PROBABILITY AND STATISTICS

[Mathematical Calculation of happening or not happening of any event]

[$\frac{\text{No. of favourable cases}}{\text{Total no. of cases.}}$]

[Denoted by "p" small
$0 \leq p \leq 1$]

[DICE , CARDS]

[EXCLUSIVE OUTCOMES]

[RANDOM EXPERIMENT]
- Result may be all together different even on performing identical conditions.

[TRIAL AN EVENT]
- REPETITION OF SAME EVENT.

[EQUALLY LIKELY EVENTS]
- one can not be prefered over others.

[INDEPENDENT EVENT]
- (when the one's happening doesn't influnced by other event)

[FAVOURABLE EVENT)
- Req. EVENT.

[CONDITIONAL PROBABILITY]
- Depends on one another. (min. two events)

Que: From a pack of 52 cards, 1 is drawn at random. Find the probability of getting a king.

Total No. of out comes = 52

No. of favourable outcomes = 4

$$p = \frac{4}{52} = \frac{1}{13}.$$

① Addition Law of Probability

(prepare it by our own)

Prove that type quest

If A and B are two events associative with experiment then,

$$P((A)\cup(B)) = P(A) + P(B) - P(A \cap B)$$

(By vien diagram)

Que: An urn contains 10 W and 10 B balls. Find the prob. of drawing two balls of same colour.

Prob (Drawing first ball) $= \frac{1}{20}$

2

* Probability. Drawing two same colour balls $= \frac{^{10}C_2}{^{20}C_2} \times 2$ ← can be white or black $= \frac{9}{19}$

Que. A bag contains 4 w and 2 b balls and a second bag contains 3 of each colour. A bag is selected at rand. at random and then a ball is drawn at random. Prob. of the ball drawn is white?

Prob of selecting a bag

$$= \frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{3}{6}$$

$$= \frac{1}{3} + \frac{3}{12}$$

$$= \frac{7}{12}$$

② Multiplication law of Probability

(Prepare it by own)

If there are two ind. events the respective prob's of which are known then the prob. that both will happen, is the product of respective prob.

$$P(A \cdot B) = P(A) \times P(B)$$

Que: An article manufactured by a company consist of two parts 'A' & 'B' in the process of manufacture of 'A', 9/100 are likely to be defective. Similarly, 5/100 are likely to be defective in the manufacture of 'B'. Prob. of assembled article will not be defective.

A: $P(\text{Defective } A) = \frac{9}{100}$

A': $P(\text{Not Defective } A) = 1 - \frac{9}{100} = \frac{91}{100}$

B: $P(\text{Defective } B) = \frac{5}{100}$

B': $P(\text{Not Defective } B) = 1 - \frac{5}{100} = \frac{95}{100}$

$P(A'.B')$ Req Probability $(P) = \frac{91}{100} \times \frac{55}{100} = \frac{8645}{10000}$

$= 0.8645$

Que: Prob that machine 'A' will be performing an usual funct^n in 5 years. Time is $\frac{1}{4}$. while the prob. that machine 'B' will still be operating usually at the end of the same period is 1/3. Find the prob in 5 years times!

(1) Both good (3) ONLY 'B'

(2) Neither good (4) ...

Conditional Probability.

Let A and B be two events of a sample space 'S' and let $P(B) \neq 0$. The conditional Prob of A given by:

$$\boxed{P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}}$$ *

Bayes's Theorem

Que: State and prove Bayes's Theorem

If $B_1, B_2, B_3, \ldots, B_n$ are mutually exclusive events if $(B_i) \neq 0$; $(i = 1, 2, 3, \ldots, n)$ of a random experiment. Then for any arbitary event A of the sample space of the above experiment with $P(A) > 0$, we have,

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i) \times P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\, P(A/B_i)}$$ for $n = 3$??

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Que: An urn 1st contains 3W and 4R balls and an urn 2nd contains 5W and 6R balls, 1 ball is drawn at random from one of the urns and is found to be white, find the prob that it was drawn from 1st urn.

U_1 : Ball from 1st

$P(U/B) =$ U_2 : Ball from 2nd

W : Pof getting a W ball

$\frac{3}{7} \times \frac{5}{11}$

$$P(U_1/W) = \frac{P(U_1) \times P(W/U_1)}{P(U_1)\, P(W/U_1) + P(U_2)\, P\left(\frac{U_2}{W}\right)}$$

$\frac{15}{77} \times$

$\frac{1}{2} \times \frac{3}{}$ 3 3 3

Binomial Probability Distribution -

Let 'n' an independent trials in an experiment, Let a random variable 'X' denote the no. of sucessive in each these trials.

Let 'p' be the prob. of a sucess and 'q' that of failure in an single trial. so that $p+q=1$.

Let the trials be independent and 'p' be constant for every trial.

Let us find the prob. of 'r' successes in 'n' trials

r successes can be optained in trials in ${}^nC_r$ ways.

$$\therefore \quad P(X=r) = {}^nC_r P(\underbrace{SSS\ldots S}_{(r)\text{ times}}\ \underbrace{FFF\ldots F}_{(n-r)\text{ times}})$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$= {}^nC_r \underbrace{P(S)P(S)P(S)\ldots P(S)}_{r\text{ factor}}\ \underbrace{P(F)P(F)\ldots P(F)}_{(n-r)\text{ factor}}$$

$$= {}^nC_r \underbrace{ppp\ldots p}_{(r)\text{ factor}}\ \underbrace{qqq\ldots q}_{(n-r)\text{ factor.}}$$

$$= {}^nC_r\, p^r q^{n-r} \qquad [\text{here, } p+q=1]\ [0 \le r \le n]$$

This distribution is called binomial probability distribution.

X : The Binomial varient.

Note: The successive probabilities $p(r)$ for $(0 \le r \le n)$ ${}^nC_0 q^n$, ${}^nC_1 q^{n-1} p$, ${}^nC_2 q^{n-2} p^2 \ldots {}^nC_n p^n$; which are the successive terms of the binomial expansion of $(q+p)^n$. Therefore, this distribution is called binomial dist

Que. Prove that, in binomial prob. distribution, mean & variance are given by np, npq respectively.

Mean :

The Binomial Prob. dist. of r successes is :

$$P(r) = {}^nC_r\, q^{n-r} p^r. \qquad (0 \le r \le n)$$

$$\boxed{\text{Mean } (\mu) = \sum_{r=0}^{n} r P(r)}$$

$$= \sum_{r=0}^{n} r\, {}^nC_r\, q^{n-r} p^r$$

$$= 0 + 1.{}^nC_1 q^{n-1} p + 2.{}^nC_2\, q^{n-2} p^2 + 3.{}^nC_3 q^{n-3} p^3 + \cdots + n\, {}^nC_n\, p^n.$$

$$\mu = nq^{n-1}p + 2.\frac{n(n-1)}{2.1} q^{n-2} p^2 + \cdots + np^n.$$

$$= nq^{n-1}p + n(n-1) q^{n-2} p^2 + \cdots + np^n$$

$$= np\left[q^{n-1} + (n-1) q^{n-2} p + \cdots + p^{n-1}\right]$$

$$= np\left[\underbrace{{}^{n-1}C_0\, q^{n-1} + {}^{n-1}C_1\, q^{n-2} p + {}^{n-2}C_2\, q^{n-3} p^2 + \cdots + {}^{n-1}C_{n-1}\, p^{n-1}}_{(q+p)^{n-1}}\right]$$

$$= np\,(p+p)^{n-1}$$

$$\mu = np.$$

$$\boxed{\text{varience } (\sigma^2) = \sum_{r=0}^{n} r^2 p(r) - \mu^2}$$

$$= \sum_{r=0}^{n} \{r + r(r-1)\}\, p(r) - \mu^2$$

$$= \sum_{r=0}^{n} r p(r) + \sum_{r=r}^{n} r(r-1)\, p(r) - \mu^2$$

$$= \mu + \underbrace{\sum_{r=2}^{n} r(r-1)\, {}^nC_r\, p^r q^{n-r}}_{\text{expansion same as mean (say S)}} - \mu^2 \quad \rightarrow \text{Eq } 1$$

$$S = \sum_{r=2}^{n} r(r-1)\, {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= 2(1)\,\frac{n(n-1)}{2}\, p^{2} q^{n-2} + 3(2)\,\frac{(n)(n-1)(n-2)}{3\times 2\times 1}\, p^{3} q^{n-3} + \cdots$$

$$\cdots + 1\, p^{n}$$

$$= n(n-1)\, p^{2} q^{n-2} + (n)(n-1)(n-2)\, p^{3} q^{n-3} + \cdots + n(n-1)\, p^{n}$$

Putting value in Eq (1)

$$\sigma^{2} = \mu + (n-1)n\, p^{2} q^{n-2} + n(n-1)(n-2)\, p^{3} q^{n-3} + \cdots n(n-1)p^{n} - \mu^{2}$$

$$\sigma^{2} = np + n(n-1)p\,(q^{n-2}p + (n-2)p^{2}q^{n-3} + \cdots p^{n-1}) - (np)^{2}$$

$$\sigma^{2} = np + np\,((n-1)q^{n-2}p + (n-1)(n-2)p^{2}q^{n-3} + \cdots (n-1)p^{n-1}) - (np)^{2}$$

$$\sigma^{2} = np + np\,($$

$(p+q)^{n-2}$

$$\sigma^{2} = npq$$

$$\left[\begin{array}{l} np - np^{2} + n(n-1)p^{2} \\ np - (np)^{2} + n^{2}p^{2} - np^{2} \\ np(1-p) \\ npq \end{array}\right]$$

Que. Assume that only the avg. 1 telephone no. out of 15 called b/w 2pm to 3pm is busy. what is the prob. that is 6 randomly selected telephone no. are called.
(1) Not more than three.
(2) Atleast 3 of 'em will be busy.

p be the the prob. of a telephone no. being busy b/w 2pm and 3pm on week days. = $1/15$

q be the prob. of a telephone no. not being busy b/w 2pm & 3pm on week days = $14/15$

(Basic req. p, q & n)

Here, $n = 6$

Case 1: The prob. that not more than 3 will be busy $P(r)$ $(0 \le r \le 3)$

$= P(0) + P(1) + P(2) + P(3)$.

$= {}^6C_0 p^0 q^{6-0} + {}^6C_1 p^1 q^5 + {}^6C_2 p^2 q^4 + {}^6C_3 p^3 q^3$

(value substitute p & q)

$= 0.9997$

Case 2: The prob. that atleast 3 of them are busy $P(r)$ $(3 \le r \le 6)$

$= P(3) + P(4) + P(5) + P(6)$

$= {}^6C_3 p^3 q^3$ - - -

$= 0.005$

Que. If the prob. of getting a target is 10% and 10 shots are fired independently. What is the prob that the target will be hit atleast once.

Let p be the prob of hitting the target. $= \frac{1}{10}$

Let q be the prob of not hitting the target. $= \frac{9}{10}$

$[n = 10]$

Atleast once hitting. $P(r)$ $(1 \le r \le 10)$

$= P(1) + P(2) + P(3) + \cdots + P(10)$.

$= {}^{10}C_1 q^9 p + {}^{10}C_2 q^8 p^2 + {}^{10}C_3 p^3 q^7 + \cdots + {}^{10}C_{10} p^{10}$

Time taking. $= 0.6513$

easy method

None shots

$= P(0)$

$= {}^{10}C_0 q^{10}$

$= 0.34$

So, req. ans. $=$ 1 - None shots

$= 1 - 0.34$

$= 0.6513$

Poision's Prob. Distrib-

Que. Prove that Poision distr. is a limiting case of binomial distribution.

If the parameters 'p' 'n' of a binomial dist are known we can find the dist. But in situations when n is very large and p is very small. Application of Binomial dist is difficult.

However if we assume that, if $n \to \infty$ and $p \to 0$. Such that np always remains finite (say λ)

Then we get the poisions approximation to the binomial distribution.

(starts with B D).

For a binomial distribution,

$$P(X=r) = {}^nC_r\, q^{n-r} p^r$$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}(1-p)^{n-r}p^r$$

$(\lambda = np)$

$(p = \lambda/n)$

$$= \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}(1-\lambda/n)^{n-r}(\lambda/n)^r$$

$$= \frac{\lambda^r}{r!}\,\frac{(n)(n-1)(n-2)\cdots(n-r+1)}{n^r}\,\frac{(1-\lambda/n)^n}{(1-\lambda/n)^r}$$

$$= \frac{\lambda^r}{r!}\left\{\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\cdots\frac{(n-r+1)}{(n)}\right\}\frac{(1-\lambda/n)^n}{(1-\lambda/n)^r}$$

$$= \frac{\lambda^r}{r!}\left\{1\,(1-1/n)(1-2/n)\cdots\left(1-\frac{(r-1)}{n}\right)\right\}\frac{(1-\lambda/n)^{-n}}{(1-\lambda/n)}$$

$$\boxed{P(r) = \frac{\lambda^r}{r}e^{-\lambda}}$$

$(0 \le r \le \infty)$

As, $n \to \infty$, $1-\frac{1}{n} \to 1$, $1-2/n \to 1 \cdots 1-\frac{(r-1)}{n} \to 1$

$\lim_{x \to \pm\infty}(1+1/x)^{-x} = e$

Que. Prove that in Poision's Prob. dist. mean and variance both are given by λ.

We know that, r success in poision's prob. dist. is given by;

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad (r = 0, 1, 2 \cdots \infty)$$

$$\text{Mean } (\mu) = \sum_{r=0}^{\infty} r P(r)$$

$$= \sum_{r=0}^{\infty} r \frac{e^{-\lambda}\lambda^r}{r!}$$

$$= \sum_{r=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!}$$

$$= \cancel{\frac{-e^{-\lambda}}{\lambda}} \; e^{-\lambda}\left(\frac{\lambda}{1} + \frac{\lambda^2}{1} + \frac{\lambda^2}{1} \cdots\right)$$

$$= e^{-\lambda}\lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right)$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$\boxed{\mu = \lambda}$$

Now, for variance $(\sigma^2) = \sum_{r=0}^{\infty} r^2 (P_r) - \mu^2$

$$= \sum_{r=0}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r!} - \lambda^2$$

$$= \sum_{r=0}^{\infty} r \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \cancel{\lambda^2 \left[-1 \right.} + -\lambda^2 + \frac{e^{-\lambda}\lambda}{1} - \lambda^2 + \frac{e^{-\lambda}\lambda^2}{1} - \lambda^2$$

$$- - -$$

$$\sigma^2 = \lambda$$

Que Assume that the prob of an individual accident pole minor ring field in a mine during a year is 1/2400. Use Poision's Prob. dist. to calculate the prob. that in a mine employing 200 minors, there will be at least one fetal accident in a year.

p → be the prob on incident coal minor being $= \frac{1}{2400}$

How many minors are $n = 200$

p is very small
& n is compartively large

Now, $\lambda = np$

$= 200 \times \frac{1}{2400} = \frac{1}{12}$

Now,

$$P(r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

$$P(r) = \frac{e^{-1/12} \cancel{x}^{\frac{1}{12}}}{r!}$$

($r \to 1$ to 200
so much time consuming)

P(at least one fetal accident) = 1 - Prob on no fetal accident

$$P(r) = 1 - P(0)$$

$$= 1 - \frac{e^{-1/12}}{0!}$$

$$= 1 - e^{-1/12}$$

$$= 1 - 92$$

$$= 0.08$$

Que. Find the probability that atmost 5 defective components will be found in a load of 200 components. If experience shows that 2% of such components are deffective, also find the prob of more than 5 deffective components

$p \rightarrow$ be the prob. of finding a deffective components. $= \frac{2}{100} = \frac{1}{50} = 0.02$

Total no. of such components $(n) = 200$

$\boxed{\because p << n}$ $\lambda = np = 4$

Prob. of atmost 5 deffective component $P(r) = \sum_{r=0}^{5} \frac{\lambda^r e^{-\lambda}}{r!}$

$P(r) = 0.7715$ (after calculation)

Prob. of more than 5 component,

$= 1 - P(r)$

$= 1 - 0.7715$

$= 0.2285$.

Curve Fitting

It is the exact relationship b/w two variables by algebric eq^n, infact these relationship is the eq^n of curve. Therefor, CF is to form the eq^n of the curve from given set of data.

Fitting a straight line

Let (x_i, y_i) where $i = 1, 2, 3, \ldots, n$ be n sets of observations of related data and

$y = a + bx \rightarrow (1)$ be the straight line

when 'a' and 'b' are constants which are obt. by solving following normal equations:

(1) ~~Take~~ Σ throughout eq^n

(2) multiply x then take Σ.

(1) $\Sigma y = na + b\Sigma x \rightarrow (2)$

(2) $\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (3)$

Fitting a IInd degree parabola -

Let (x_i, y_i) where $(i = 1, 2, \ldots n)$ be 'n' sets of observations of related data.

$y = a + bx + cx^2 \rightarrow (1)$ (IInd degree parabola to be fitted)

where, a, b, c are constants which are obt. by solving following normal equations:

(1) Take Σ

(2) $\times x$ then take Σ

(3) $\times x^2$ then take Σ

$\Sigma y = an + b\Sigma x + c\Sigma x^2 \rightarrow (2)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \rightarrow (3)$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \rightarrow (4)$

Note -
In case of change of origin.

① If n is odd -
Then, ne variable, $u = \dfrac{x - (\text{middle term})}{\text{length of the interval}}$ ↳ h

② If n is even -

Then $\mu = \dfrac{x - (\text{mean of two middle terms})}{(\text{Length of the interval}/2)}$ ↳ h/2

Que. By the method of least sqs. find the st line for the given data -

x	y	xy	x^2
1	14	14	1
2	27	54	4
3	40	120	9
4	45	180	16
5	68	340	25
$\Sigma x = 15$	$\Sigma y = 194$	$\Sigma xy = 708$	$\Sigma x^2 = 45$

Let the st. line for the given set of data be

$y = a + bx$ → ①

where a, b are constants which are obtained by solving following normal eqn

So,

$\Sigma y = an + b\Sigma x$ → ②

$\Sigma xy = a\Sigma x + b\Sigma x^2$ → ③

Here (n = 5)

Putting all values in ① & ②

$194 = a5 + b\,15$

$708 = a15 + b\,45$

By solving these, we get $a = 0$, $b = 13.6$

So, eqn will be $\boxed{y = 13.6x}$

Que. If a second degree for the given data

x	y	Σxy	Σx^2	x^3	x^4	x^2y
0	1	0	0	0	0	
1	4	4	1	1	1	
2	10	20	4	8	16	
3	17	51	9	27	81	
4	30	120	14	64	256	
$\Sigma x = 10$	$\Sigma y = 62$	$\Sigma xy = 195$	$\Sigma x^2 = 28$	$\Sigma x^3 = 100$	384	

Let the eq^n of IInd degree parabola be

$y = a + bx + cx^2 \rightarrow ①$

where, a, b, c are constants obtained by normal fitting.

So,

$\Sigma y = an + b\Sigma x + c\Sigma x^2 \rightarrow ②$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \rightarrow ③$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \rightarrow ④$

[Here, $n = 5$]

$62 = 5a + 10b + 28c$

$195 = 10a + 28b + 100c$

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Co-relation analysis

Whenever two variables 'x' and 'y' are so related that an increase in the one is accompanied by an inc. in the other, then the variables are said to be co-related.

eg. The yeild of crop varies with the amount of rainfall.

Types of Co-relation -

(1) +ve co-relation -

If an inc. in the value of one variable 'X' results in the corresponding inc. in the value of other variable 'Y', on an average. (same for decrease) then the co-relation is said to be positive. (directly proportional)

(2) -ve co-relation -

If an inc. in the value of one variable 'X' results in a corresponding dec. in the value of other variable 'Y'. (vice-versa) is said to be negative co-relation (inversly proportional)

Linear Co-relation -

When All the plotted points lie approximately on the straight line then the co-relation is said to be linear.

Perfect Co-relation -

When the value of both the variables under study changes at a constant ratio irrespective of their directions.

Carl- Pearsion's coefficient of Corelation - (r)

'r' b/w two variables (x) and (y) is defined by the relation -

$$\star \boxed{r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 (\Sigma Y^2)}}} = \frac{P}{\sigma x \, \sigma y} = \frac{\text{co. variance}(x,y)}{\sqrt{\text{var}\,x}\,\sqrt{\text{var}\,y}}$$

X and Y are the deviation meseured from thier respected mean. $X = x - \bar{x}$
$Y = y - \bar{y}$

P is the co. variance, i.e. $\left(\frac{\Sigma XY}{n}\right)$

$\star \boxed{-1 \leq r \leq 1}$

Que: Ten students got the following % of marks in eco. and stats -

Roll No.	Marks in eco.	Marks in stast.	$x-(\bar{x})$ X	$y-(\bar{y})$ Y	X^2	Y^2	XY
1.	78	84	13	+18	169	324	
2.	36	51	-21	-15	441	225	
3.	98	91	33	25	1089	625	
4.	25	60	-40	-6	3364		
5.	75	68	10	2	100		
6.	82	62	17	-4	289		
7.	90	86	25	20	625		
8.	62	58	-3	-8	9		
9.	65	53	0	-13	0		
10.	39	47	-26	-19	676		
	650						2704

Calculate the coefficien of corelation (r).

Let the marks of two subjects be denoted by x, y respectively.

Then the mean for x marks be $\bar{x}$

$$\bar{x} = \frac{650}{10} = 65$$

and the mean for y marks be $\bar{y}$

$$\bar{y} = \frac{660}{10} = 66$$

If X and Y are deviations of x and y marks from their respective means, then, the data maybe arrangend in the following form -

$X = x - (\bar{x} = 65)$ $\quad\quad$ $Y = y - (\bar{y} = 66)$

$\vdots$ $\quad\quad$ $\vdots$

we know that,

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \Sigma Y^2}} =$$

$$= 0.78$$

Solution of Equation

Equation-
Suppose we have expression-

[For Good marks : (all terms must be clear and well defined)]

$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$

($a_0, a_1, a_2, \ldots a_n$ are constants)

on equating this polynomial with zero, is called equation.

algebric sin/cos log/e

General Properties :

1. If α is a root of equation of $f(x)=0$ then the polynomial $f(x)$ is exactly divisible by $(x-\alpha)$ and conversly

2. Every eq^n of n^{th} degree has 'n' roots (roots maybe real or imaginary)

3. Intermediate value property-
If $f(a)$ and $f(b)$ have diff signs then the equation $f(x)=0$ has atleast one root b/w $x=a$ and $x=b$.

4. In an eq^n with real coefficients, imaginary roots occur in conjugate figures.
i.e. if $(\alpha + i\beta)$ is a root of $f(x)=0$ then $(\alpha - i\beta)$ must also be its root.
|||ly if $(a+\sqrt{b})$ is an irrational root of an eq^n then $(a-\sqrt{b})$ must also be its root.

Relation b/w Roots and coefficient.

If $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ are the roots of the eqn

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0$$

① Sum of the Roots -

$$\Sigma \alpha_1 = \frac{-a_1}{a_0}$$

$$\Sigma \alpha_1 \alpha_2 = \frac{a_2}{a_0}$$

$$\Sigma \alpha_1 \alpha_2 \alpha_3 \dots = \frac{-a_3}{a_0}$$

$$\Sigma \alpha_1 \alpha_2 \alpha_3 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

Que. Solve : $2x^3 + x^2 - 13x + 6 = 0$

(Check given poly is proper or improper)

It is given that,

$$2x^3 + x^2 - 13x + 6 = 0 \quad \rightarrow \text{Eq ①}$$

By Error and Trial method,

$x = 2$ is a root of given eqn ①

So, $(x-2)$ will be a factor of Eq ①

So, $(x-2) \overline{)2x^3 + x^2 - 13x + 6} \,(2x^2 + 5x - 3$

$$\begin{array}{l} 2x^3 - 4x^2 \\ \hline 5x^2 - 13x + 6 \\ 5x^2 - 10x \\ \hline -3x + 6 \\ -3x + 6 \\ \hline 0 \end{array}$$

Now, for the remaining roots, equate with

$$2x^2 + 5x - 3 = 0$$

i.e. $x = \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm 7}{3}$

$$x = \frac{1}{2}, -3$$

So,
Roots are $\frac{1}{2}, 2, -3$

Que. Solve the equation: $3x^3 - 4x^2 + x + 88 = 0$ one root being $(2 + i\sqrt{7})$.

Given that,

$$3x^3 - 4x^2 + x + 88 = 0 \rightarrow \text{Eq } ①$$

$\because$ $(2+i\sqrt{7})$ is a root of given eq^n

$\therefore$ $(x-2-i\sqrt{7})$ must be a factor of Eq ① and will completely divisible the eq.

$$(x-2-i\sqrt{7}) \overline{)\, 3x^3 - 4x^2 + x + 88 \,}\; 3x^2 + (2+i3\sqrt{7})x$$
$$3x^3 - 6x^2 - i3\sqrt{7}x^2$$
$$2x^2 + i3\sqrt{7}x^2 + x + 88$$
$$(2+i3\sqrt{7})x^2 +$$

Since one root is $(2+i\sqrt{7})$ of eq^n ①
So, $(2-i\sqrt{7})$ will also be a root of eq^n ②

$\therefore$ $(x-2-i\sqrt{7}) \times (x-2+i\sqrt{7})$ will divide eq ① completely.

i.e. $(x^2-4x+11)$ completely divide the given Eq.

On dividing,

$$x^2-4x+11 \overline{)\, 3x^3 - 4x^2 + x + 88 \,}\; 3x + 8$$
$$3x^3 - 12x^2 + 33x$$
$$8x^2 - 32x + 88$$
$$8x^2 - 32x + 88$$
$$0$$

(can be done by sum and product of roots)

So, the third root of the given eq^n will be $x = -8/3$

So, all three roots will be,
$x = -8/3,\ 2+i\sqrt{7},\ 2-i\sqrt{7}$.

Que. Solve: $x^3-7x^2+36=0$ given that one root is double of another

given that,

$$x^3-7x^2+0x+36=0 \rightarrow Eq \ (1)$$

By Relation of roots and coefficient,

Sum of roots $= (-1)^{2}\dfrac{a_2}{a_0} \quad -\dfrac{(-7)}{1} = 7$

Solution of Cubic Equation

~By Carden's method.

Suppose we have an eqn:

$$ax^3+bx^2+cx+d=0 \quad \rightarrow Eq \ (1)$$

Dividing by 'a' we get an eqn of the form

$$x^3+lx^2+mx+n=0 \quad \rightarrow Eq \ (2)$$

$(l=\frac{b}{a})$ To remove the 'x^2' term by putting

$\boxed{y=x-(-l/3)}$ or $\boxed{x=y-l/3}$

So, we get,

$$y^3+Py+q=0 \quad \rightarrow Eq \ (3)$$

Now, To solve eq ②.

Put $\boxed{y = u+v}$

$$y^3 = (u+v)^3$$

$$y^3 = u^3+v^3+3uvy$$

$$y - 3uvy - (u^3+v^3) = 0 \rightarrow \text{Eq ④}$$

on comparing Eq ③ and Eq ④

we get,

$uv = -p/3$, $\boxed{u^3+v^3 = -q}$ Sum of roots

$\Rightarrow \boxed{u^3v^3 = -p^3/27}$

Product of roots

u^3 and v^3 are roots of eqⁿ -

$$t^2 + \overset{\text{sum}}{q}t - \overset{\text{product}}{p^3/27} = 0$$

which gives,

$$u^3 = \frac{1}{2}\left(-q+\sqrt{q^2+4p^2/27}\right) = \lambda^3 \text{ say}$$

$$v^3 = \frac{1}{2}\left(-q-\sqrt{q^2+4p^2/27}\right)$$

which implies,

$[u = \lambda, \lambda\omega, \lambda\omega^2]$

[where, ω is one of the imaginary cube root of unit.]

$\left[v = -\frac{p}{3\lambda}, -\frac{p\omega^2}{3\lambda}, -\frac{p\omega}{3\lambda}\right]$

∴ 3 roots of eq ③ $(\lambda\omega^2 - p\omega/3\lambda)$

$y = u+v$ $(\lambda - p/3\lambda)$ $(\lambda\omega - p\omega^2/3\lambda)$

Note: ① If one value of u is found to be a rational no. then find the corresponding value of v giving one root. $y = u+v$ then find $x = \alpha$, finally divide the LHS by $(x-\alpha)$ giving the remaining quadratic eqⁿ from which other two roots can be found.

(2) If u^3 and v^3 turn out to be conjugate complex no. the roots of the given cubic can be obtain using De-mroger's theorem

11/12/23

Fuzzy Set Theory

Set: Collection of well defined object -

→ Roster form → By property

(1) Characteristics functions -

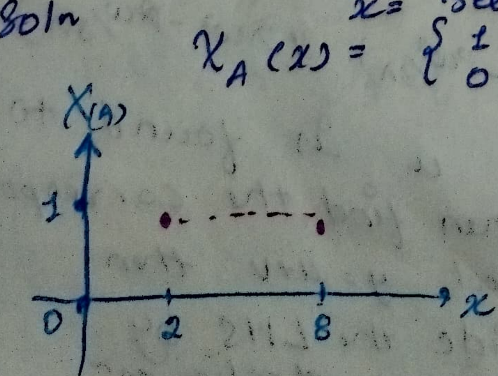
Let 'X' be an universal set, the characteristic functn of A denoted by χ_A is defined for each $x \in X$ as

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

eg. Let the universal set X with the set of all non neg. (−) real no's and let A is the set having element x, x is a real no. and $2 \le x \le 8$ be a subset of X then write characteristic funct for A.

Soln x = Set of all non-ve Real no.

$$\chi_A(x) = \begin{cases} 1 & 2 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$$



② Membership Function

A function similar to the characteristic funt is called a membership funt. It assigns to each element $x \in X$ a number $\mu_{A(x)}$, $[0,1]$ that characterise the degree of membership of 'x' in 'A'. Thus membership funtn are funct of the form $\mu_A : X \to [0,1]$

The membership functn of x in A is also called grade of membership or degree of complitivity or degree truth of x in A.

③ Union intersection

If A and B are two sets then we have

$$\mu_{(A \cup B)}(x) = \max[\mu_A(x), \mu_B(x)] \ \forall x \in X.$$

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] \ \forall x \in X.$$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x), \ \forall x \in X.$$

④ Fuzzy Set -

Let X be a collection of object or universal set Then a Fuzzy set 'A' in 'X' is defined as the set of ordered pairs -

$$A = \{(x, \mu_A(x)) : x \in X\}$$

where $\mu_A \to$ membership function

(grade of x in A)

$\mu_A(x) = 1 \Rightarrow$ full membership

$\mu_A(x) = 0 \Rightarrow$ none membership

$\mu_A(x) = (0,1) \Rightarrow$ intermediate membership

Representation of Fuzzy sets -

① ordered pair -

$X = \{x_1, x_2, x_3, \ldots, x_n\}$

$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \ldots (x_n \mu_A(x_n))\}$

② $A = \left\{\frac{\mu_A(x_1)}{x_1}, \frac{\mu_A(x_2)}{x_2}, \frac{\mu_A(x_3)}{x_3}, \ldots \frac{\mu_A(x_n)}{x_n}\right\}$

NOTE - The elements with zero degree of membershi are normally not listed.

Basic Fuzzy set operation -

① Complement of a Fuzzy set -

Let A be a Fuzzy set defined on an universal set X. The complimen of A is denoted by A' OR $\bar{A}$ OR A^c and is defined as:

$$A^c = \{(x, \mu_{\bar{A}}(x)) : x \in X\}$$

where,

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x), \quad \forall x \in X.$$

② Subset of a Fuzzy set -

Let 'A' and 'B' be two Fuzzy set then

A ⊂ B (subset) if and only if $\mu_A(x) \leq \mu_B(x), \forall x \in X$.

③ Union of two Fuzzy Sets

Let 'A' and 'B' be two fuzzy sets defined on an universal set 'X'. The union of A and B is denoted by $A \cup B$ and defined as -

$$(A \cup B)(x) = \{(x, \mu_{A \cup B}(x)) : x \in X\}$$

where,

$$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)] \ \forall x \in X$$

→ For All

④ Intersection of two Fuzzy sets -

Let 'A' and 'B' be two fuzzy sets defined on a universal set X. The intersection of A and B denoted by $A \cap B$ and is defined as -

$$A \cap B(x) = \{(x, \mu_{A \cap B}(x)) : x \in X\}$$

where,

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] \ \forall x \in X$$

⑤ Equality of two Fuzzy sets

Let A and B be two fuzzy sets then $A = B$ if and only if $\mu_A(x) = \mu_B(x), \forall x \in X$.

Some fundamental Properties of Fuzzy sets

If X be the universal set, ϕ is the null fuzzy set and A, B, C are three fuzzy sets Then:

① Identity law -

ⓐ $A \cap X = A$ and $A \cup \phi = A$

ⓑ $A \cup X = X$ and $A \cap \phi = \phi$

② Idempotent Law -

ⓐ $A \cup A = A$ and $A \cap A = A$

③ commutative law -

ⓐ $A \cap B$ or $B \cap A$ are equal.

ⓑ $A \cup B$ or $B \cup A$ are equal.

(4) Associative law -

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

(5) Distributive law -

(a) $(A) \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(6) Transitive law,

If $A \subset B$ (subset) and $B \subset C$ then $A \subset C$

(7) De-Morgan's law -

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

Algebric Operations of Fuzzy sets -

(1) Sum -

The algebric sum of two fuzzy sets A and B is denoted by A + B and is defined by -

$$A + B = \{(x, \mu_{A+B}(x)) : x \in X\}$$

$1 \leq \mu_{A+B} \leq 1$

where,

$$\boxed{\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)}, \forall x \in X$$

(2) Product -

The algebric product of two fuzzy set A and B is denoted by A . B and is defined as -

$$A \cdot B = \{(x, \mu_{A \cdot B}(x)) : x \in X\}$$

where,

$$\boxed{\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)}, \forall x \in X$$

(3) Power -

The nth power of a fuzzy set 'A' is defined

Let $X = \{x_1, x_2, x_3, x_4\}$ and two fuzzy sets A & B are:
$A = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0.7), (x_4, 1)\}$
$B = \{(x_1, 0.6), (x_2, 1), (x_3, 0.4), (x_4, 0.3)\}$

Then find $A \cup B$, $A \cap B$ and check is A is subset of B ($A \subset B$).

We know that,
If A and B are two fuzzy sets then,
$A \cup B$ is denoted by $A \cup B$ and defined as -
$(A \cup B)(x) = \{(x, \mu_{A \cup B}(x)), \forall \in X\}$
where,
$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)], \forall x \in X$

Now,
(i) $\mu_{A \cup B}(x_1) = \max[\mu_A(x_1), \mu_B(x_1)]$
$= \max[0.2, 0.6]$
$= 0.6$

(ii) $\mu_{A \cup B}(x_2) = \max[\mu_A(x_2), \mu_B(x_2)]$
$= \max[0.5, 1]$
$= 1$

(iii) $\mu_{A \cup B}(x_3) = \max[\mu_A(x_3), \mu_B(x_3)]$
$= \max[0.7, 0.4]$
$= 0.7$

(iv) $\mu_{A \cup B}(x_4) = \max[\mu_A(x_4), \mu_B(x_4)]$
$= \max[1, 0.3]$
$= 1$

So, $A \cup B = \{(x_1, 0.6), (x_2, 1), (x_3, 0.7), (x_4, 1)\}$

Now, Calculate $A \cap B$.

we know that,

If A and B are two fuzzy sets then, A intersectio[n] B is denoted by $A \cap B$ and is defined as -

$$A \cap B = \{(x, \mu_{A \cap B}(x))\} \ \forall x \in X.$$

where,

$$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$$

Now,

(i) $\mu_{A \cap B}(x_1) = \min[0.2, 0.6]$
$= 0.2$

(ii) $\mu_{A \cap B}(x_2) = \min[0.5, 1]$
$= 0.5$

(iii) $\mu_{A \cap B}(x_3) = \min[0.7, 0.4]$
$= 0.4$

(iv) $\mu_{A \cap B}(x_4) = \min[1, 0.3]$
$= 0.3$

So,

$$A \cap B = \{(x_1, 0.2), (x_2, 0.5), (x_3, 0.4), (x_4, 0.3)\}$$

Now,

we also know that,

If A and B are two fuzzy sets then, A is a subset if and only if $\mu_A(x) \leq \mu_B(x), \forall x \in X$.

Here,

(i) $\mu_A(x_1) = 0.2$ and $\mu_B(x_1) = 0.6$

$\mu_A(x_1) \leq \mu_B(x_1)$

ii) $\mu_A(x_2) = 0.5$ and $\mu_B(x_2) = 1$

$$\mu_A(x_2) \leq \mu_B(x_2)$$

iii) $\mu_A(x_3) = 0.7$ and $\mu_B(x_3) = 0.4$

$$\mu_A(x_3) \geq \mu_B(x_3)$$

(iv) $\mu_A(x_4) = 1$ and $\mu_B(x_4) = 0.3$.

$$\mu_A(x_4) > \mu_B(x_4)$$

$\because$ case (iii) and (iv) are not satisfied

$\therefore$ A is not a subset of B.