Bayes Theorem

Let $E_1, E_2 ---E_n$ be ~~the parti~~ a set of events associated with a sample space S, where all the events $E_1, E_2 ---E_n$ have non zero probability of occurence and they form a partition of S.

Let A be any event associated with S then a/c to Bayes theorem

$$\boxed{P(E_i/A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{k=1}^{n} P(E_k)\,P(A|E_k)}}$$

Prof :-

A/c to Condition Probabity.

$$\boxed{P(E_i/A) = \frac{P(A \cap E_i)}{P(A)}} \quad (1) \quad ; \quad \boxed{P(A/E_i) = \frac{P(A \cap E_i)}{P(E_i)}} \quad (2)$$

$$\boxed{P(A \cap E_i) = P(A/E_i)\, P(E_i)} \quad (3)$$

put eq (3) in (1)

$$\boxed{P(E_i/A) = \frac{P(A/E_i)\, P(E_i)}{P(A)}} \quad (4)$$

A/c to total probabity

$$\boxed{P(A) = \sum_{k=1}^{n} P(E_k)\, P(A|E_k)} \quad (5)$$

Put eq (5) in (4)

$$\boxed{P(E_i/A) = \frac{P(A|E_i)\, P(E_i)}{\sum_{k=1}^{n} P(E_k) P(A/E_k)}}$$

Hence proved

Q.2 An anti-aircraft gun can take a maximum of four shots on enemy's plane moving from it. The probabilities of hitting the plane at first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. Find the probability that the gun hits the plane. 5

Solution:

To find the probability that the gun hits the plane, we can use the complement rule -

P(hit) = 1 - P(miss)

where P(miss) is the probability that all 4 shots miss the plane.

Probability of all 4 shots missing the plane:

P(miss) = 0.6 x 0.7 x 0.8 x 0.9

= 0.3024

Using the complement rule,

P(hit) = 1 - P(miss)

= 1 - 0.3024

= 0.6976

Therefore, the probability that the gun hits the plane is 0.6976, which is between 0.69 and 0.7.

1. Solve by Cardan's method $x^3 - 3x^2 + 12x + 16 = 0$

Soln. Given eqn :- $x^3 - 3x^2 + 12x + 16 = 0$ — (1)

To remove x^2 term, we substitute $x = y - l/3$

$$\left(y - \frac{l}{3}\right)^3 - 3\left(y - \frac{l}{3}\right)^2 + 12\left(y - \frac{l}{3}\right) + 16 = 0 \quad (l = -3)$$

$\Rightarrow \quad (y+1)^3 - 3(y+1)^2 + 12(y+1) + 16 = 0$

$\Rightarrow \quad y^3 + 9y + 26 = 0$ — (2)

$\Rightarrow y = u+v$

$\Rightarrow y^3 - 3uv.y - (u^3+v^3) = 0$ — (3)

compare (2) & (3)

we get, $uv = -3$, $u^3+v^3 = -26$

$u^3v^3 = -27$, $u^3+v^3 = -26$

$\therefore u^3, v^3$ are the roots of the eqn $t^2+26t-27=0$

$\therefore t^2+26t-27=0$

$(t-1)(t+27)=0$

$t = -27, 1$

$u^3 = -27$ i.e., $u = -3$ & $v^3 = 1$, i.e. $v = 1$

$y = u+v = -3+1 = -2$ & $x = y+1 = -1$

Dividing L.H.S of (i) by $x+1$, we obtain

$$\begin{array}{r|l} & x^2-4x+16 \\ x+1 & x^3-3x^2+12x+16 \\ & \underline{x^3+x^2} \\ & -4x^2+12x+16 \\ & \underline{-4x^2-4x} \\ & 16x+16 \\ & \underline{16x+16} \\ & R \to 0 \end{array}$$

$x^2-4x+16=0$

or $x = \frac{4 \pm \sqrt{16-64}}{2} = 2 \pm i2\sqrt{3}$

Hence the required roots of the given eqn are $-1, 2 \pm i2\sqrt{3}$.

Q.4 Let $X = \{x_1, x_2, x_3, x_4,\}$ and two fuzzy sets A and B are 5

$A = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0.7), (x_4, 1)\}$

$B = \{(x_1, 0.6), (x_2, 1), (x_3, 0.4), (x_4, 0.3)\}$ then find $A \cup B$ and $A \cap B$. Is A is subset of B.

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1. Let $X = \{x_1, x_2, x_3, x_4\}$ and two fuzzy sets be
$A = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0.7), (x_4, 1)\}$
$B = \{(x_1, 0.6), (x_2, 1), (x_3, 0.4), (x_4, 0.3)\}$. Then find $A \cup B$, $A \cap B$ & is $A \subseteq B$.

Solu^n i. $A \cup B$; Let/Given A & B are two fuzzy sets defined on an universal set X, the union of A & B is defined as

$t X$.

A^c.

$$A \cup B(x) = \{(x, \mu_{A \cup B}(x)) : x \in X\},$$

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X.$$

Now,

$\mu_{A \cup B}(x_1) = \max\{\mu_A(x_1), \mu_B(x_1)\} = \max\{0.2, 0.6\} = 0.6$

$\mu_{A \cup B}(x_2) = \max\{\mu_A(x_2), \mu_B(x_2)\} = \max\{0.5, 1\} = 1$

$\mu_{A \cup B}(x_3) = \max\{\mu_A(x_3), \mu_B(x_3)\} = \max\{0.7, 0.4\} = 0.7$

$\mu_{A \cup B}(x_4) = \max\{\mu_A(x_4), \mu_B(x_4)\} = \max\{1, 0.3\} = 1$

$\therefore A \cup B(x) = \{(x_1, 0.6), (x_2, 1) (x_3, 0.7), (x_4, 1)\}$

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ii $A \cap B$; Given A & B are two fuzzy sets defined on an universal set X, the intersection ~~union~~ of A & B is defined as,

$$A \cap B(x) = \{(x, \mu_{A \cap B}(x)) : x \in X\}$$

where $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$

Now,

$\mu_{A \cap B}(x_1) = \min\{\mu_A(x_1), \mu_B(x_1)\} = \min\{0.2, 0.6\} = 0.2$

$\mu_{A \cap B}(x_2) = \min\{\mu_A(x_2), \mu_B(x_2)\} = \min\{0.5, 1\} = 0.5$

$\mu_{A \cap B}(x_3) = \min\{\mu_A(x_3), \mu_B(x_3)\} = \min\{0.7, 0.4\} = 0.4$

$\mu_{A \cap B}(x_4) = \min\{\mu_A(x_4), \mu_B(x_4)\} = \min\{0.3, 1\} = 0.3$

$\therefore A \cap B(x) = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.3)\}$

iii Given A & B be two fuzzy set, then $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ $\forall x \in X$

$\because \mu_A(x) = \mu_B(A)$

$\therefore$ A is a subset of B $(A \subseteq B)$.

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