarrow$ Eq ①

In the process of finding CF first we find AE by replacing RHS → 0, D → m, D' → 1 and z → 1

∴ AE ⇒ $(m^2 - 3m + 2) = 0$

$m = 1, 2.$

∴ CF = $F_1(y+x) + F_2(y+2x) \rightarrow$ Eq ②

Now, $PI = \frac{1}{D^2-3DD'+2D'^2}\left(e^{2x-y} + e^{x+y} + \cos(x+2y)\right)$

$$PI = \frac{1}{D^2-3DD'+2D'^2}e^{2x-y} + \frac{1}{D^2-3DD'+2D'^2}e^{x+y} + \frac{1}{D^2-3DD'+2D'^2}\cos(x+2y)$$

$PI = PI_1 + PI_2 + PI_3$

$PI_1 = \dfrac{1}{D^2 - 3DD' + 2D'^2} e^{2x-y}$

$\Rightarrow D \to 2 \ , \ D' \to 1$

$PI_1 = \dfrac{1}{12} e^{2x-y}$

$PI_2 = \dfrac{1}{D^2 - 3DD' + 2D'^2} e^{x+y}$

$\Rightarrow D \to 1 \ , \ D' \to 1$

$PI_2 = \dfrac{1}{1-3+2=0} e^{x+y}$ (rule fails)

$= x \dfrac{1}{2D - 3D'} e^{x+y}$

$= x \dfrac{1}{2-3} e^{x+y}$

$PI_2 = -x e^{x+y}$

$PI_3 = \dfrac{1}{D^2 - 3DD' + 2D'^2} \cos(x+2y)$

$D^2 = -1 \ , DD' = -2 \ , D'^2 = -4$

$PI_3 = \dfrac{1}{-1+6-8} \cos(x+2y)$

$= \dfrac{-1}{3} \cos(x+2y)$

$PI = \dfrac{1}{12} e^{2x-y} + (-x e^{x+y}) + \left(-\dfrac{1}{3} \cos(x+2y)\right) \rightarrow$ Eq (3)

So, from eq (2) and (3)

$CS = F_1(y+x) + F_2(y+2x) + \dfrac{1}{12} e^{2x-y} - xe^{x+y} - \dfrac{1}{3} \cos(x+2y)$

(2) Solve: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2y^2$

given PDE can be written in the differential opperator form by $\frac{\partial}{\partial x} = D$, $\frac{\partial}{\partial y} = D'$; it follows:

$$z(D^2 + D'^2) = x^2y^2 \quad \rightarrow \text{Eq } ①$$

In the process of finding CF first we find AE by replacing RHS $\rightarrow 0$, $z \rightarrow 1$, $D' \rightarrow 1$, $D \rightarrow m$

$\therefore$ AE $\Rightarrow m^2 + 1 = 0$

$$m = \pm i$$

$\therefore$ $CF = F_1(y + ix) + F_2(y - ix) \quad \rightarrow \text{Eq } ②$

Now,

$$PI = \frac{1}{D^2 + D'^2} x^2y^2$$

$$= \frac{1}{D^2}\left(1 + \frac{D'^2}{D^2}\right)^{-1} x^2y^2$$

$$= \frac{1}{D^2}\left(1 - \frac{D'^2}{D^2} + \cdots\right) x^2y^2 \quad \text{(Binomial equation)}$$

$$= \frac{1}{D^2}\left(x^2y^2 - \frac{1}{D^2} D'^2 (x^2y^2)\right)$$

$$= \frac{1}{D^2}\left(x^2y^2 - \frac{1}{D^2} 2x^2\right)$$

$$= \frac{1}{D^2} x^2y^2 - \frac{1}{D^4} 2x^2$$

$$= \frac{x^4y^2}{12} - \frac{2x^6}{3 \times 4 \times 5 \times 6}$$

$$= \frac{x^4y^2}{12} - \frac{x^6}{180} \quad \rightarrow \text{Eq } ③$$

$\frac{dy^2}{dy^2} = 2$

$\frac{dy^2}{dy^4}$

$\frac{dy^2}{dy^2} \rightarrow 1$, $\frac{d2}{dy^2} \rightarrow 0$

So, from Eq ② and Eq ③

$$CS = F_1(y + ix) + F_2(y - ix) + \frac{x^4y^2}{12} - \frac{x^6}{180}$$

(3) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y-1)e^x$

Given PDE can be written as -

$z(D^2 - DD' - 2D'^2) = (y-1)e^x \rightarrow$ Eq ①

For CF, first we find AE.
RHS → 0 , D → m , D' → 1 , z → 1

AE : $(m^2 - m - 2) = 0$

$m = -1, 2$

$\therefore$ CF $= F_1(y-x) + F_2(y+2x)$

Now,

$PI = \frac{1}{D^2 - DD' - 2D'^2} \underline{(y-1)e^x}$; Any funct^n in x or y

$PI = \frac{1}{(D+D')(D-2D')} (y-1)e^x$

1st factor $\Rightarrow \frac{1}{(D-2D')}(y-1)e^x = \int \underbrace{(c-2x-1)e^x}_{\text{int by parts}} dx$ $\quad$ $\left[\begin{array}{l} y = c - \overset{2\text{ here}}{m}x \\ y = c - 2x \end{array}\right.$

$\frac{1}{D-2D'}(y-1)e^x = \cancel{ex - x^2 ... e^x} - \int -2e^x dx$

$= ce^x - 2xe^x + e^x \quad [c = y+2x]$

$\frac{1}{D-2D'}(y-1)e^x = (y+2x)e^x - 2xe^x + e^x$

$\Rightarrow \frac{1}{D-2D'}(y-1)e^x = (y+1)e^x$ $\quad$ $(m=-1)$ $[y = c+x]$

Now, 2nd factor, $\frac{1}{D+D'}(y+1)e^x = \int (c+x+1)e^x dx$

$= ce^x + \quad + e^x$

$PI = ye^x$

$CS = F_1(y-x) + F_2(y+2x) + ye^x$

Method of Seperation of variables -

In this method, we assume that, the dependent variable is the product of two functions, each of which involves only one of the independent variables. So two ordinary DE are formed.

(Que) Use the method of seperation of variables to solve the give PDE :-

$3\mu_x + 2\mu_y = 0$, when $\mu_x = \frac{\partial u}{\partial x}$; $\mu_y = \frac{\partial u}{\partial y}$

We have,

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0 \rightarrow Eq\ ①$$

independent

Let $\mu(x,y) = X(x)\,Y(y)$.

(OR)

$$u = X \cdot Y \rightarrow Eq\ ②$$

f(x) f(y)

soln of Eq ①

Differentiate Eq ② Partially wrt x.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(X \cdot Y)$$

$$\frac{\partial u}{\partial x} = X' \cdot Y \rightarrow Eq\ ③$$

[$X' \rightarrow$ ordinary
$X' = \frac{dX}{dx}$]

Now,

Diff. Eq ② Partially wrt y..

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(X \cdot Y)$$

$$\frac{\partial u}{\partial y} = X \cdot Y' \rightarrow Eq\ ④$$

[$Y' = \frac{dY}{dy}$]

Substitute value from 3, 4 in eq ①

$3X'Y + 2XY' = 0$ → Eq ⑤

Divide by XY. by both side.

$$\frac{3X'}{X} + \frac{2Y'}{Y} = 0$$

$$\frac{3X'}{X} = -\frac{2Y'}{Y} \rightarrow \text{Eq ⑥}$$

RHS is const for LHS ∴ So we take both eqns are equal to some const. say 'k'.

$$\frac{3X'}{X} = -\frac{2Y'}{Y} = K \rightarrow \text{Eq ⑦}$$

So, And.

$$\frac{3X'}{X} = k \qquad\qquad -\frac{2Y'}{Y} = k$$

$$\frac{X'}{X} = \frac{k}{3} \qquad\qquad \frac{Y'}{Y} = -\frac{k}{2}$$

$$\frac{1}{X}\,dX = \frac{k}{3}\,dx \qquad\qquad \frac{1}{Y}\,dY = -\frac{k}{2}\,dy$$

Int both side   Int. both side

$$\log X = \frac{kx}{3} + \log c_1 \qquad\qquad \log Y = -\frac{k}{2}y + \log c_2$$

$$X = c_1 e^{kx/3} \rightarrow \text{Eq ⑧} \qquad\qquad Y = \frac{c_2}{e^{\frac{k}{2}y}}$$

$$Y = c_2 e^{-\frac{ky}{2}} \rightarrow \text{Eq ⑨}$$

From Eq ⑧ & ⑨ Substitute value in Eq ②

$$u(x,y) = c_1 c_2 e^{\left(\frac{kx}{3} - \frac{ky}{2}\right)}$$

[If cond. given, we might have to find k, c1 or c2]

(Que) Solve: The given PDE:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad ; \text{ where } \quad u(x,0) = 6e^{-3x}$$

We have

$$\frac{\partial u}{\partial x} - \frac{2\partial u}{\partial t} - u = 0 \quad \rightarrow Eq\ ①$$

Let $u(x,y) = X \cdot T \quad \rightarrow Eq\ ②$

Diff both side partially wrt x

$$\frac{\partial u}{\partial x} = X' \cdot T \quad \rightarrow Eq\ ③$$

Diff Eq ② wrt ~~t~~ partially.

$$\frac{\partial u}{\partial y} = X \cdot T' \quad \rightarrow Eq\ ④$$

Sub. ③ & ④ in ①

$$X'T - 2XT' - XT = 0 \quad \rightarrow Eq\ ⑤$$

From ⑦ and ⑧ in ②

$$u(x, y) = C_1 C_2 \, e^{\left(\frac{3k \mp 1}{2}\right) x}$$

Now, Aq, Put $y \to 0$

$$u(x, 0) = C_1 C_2 \, e^{\left(\frac{3k \mp 1}{2}\right) x} = 6 e^{-3x}$$

So, $C_1 C_2 = 6$; $\frac{3k \mp 1}{2} = -3$

$$3k \mp 1 = -6$$

$$k = -\frac{5}{3}$$

→ yaha tak exam me

(3rd unit