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Unit - 04 ELECTROMAGNETISM

1.) Linear charge density = $q = \int_0 \lambda \, dl$

2.) Surface charge density = $q = \iint_S \sigma \, da$

3.) Volume charge density = $q = \iiint_V \rho \, dV$

I Divergence $(\nabla \cdot V)$:

$$\nabla \cdot V = \left(\hat{i}\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz} \right) \cdot \left(iV_x + jV_y + \hat{k}V_z \right)$$

$$\nabla \cdot V = \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)$$

II Curl $(\vec{\nabla} \times \vec{r})$

$$(\vec{\nabla} \times \vec{V}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

Gauss

1.) Divergence theorem :

$$\iint \vec{J} \cdot \vec{da} = \iiint \underset{(\vec{\nabla} \cdot \vec{J})}{\operatorname{div} \vec{J}} \, dv$$

2.) Stok's theorem :

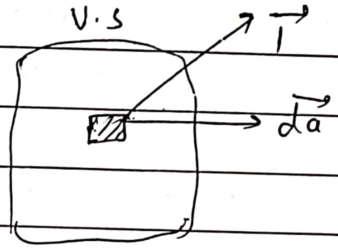
$$\oint_C \vec{B} \, dl = \iint \operatorname{curl} \vec{B} \, da = \iint (\vec{\nabla} \times \vec{B}) \, \vec{da}$$

Continuity equation for charge and current.

If the value of current density J does not change with time current in the system remains constant and it is called steady current. Steady current obey the law of conservation of charge i.e charge can neither be created nor be destroyed. Mathemical exp. of the law of conservation of charge is eqn of continuity

Steady Current :

Small area $\vec{da}$ current $I\vec{da}$



Vs current : $I = \iint \vec{J}\,\vec{da}$ — (1)

for steady current, no accumulation of charge

$0 = \iint I \cdot da$ — (2)

Gauss divergence theorem

$\iint \vec{J}\,\vec{da} = \iiint_v \text{div}\,.\,\vec{J}\cdot dv$ — (3)

or

$\text{div}\,\vec{J} = 0$ — (4)

The net flux of current density to the entire vol. is zero. and this called the eqn of continuity for steady current time independent

Continuity eqⁿ for time varying current:

If the current is not steady current density J will not depend only in position xyz but will also depend on time t: In such case acc to law of conservation of charge at any instant the rate of charge entering in the vol. V enclosed by surface s. = rate of charge leaving out the vol. V enclosed by surface S.

$$\iint \vec{J}\cdot\vec{da} = -\frac{dq}{dt} \quad -(1)$$

V,S

$$q = \iiint \rho\, dv \quad -(2)$$

$$\iint \vec{J}\,\vec{da} = -\frac{d}{dt}\iiint \rho\, dv \quad -(3)$$

$$= -\iiint \frac{d\rho}{dt}\, dv \quad -(4)$$

Gauss divergence theorem,

$$\iiint \mathrm{div}\,\vec{J}\, dv = -\iiint \frac{d\rho}{dt}\, dv \quad -(5)$$

or $\boxed{\mathrm{div}\,\vec{J} = -\frac{d\rho}{dt}}$ or $\vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}$ — (6)

The divergence of current density is equal to the rate of change of volume charge density of charge.

Time independent:

$$\frac{\partial\rho}{\partial t} = 0 \quad -(7)$$

$$\mathrm{div}\,\vec{J} = 0 \quad -(8)$$

Ampere's law: Acc. to this law line integration of magnetic field $\vec{B}$ along closed curve in magnetic field produced due to the flow of current in the conductor is equal μ_0 times the sum of all currents enclosed within that curve.

~~List~~ But for steady current the total current ~~et~~ enclosed within a close curve is equal to the flux of current density linked with the surface enclosed by that curve.

$$\oint_C B \cdot dl = \mu_0 I \quad - (1)$$

$$I = \iint \vec{J} \cdot \vec{da} \quad - (2)$$

$$\oint_C \vec{B}\, dl = \iint \mu_0 \vec{J} \vec{da} \quad - (3)$$

Stok's law
$$\iint \text{curl } \vec{B}\, da = \iint \mu_0 \vec{J} \vec{da} \quad - (4)$$

or
$$\text{curl } \vec{B} = \mu_0 \vec{J}. \quad - (5)$$

or
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad - 5$$

take divergence equation (5)

$$\text{div} [\text{curl } \vec{B}] = \text{div} [\mu_0 \vec{J}] \quad - (6)$$

$$\text{div } \vec{J} = 0 \quad - (7)$$

displacement current density:

Ampere's law ~~for a time~~ modify for the time varying field, maxwell's assumes that magnetic field around a conductor is not produced due by current flowing in it (free e^-) but apart from it a magnetic field is also produced due to varying magnetic field in vaccum or

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dielectric.

$$B = B' + B'' \quad -(1)$$

$$\text{curl } B' = \mu_0 \vec{J}, \quad \text{Curl } B'' = \mu_0 \vec{J_d} \quad -(2)$$

$$\text{Curl } B = [\mu_0 \vec{J} + \mu_0 J_d] \quad -(3)$$

B' = free electron
B'' = displacement of current density

Faraday's law: Acc. to this law an electric field is produced by magnetic field, similarly magnetic field is also produced by varying electric field. Acc. to ~~fo~~ this law of electromagnetic induction emf ~~is~~ induced in coil depends on the rate of change of flux ϕ linked with it.

Consider a coil kept in a time varying field. Let magnetic field at any instant at that place be the magnetic flux ϕ linked with the coil at that instant is

$$\text{flux}, \ \phi = \iint \vec{B} \cdot \vec{da} \quad -(4)$$

$$\text{EMF}, \ \varepsilon = \frac{d\phi}{dt} = \frac{d}{dt} \iint B \, da \quad -(6)$$

$$e = -\iint \frac{d\vec{B}}{dt} da \quad -(7)$$

But if the electric field produced due to varying magnetic field is ε then the line integral of electric field along the loop of a coil will be equal to the emf induced in it.

$$e = \oint_c \vec{E}\, dt \quad -(8)$$

$$\quad -(9)$$

$$-\iint \frac{d\vec{B}}{dt}\, da = \oint_c \vec{E} dt$$

Stok's theorem,

$$\iint \text{curl } \vec{E}\, da = -\iint \frac{d\vec{B}}{dt}\, da \quad -(10)$$

$$\boxed{\text{curl } \vec{E} = -\frac{d\vec{B}}{dt}} \quad -(11)$$

→ 3rd maxwell eqⁿ.

~~Similarly~~

Rate of change of magnetic field can produce electric field.

Similarly, $\text{curl } B'' = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \quad -(12)$

Comparing (3) and (12)

$$\mu_0 \vec{J_d} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \quad -(13)$$

$$\vec{J_d} = \varepsilon_0 \frac{d\vec{E}}{dt} \quad -(14)$$

or: $\boxed{\vec{J_d} = \frac{d\vec{D}}{dt}}$

$$\vec{D} = \vec{\varepsilon_0} \vec{E}$$

↳ Electric displacement

Now,

$$\boxed{\text{curl } B = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}} \quad -(16)$$

↳ (4th) Maxwell eqⁿ

↓ time varying current

Take divergence of eq^n (16)

$$\operatorname{div}[\operatorname{curl}\vec{B}] = \operatorname{div}\left[\mu_0\vec{J} + \mu_0\varepsilon_0\frac{d\vec{E}}{dt}\right]$$

$$0 = \operatorname{div}(\mu_0\vec{J}) + \operatorname{div}\left[\mu_0\varepsilon_0\frac{d\vec{E}}{dt}\right]$$

$$= \mu_0[\operatorname{div}\vec{J}] + \mu_0\varepsilon_0\left[\operatorname{div}\frac{d\vec{E}}{dt}\right]$$

$$= \mu_0[\operatorname{div}\vec{J}] + \mu_0\varepsilon_0\frac{d[\operatorname{div}\vec{E}]}{dt}$$

$$0 = \cancel{\mu_0}\,\operatorname{div}\vec{J} + \frac{\varepsilon_0\times d\rho}{\varepsilon_0\times dt}$$

$$\boxed{\operatorname{div}\vec{J} = -\frac{d\rho}{dt}}$$

Maxwell's theory:

1.) $\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$ 2) $\operatorname{div}\vec{B} = 0$ 3.) $\operatorname{curl}\vec{E} = -\frac{d\vec{B}}{dt}$

4.) $\operatorname{Curl}\vec{B} = \mu_0\left[\vec{J} + \varepsilon_0\frac{d\vec{E}}{dt}\right]$

The relation expressing the divergence and curl of magnetic field are known as maxwell's equation.

Consider a volume V enclosed by a surface S in vaccum acc. to gauss law the total electric flux passing normally through a closed surface is equal to $1/\varepsilon_0$ times the total charge enclosed within that curve

$$\iint \vec{E}\cdot da = \frac{q}{\varepsilon_0} \quad -(1)$$

$$q = \iiint \rho\, dv \quad -(2)$$

$$\iint \vec{E}\cdot \vec{da} = \iiint \frac{\rho}{\varepsilon_0}\, dv \quad -(3)$$

GDT

$$\iiint div.\vec{E}\cdot dv = \iiint \frac{\rho}{\varepsilon_0} \quad -(4)$$

or $$div\ \vec{E} = \frac{\rho}{\varepsilon_0} \quad -(4)$$

$$div\ \vec{D} = \rho \quad -(5)$$

(Electric displacement)

2) $div\ \vec{B} = 0$. (magnetic monopoles do not exist)

Magnetic lines of force r closed curves, their is no existence of magnetic pole, thus the no. of magnetic lines of force entering a surface in a magnetic field r equal to no. of lines leaving the field. Hence magnetic flux linked with a closed surface in a magnetic field is always zero.

$$\iint \vec{B}\cdot \vec{da} = 0 \quad -(1)$$

G.D.T

$$\iiint div\ \vec{B}\cdot dv = 0 \quad -(2)$$

or $$div\ \vec{B} = 0 \quad -(3)$$

3^{rd} and 4^{th} eq^n also needs to be proved.

Wave eq^n for electric field (E) and magnetic field (H).

In the free space their is no charge ($\rho = 0$) & hence their is no current

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$(J = 0)$

1) $\text{div}\ \vec{E} = 0$ 2.) $\text{div}\ \vec{B} = 0$, 3.) $\text{curl}\ \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}$

4) $\mu_0\ \text{curl}\ \vec{H} = \mu_0 \left[0 + \varepsilon_0 \frac{d\vec{E}}{dt}\right]$ $\text{curl}\ \vec{H} = \varepsilon_0 \frac{dE}{dt}$

Take curl of eq^n (3)

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu_0 \frac{\partial}{\partial t}[\vec{\nabla} \times \vec{H}] \quad - (5)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E} \quad - (6)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \vec{\nabla}^2\vec{E} = -\mu_0 \frac{\partial}{\partial t}[\vec{\nabla} \times H] \quad - (7)$$

$$0 - \vec{\nabla}^2\vec{E} = -\mu_0 \frac{\partial}{\partial t}\left[\varepsilon_0 \frac{\partial E}{\partial t}\right] \quad - (8)$$

$$\nabla^2\vec{E} = \varepsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2} \quad - (9)$$

↓

wave eq^n for electric field.

wave eq^n for magnetic field:

Take curl of eq^n (4),

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{H}] = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) \quad - (10)$$

$$\vec{\nabla}(\vec{\nabla} \times \vec{H}) - \vec{\nabla}^2\vec{H} = \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) \quad - (11)$$

$$0 - \vec{\nabla}^2\vec{H} = \varepsilon_0 \frac{d}{dt}\left(-\mu_0 \frac{d\vec{H}}{dt}\right) \quad - (12)$$

$$\nabla^2\vec{H} = \varepsilon_0\mu_0 \frac{\partial^2 H}{\partial t^2} \quad - (13)$$

wave eq^n for magnetic field

(1D) $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ ← Plane progressive wave equation

(3D) $\nabla^2 y = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ — (14)

from eq^n (14), (9), ~~(8)~~ & (13),

$\frac{1}{c^2} = \mu_0 \varepsilon_0 \Rightarrow \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$ — (15)

In Dielectric med.

+ There are no fundamental charge carriers [in] dielectric medium hence the current densi[ty] in dielectric med is zero.

+ If the dielectric is isotropic there is no charge volume distribution of charge in the medium that means volume charge density is zero ($\rho = 0$)

+ In an isotropic dielectric med: the direction of electric disp. vector D is ~~to~~ same as the electric field $\vec{E}$. $D = \varepsilon \vec{E}$ ε = permittivity in dielectric

+ In an isotropic dielectric med. $B = \mu H$ where μ = permeability in dielectric med.

div $\vec{E} = 0 = \vec{\nabla} \cdot \vec{E}$

div $\vec{B} = 0 = \vec{\nabla} \cdot B$

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3.) $\text{curl } \vec{E} = -\mu_0 \frac{d\vec{H}}{dt}$

4.) $\text{curl } \vec{H} = \frac{\varepsilon dE}{dt}$

~~For~~ Take curl of eq^n (3)

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu_0 \frac{d}{dt}[\vec{\nabla} \times \vec{H}] \quad -(5)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\nabla \cdot \vec{E}) - \vec{\nabla}^2 \vec{E} \quad -(6)$$

$$\vec{\nabla}[\vec{\nabla} \cdot \vec{E}] - \nabla^2 E = -\mu_0 \frac{d}{dt}[\vec{\nabla} \times H] \quad -(7)$$

$$0 - \nabla^2 \cdot E = -\mu \frac{d}{dt}\left[\varepsilon \frac{dE}{dt}\right] \quad -(8)$$

$$\nabla^2 \vec{E} = \mu \varepsilon \frac{d^2 \vec{E}}{dt^2} \quad -(9)$$

wave eq^n for electric field

Take curl of eq^n (4)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \varepsilon \frac{d}{dt}(\vec{\nabla} \times \vec{E}) \quad -(10)$$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{H}) - \vec{\nabla}^2 H = \frac{\varepsilon d}{dt}[\vec{\nabla} \times E) \quad -(11)$$

$$0 - \nabla^2 H = \varepsilon\left[-\mu \frac{d^2 H}{dt^2}\right] \quad -(12)$$

$$\vec{\nabla}^2 \vec{H} = \mu \varepsilon \frac{d^2 \vec{H}}{dt^2} \quad -(13)$$

wave eq^n for magnetic field

1D $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$ — Plane progression wave equation

$$\nabla^2 y = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad -(14)$$

from eq^n (14), (9) & (13)
(15)

$$v^2 = \frac{1}{\varepsilon\mu} = \frac{1}{\varepsilon_0\varepsilon_r\mu_0\mu_r}$$

$$v^2 = \frac{c^2}{\varepsilon_r\mu_r} \quad - (16)$$

$$v = \frac{c}{n} \quad - (17) \qquad \boxed{n = \sqrt{\varepsilon_r\mu_r}}$$

Energy density in electromagnetic field :

Consider a vol. V enclosed by closed surfaces wi[thin] a medium in which an electromagnetic wave propogates..

1.) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

2.) $\vec{\nabla} \times \vec{H} = \left[J + \varepsilon \frac{\partial E}{\partial t} \right]$

$$H \cdot (\vec{\nabla} \times \vec{E}) = -H\frac{\partial B}{\partial t} \quad - (3)$$

$$E \cdot (\vec{\nabla} \times \vec{H}) = E \cdot J + E\varepsilon \frac{dE}{dt} \quad - (4)$$

(4) - (3)

$$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) - \vec{H} \cdot (\vec{\nabla} \times \vec{E}) = E \cdot J + E\varepsilon\frac{dE}{dt} + H\frac{dB}{dt} \quad (5)$$

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A(\vec{\nabla} \times B) \quad - (6)$$

$$\nabla \cdot (E \times H) = H \cdot (\nabla \times E) - \vec{E}(\vec{\nabla} \times \vec{H}) \quad - (7)$$

$$-\nabla\cdot(\vec{E}\times\vec{H}) = E\cdot J + \varepsilon E\frac{dE}{dt} + \mu H\frac{dH}{dt} \quad -(8)$$

$$= \vec{E}\cdot\vec{J} + \frac{d}{dt}\left(\frac{1}{2}\varepsilon\vec{E}\right) + \frac{d}{dt}\left(\frac{1}{2}\mu H^2\right)$$

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{d}{dt}\left[\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right] \quad -(7)$$

V.S.

$$\iiint \vec{E}\cdot\vec{J}\,dv + \iiint \frac{d}{dt}\left[\frac{1}{2}\varepsilon\vec{E}^2 + \frac{1}{2}\mu H^2\right]dv = -\iiint \vec{\nabla}\cdot(E\times H)\,dv \quad -(10)$$

$\cancel{H\cdot[\vec{\nabla}\times\vec{E} = \dot{dB}]}$

$\cancel{\frac{1}{2}\varepsilon E^2}$ v, dq, F, dl

Lorenz's force

$$F = dq\left[\vec{E} + \vec{v}\times\vec{B}\right] \quad -(11)$$

work done

$$dw = Fdl = dq\left[\hat{E} + v\times B\right]dl \quad -(12)$$

$$= dqEdl + dq(v\times B)\,dl$$

$$dw = dqEdl \quad -(13)$$

$$dq = \rho dv, \qquad \rho v = J$$

work done on charge v, s (↑ in mechanical energy)

$$dw = \iiint E\cdot J\,dv\,dt$$

$$\text{Rate work done} = \frac{dw}{dt} \underbrace{\iiint E\cdot J\,dv}_{U_m} \quad -(14)$$

$\frac{1}{2}\varepsilon E^2$ represent rate of work done against the repulsive force acting on like charges in a static ~~are~~ charge distribution.

$\frac{1}{2}\mu H^2$ represent a rate of work done against the magnetic field in a dielectric med.

Total $\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2$ represent rate of total energy stored in the electromagnetic field.

Poynting vector measures the energy flux density in em wave.

$$E \times H = S$$

Poynting vector is amount of energing flowing per second through a unit area normal to the direction of propogation of em wave.