Unit -1
Matrix And Determinants

Matrix And Determinants

This system of no. arranged in rectangular array (in r & c) and bounded by a bracket is called matrix.

Various types of matrix :-

(i) Row Matrix :- If a matrix has only one row and any no. of column then it's called row m.

Ex :-

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$A_{1\times n} = [a_{ij}]_{i,j=1 \text{ to } n}$$

(ii) Column M :- A mat. having one colmn and any no. of row is called a colmn matric. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(iii) Null M :- Any Mat. in which all the elements are zero is called zero mat.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(iv) Sq. Mat :- {row no. = no. of column}

Ex :- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\{3\times 3, 4\times 4\}$

(v) Diagonal Mat :- A sq. mat. is called a diagonal Mat. if all non-diagonal Mat. are zero.

$$A_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(vi) Scalar :- A diagonal Mat. in which all the elements are equal to some scalar

$$\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

(vii) Unit Mat :- A sq. Mat. is called a unit matrix if all the diagonal elements are unity and non-diagonal elements are zero.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(viii) Symmetric Mat :- A sq. Mat. is called a symm. Mat if are all values :- ~~i and j~~

$a_{ij} = a_{ji}$ then it is sy. mat

$A = A'$.

(ix) Skew sym. :- A sq. Mat. is called skew sym. matrix if writing I^{st}

(i) $A_{ij} = -a_{ji}$ $i, j = A'$

(ii) All diagonal elements are zero.

$$A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$$

(x) Triangular Mat:- A sq. Mat, all of whose elements below the leading diago. are zero is called a upper triangular Mat. :-

$$A = \begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

low. Trig. Mat:-

$$\begin{bmatrix} a & 0 & 0 \\ h & b & 0 \\ g & f & c \end{bmatrix}$$

(xi) Orthogonal Mat:- A sq. mat. 'A' is called an ortho. mat. if $A \times A' = I$

$$\boxed{AA' = I}$$

(xii) Idempotent Mat:-

A mat. A is called an idempo. mat. if $\{A^2 = A\}$

(xiii) Periodic Mat:- A mat. A is called a periodic Mat, if

$A^{k+1} = A$ where k is the least positive integer for which $\{A^{k+1} = A\}$, then k is called the period of 'A'.

(xiv) Nilpotent Mat:-

$\{A^k = 0\}$ k is pos. integer

if A is the least pos. for $A^k = 0$ then k is the index of nilpot. mat.

(xv) Involuntary Mat :-

A mat. A is called Invo. Mat if $\{A^2 = I\}$

Elementary transformation :-

There are 3 types of elem. transformation.

(i) Interchanging any two rows or columns, this transfo. is indicated as $\{R_{ij} \Leftrightarrow R_{ji}\}$ $C_i \rightarrow C_i \leftrightarrow C_j$

$R_i \longrightarrow R_i \Leftrightarrow R_j$

(ii) Multiplication of the elements R_{ij} by any k

kR_i / kC_i

(iii) Add^n of the const. multiplication other row R_i to the another row kR_j

$$\boxed{R_i \rightarrow R_i + kR_j}$$

(iv) If a matrix 'B' is obtained from a matrix 'A' by one or more elementary operations or transformation then 'B' is said to be equivalent to 'A'

$A \sim B$

* The rank of a matrix is said to be 'r' it has atleast one zero minor of order

Every minor 'A' higher than R is

Non zero row is that row in which all the elements are non-zero.

The rank of prod. matrix A·B of & mat. A & B is less than the rank of A & B matrix.

(ii) Corresponding to every non-sing. matr. 'A' of rank 'R' their exist non-sing. mat. 'P' and 'Q' such that P and Q

$$PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$ r is order of unit matrix,

Rank of a matrix is equals to no. of non-zero row in upp. triag. matrix.

(iii) Normal form :- By performing elementary transformation any non-zero matr. 'A' can be reduced to one of the following 4 form

- $[I_r]$
- $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$
- $\begin{bmatrix} I_r & 0 \end{bmatrix}$
- $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

The no. r is rank of 'A'

$\rho(A) = r$

$A^{-1} = QP$

DATE
PAGE

Que:- find the rank of given matrix $\begin{bmatrix} 4 & 1 & 2 & 1 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

$\rho(A) = 3$

$\rho(A) = 3$

In this method we will use only elementary row transformation.

Que:- find the rank of ~~triang~~ given matrix converting it into the normal form.

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 1 & 3 & 4 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

$\rho(A) = 3$ on solu.

Q:

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

find two sing. matrix P and Q such that PAQ is the normal form and find A^{-1} if it exist otherwise give reason.

$A_{3\times3} = I_{3\times3}\, A\, I_{3\times3}$

opr. $R_1 \to R_1 - R_2$

Opt at $R_2 \to R_2 - 2R_1$

opt. at $C_2 \to C_2 + C_3$

opt. at $C_3 \to C_3 - 4C_1$

$I_3 = PAQ$

where P is $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ & & 1 \end{bmatrix}$ $|P| \neq 0$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 1 & -3 \end{bmatrix}$$

$|Q| \neq 0$

$A^{-1} = QP$

DATE

PAGE

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots a_{1n}x_n = b_1$

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

Since we have to apply rank theory so we construct a new matrix with the help of A, and B which is known as obedient Matrix

$\boxed{\rho(A) = \rho(C)}$ then system is consistent.

If
- $\rho(A) = \rho(C) = (r)$ ← same no. = total no. of unknowns then system is consistent with unique sol^n.

If
- $\rho(A) = \rho(C) = (r < n)$, then sys. is consistent but ∞ many $sol^n s$.

- $\rho(A) \neq \rho(C)$ then system is inconsistent and their is no sol^n.

Que:- Test consistency and hence solve the given system of equation

$$x_1 + 2x_2 + x_3 = 2$$
$$3x_1 + x_2 - 2x_3 = 1$$
$$4x_1 - 3x_2 - x_3 = 3$$
$$2x_1 + 4x_2 + 2x_3 = 4$$

Sol^n:- The given set of eq^n in the matrix form can be written as

$$AX = B \quad \ldots (1)$$

where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 4 & -3 & -1 \\ 2 & 4 & 2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

since we have to apply rank so we construct a new matrix with the help of the elements of A and B which is known as

$$C = [A, B]$$

$$= \begin{bmatrix} 1 & 2 & 1, & 2 \\ 3 & 1 & -2, & 1 \\ 4 & -3 & -1, & 3 \\ 2 & 4 & 2, & 4 \end{bmatrix}$$

$\rho(A) = 3 = \rho(C)$ which is equal to unknow so system is consistent with unique sol^n, for the sol^n we will replace

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & \end{bmatrix}$$

$$, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \text{ in eq. } I^{st}$$

Linear dependence and independence of vectors

$X_1, X_2, X_3 \dots X_n$ are said to be dependent "if"

(1) all the vectors are of the same order.

(2) n scalars $\lambda_1, \lambda_2, \lambda_3$ towards λ_n not all zero exist such that

$$\lambda_1 x_1 + \lambda_2 X_2 + \lambda_3 X_3 + \dots + \lambda_n X_n = 0$$

otherwise they are said to be linearly independent.

Qu:- Examine the following vectors for linear dependence and find the relation if it exist.

$$X_1 = (1,2,4), \ X_2 = (2,-1,3), \ X_3 = (0,1,2),$$
$$X_4 = (-3,7,2)$$

Sol^n:-

Consider the matrix eq^n

$$\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0.$$

$$\lambda_1(1,2,4) + \dots + \lambda_4(-3,7,2) = 0$$

$$(\lambda_1) + 2\lambda_2 + \lambda_4(-3) = 0 \dots \text{(1)}$$

$$2\lambda_1 - \lambda_2 + \lambda_3 + 7\lambda_4 = 0 \dots \text{(2)}$$

$$4\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 = 0 \dots \text{(3)}$$

$$\Rightarrow A\lambda = 0$$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 2 & -1 & 1 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

operation at $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & -5 & 2 & 14 \end{bmatrix}$$

operation at $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{aligned} &\lambda_3 + \lambda_4 = 0 \Rightarrow \\ &-5\lambda_2 + \lambda_3 + 13\lambda_4 = 0 \\ &\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0 \end{aligned} \right)$$

$t \neq 0$ $\lambda_4 = t$

$\lambda_3 = -t$

$\lambda_2 = \frac{12}{5}t$

$\lambda_1 = -\frac{9}{5}t$

$$-\frac{9}{5}t\,\lambda_1 + \frac{12t}{5}\lambda_2 - t\lambda_3 + t\lambda_4 = 0$$

$$\frac{-t}{5}\left(9\lambda_1 - 12\lambda_2 + 5\lambda_3 - 5\lambda_4\right) = 0 \quad \because t \neq 0$$

$$\Rightarrow 9\lambda_1 - 12\lambda_2 + 5\lambda_3 - 5\lambda_4 = 0$$

even values and even vectors

Let

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$AX = Y \quad \ldots (1)$

where A is ~~matrix~~ colm vector and Y is also column matrix.

clmn matrix is transformed into colmn vector Y, by means of sq. matrix A.

~~Let X be a such vector which transforms~~

Let X be a such vector which transforms into $\lambda(X)$, $\lambda_2 X$ by means of the transformations I.

suppose the linear transformation $Y = AX$ transform X into a scalar multiple of itself, $\lambda_2 X$.

$$AX = Y = \lambda X$$

$$AX - \lambda I X = 0$$

$$[A - \lambda I] X = 0 \quad \ldots (2)$$

Thus the unknown scalar (λ) is known as the hygen value of the matrix A and corresponding non-zero solm and correspont non-zero vector "X" is called hygen vector.

The matrix $[A - \lambda I]$ is known as characteristic matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

Characteristic polynomial :-

The determinant $|A - \lambda I|$ when expanded will give a polynomial which we call as characteristic polynomial of matrix 'A'

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}$$

$$\Downarrow$$

$$-\lambda^3 - 7\lambda^2 + 11\lambda - 5$$

Characteristic eq^n :-

The eq^n $|A - \lambda I| = 0$ is called characteristic eq^n and root of this eq^n is known as characteristic root or hygen values.

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & \end{bmatrix}$$

$$+\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$$

$$\lambda = 1, 1, 5$$

Propertie of Hy. value :

(1) Any sq. matrix A and its transpose A' have the same hygen value.

(2) The sum of the hygen values of a matrix is equal to trace of the matrix.

(3) The product of the Eygen values of a matrix 'A' is equal to determinant of the matrix.

(4) If $\lambda_1, \lambda_2, \lambda_3 \cdots \lambda_3$ ~~towards~~ are Eygen values 'A' then the Eygen values of :-

(i) kA are

$$k\lambda_1, k\lambda_2 \cdots k\lambda_n$$

(ii) A^{-1} are $\frac{1}{\lambda_1}, \frac{1}{\lambda_2} \cdots \frac{1}{\lambda_n}$

(iii) A^m are $\lambda_1^m, \lambda_2^m \cdots \lambda_n^m$.

(1) Two ideal vectors of 'X' of a matrix 'A' is not unique.

(2) If '$\lambda_1, \lambda_2 \cdots \lambda_n$' be descrint Eygen values of an matrix then corresponding diegonal vectors, $X_1, X_2 \cdots X_n$, form a linearly independent set.

(3) If two or more Eygen values are equal it may or may not be possible to get linearly indepent Eygen vectors corresponding to the equal roots.

(4) Two Eygen vectors X_1, X_2 are called orthogonal vectors if

$$\boxed{X_1' X_2 = 0}$$

(5) Eygen vectors of of symmetric matrix corresponding to diff. Eygen values are paired wise orthogonal.

is non symm. Matrice with non-repeated Eigen values-

find the Eigen value and Eigen vectors of the matrix

Given that

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

let the characteristic equation for given matrix is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0 \qquad \lambda = 2, 3, 5$$

Eigen vectors :-

The Eigen vectors of matrix 'A' corresponding to the Eigen value 'λ' is given by the non-zero sol^n of the eq^n

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{.... (a)}$$

first Eigen vector 'x_1' corresponding to Eigen value $\lambda x = 2$ in eqn 'a'

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ \end{bmatrix} \begin{bmatrix} x \\ y \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \end{bmatrix}$$

DATE
PAGE

$$\begin{cases} 1.x + 1.y + 4z = 0 \\ 0.x + 0.y + 6z = 0 \\ 0.x + 0.y + 3z = 0 \end{cases}$$

$$x_1 = k\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad k \neq 0$$

$$\boxed{\frac{x}{6} = \frac{y}{-6} = \frac{z}{0} = \frac{k}{6}}$$

~~Second~~ Eygen vector $\lambda = 3$ is obtained by replacing $\lambda = 3$ in eq^n (a).

$$\cancel{\frac{x}{6} = \frac{y}{0} = \frac{z}{0} = \frac{k}{6}}$$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix}$$

$$\boxed{\frac{x}{-2} = \frac{y}{6} = \frac{z}{6} = \frac{k}{-2}}$$

$\boxed{\lambda = 5}$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2x + y + 4z \\ 0x - 3y + 6z \\ 0 \quad 0 \quad 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$$

$$\frac{x}{6+12} = \frac{y}{+12-0} = \frac{z}{6-0} = k$$

Find the Eygen value and value

$$A = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} \text{ at } (1, 1, 3)$$

$\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$\boxed{x + y + z = 0}$

Symm. matrix with non-repeated Eigen value:-

Q:- Find the Eig. values and vectors of

$$A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2-\lambda & 5 & 4 \\ 5 & 7-\lambda & 5 \\ 4 & 5 & -2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -(2+\lambda) & 5 & 4 \\ 5 & 7-\lambda & 5 \\ 4 & 5 & -(2+\lambda) \end{bmatrix} = 0$$

$-(2+\lambda)$

$\{(7-\lambda)(2+\lambda)-25]$

$-5[-(2+\lambda)5-16]$

$+4[25-28-4\lambda]$

$-(2+\lambda)\{-(7-\lambda)(2+\lambda)$

$-(2+\lambda)[-(14+7\lambda-2\lambda-\lambda^2)$

$-25]$

$-5[-10-5\lambda-16]+4[25-3-4\lambda]$

$-(2+\lambda)[$

$\lambda = -3, -6, 12)$

$\Rightarrow -(2+\lambda)[-(14+5\lambda-\lambda^2)]-5(-10-5\lambda-16)+4(12-4\lambda)$

$3, -6, 12$

Q.

$$\Rightarrow \begin{bmatrix} (2+3) & 5 & 4 \\ 5 & 7+6 & 5 \\ 4 & 5 & -14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 5 & 4 \\ 5 & 13 & 5 \\ 4 & 5 & -14 \end{bmatrix}$$

$\Rightarrow$

Sym. matrix with repeated Eygen values :-

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix} \Rightarrow 2-\lambda\left((2-\lambda)^2-1\right) + 1\left((\lambda-2)+1\right) + 1\left(1-2+\lambda\right)$$

$$\Rightarrow (2-\lambda)\left[4+\lambda^2-4\lambda-1\right] + 1(\lambda-1) + (\lambda-1)$$

$$\Rightarrow 8-2\lambda^2-8\lambda-2-4\lambda-\lambda^3+4\lambda^2+\lambda+2\lambda-2$$

$$\Rightarrow -\lambda^3+2\lambda^2-9\lambda-12=0$$

$$\Rightarrow \lambda^3-2\lambda^2+9\lambda+12=0$$

$(\lambda = 4, 1, 1)$

~~$(\lambda^2+9)(\lambda-2)(\lambda^2-6)=0$~~

$$A\begin{bmatrix} -2 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}_{3\times 3}\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} = 1$$

$-2x - y + 2$

Since given matrix is symm. and Eygen values are repeated, so we take 3rd Eygen vector

$$x_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$$ which is pair wise orthogonal to x_1 and x_2.

$x_1' x_3 = 0$

$x_2 = \begin{bmatrix} k_1 \\ k_2 \\ k_2 - k_1 \end{bmatrix}$

$$[1 \; -1 \; 1] \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

$$x_2' x_3 = 0 \Rightarrow [1, \; 1 \; 0] \begin{bmatrix} l \\ m \\ n \end{bmatrix} = 0$$

Q:- Cayly hamilton theorem :-

Every sq. matrix satisfies its own characteristic equation

$$|A - \lambda I| = (-1^n)(\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n)$$

$$|A - \lambda I| = 0$$,

$$(\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0) \dots (1)$$

Q:- find the characteristic eq^n of the matrix 'A'

find A^{-1}.

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4-\lambda & 3 & 1 \\ 2 & 1-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{bmatrix} = 0$$

PAGE

$(1-\lambda)\left[(1-\lambda)^2 + (4)\right. \quad -2\left(3(1-\lambda)+2\right) + 1\left(-6 + \lambda - 1\right)\Big]$

$(1-\lambda)\left[1+\lambda^2-2\lambda+4 \quad -6+6\lambda-4 \quad -7+\lambda\right.$

$= (4-\lambda)\Big[$

$\lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0 \quad \ldots \quad (1)$

Acc. to ~~cal~~ cayley-hamilton theorem eq: is satisfied

A :-

$A^3 - 6A^2 + 6A - 11I = 0 \quad \ldots \quad (2)$

$A^{-1}\left(A^3 - 6A^2 + 6A - 11I\right) = A^{-1} \times 0$

$A^2 - 6A + 6I - 11A^{-1} = 0$

$\boxed{11\, A^{-1} = A^2 - 6A + 6I}$