INTERFERENCE

Superposition of two light waves.

Conditions of Interference:-

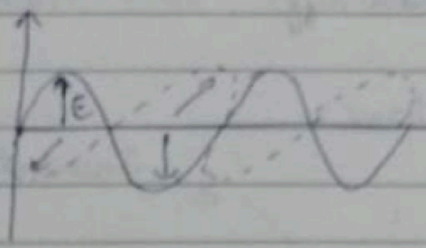
→ Coherent Sources

→ Monochromatic light

→ Phase difference (0/constant)

– Constructive /Destructive

⟶ Cond^n of superposition



(Approx same amplitude)

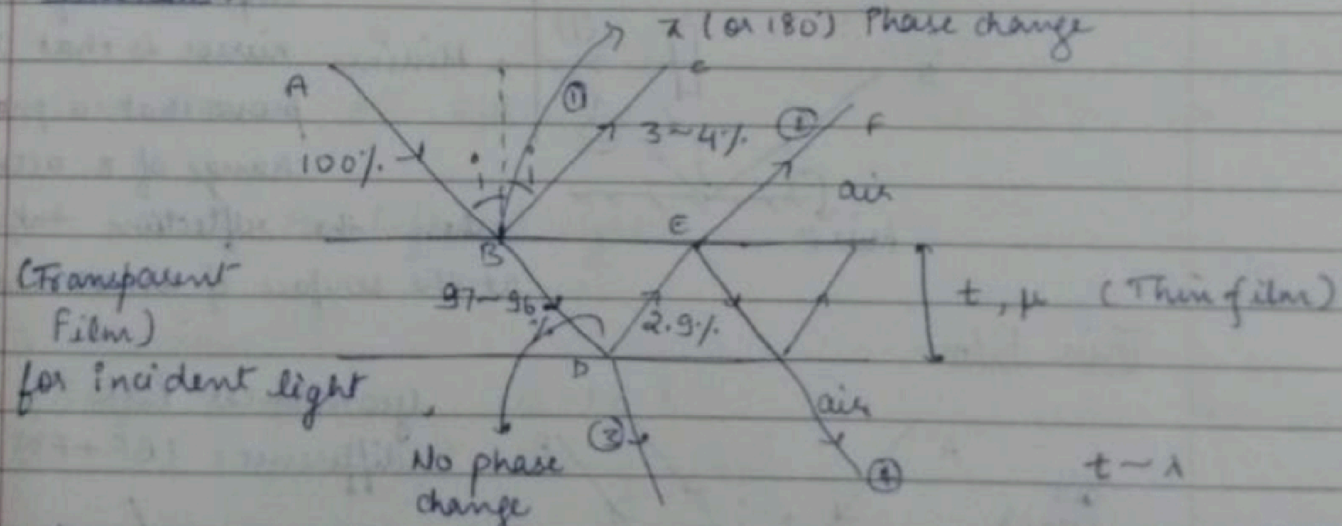
→ Amplitudes of interfering waves should be nearly equal.

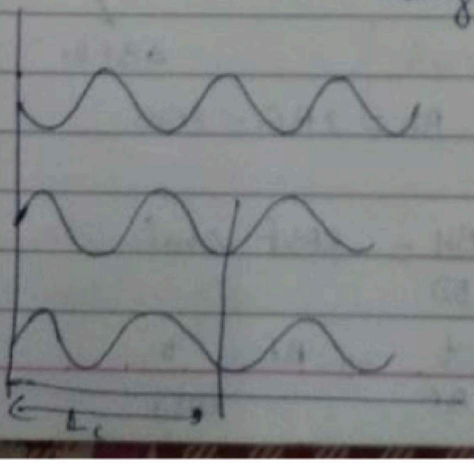
Types of Interference-

Interference

Division of Wavefront (YDSE) → Incident WF divides into 2 parts by refl^n, refr^n or diff^n	Division of Amplitude (Thin Film, Newton's Ring Interference, Michealson's Int.) Amplitude is divided into 2 parts either by parallel refl^n or refr^n

Thin Films :-





L_c → Coherence length

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$$

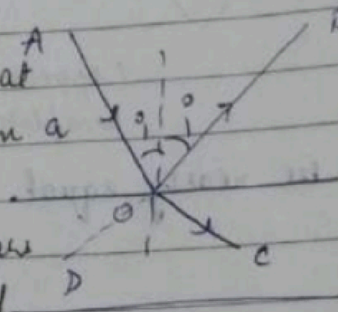
$\Delta\phi$ → Phase diff.

Δx → Path difference

Stoke's Law :-

When a light wave is reflected at the surface of an optically denser medium, it suffers a phase change of π i.e. p.d of $\lambda/2$.

No such phase diff. is introduced if the reflection takes place from the surface of rarer medium



Incident	AO, a	OB, ar	OC, at
Reflection	OB, ar	OA, ar.r	OD, atr'
Transmission	OC, at	OD, ar.t	OA', att'

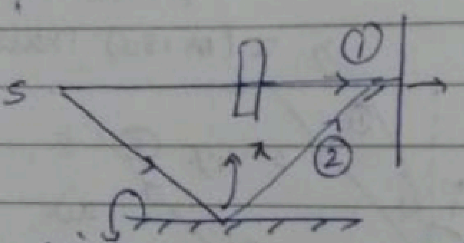
OA: $a = ar^2 + att' \Rightarrow 1 = r^2 + tt'$

OD: $0 = art + atr' \Rightarrow \boxed{r' = -r}$

↓

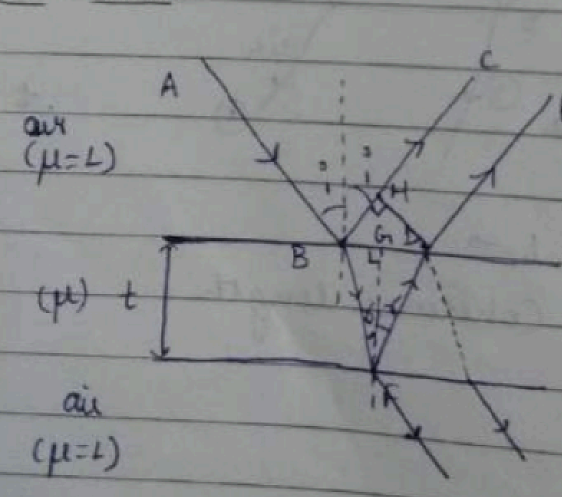
Represents Out of phase

Llyod's Mirror :-



Importance of Llyod's mirror is that it proves that a phase change of π occurs when the reflection takes place at the surface of an OD medium.

Thin Film -



Geometrical Path difference $= (BF + FD) - BH$

$(BF+FD)$ ↓ ΔBFG

BH ↓ ΔBHD

$BD = 2BG = 2GD$

$\frac{BH}{BD} = \cancel{\cos i}\ \sin i$

B

$\cos r = \frac{t}{BF} \Rightarrow BF = \frac{t}{\cos r}$

$$\frac{BG}{BF} = \sin r$$

$$BG = \sin r \cdot \frac{t}{\cos r} = t \cdot \tan r$$

$$BH = \frac{\sin i}{\cancel{\cos i}} \; 2t \tan r$$

$$\Rightarrow \quad BH = 2t \sin i \cdot \tan r$$

Geometrical Path difference $= \frac{2t}{\cos r} - 2t \sin i \cdot \tan r$

$$= \cancel{2t - 2}\; 2t \left[\frac{1 - \sin i \cdot \sin r}{\cos r} \right]$$

Optical Path difference $= \mu \left[(BF + FD) - BH \right]$

$$= \mu(BF + FD) - 1 \cdot BH$$

$$= \mu \left[\frac{2t}{\cos r} \right] - 2t \sin i \cdot \tan r$$

$$= \frac{2t}{\cos r} \left[\frac{\sin i}{\sin r} - \sin i \cdot \sin r \right] = \frac{2t \sin i}{\cos r} \left[\frac{1 - \sin^2 r}{\sin r} \right]$$

$= \cancel{2\mu t \cos r}$

$$= \frac{2t\mu \cos^2 r}{\cos r} = \boxed{2\mu t \cos r}$$

Correction $= 2\mu t \cos r \pm \frac{\lambda}{2} = \Delta$

(For Reflected Light)

(I) Maxima:
(Bright Fringe)

$$\Delta = \pm m\lambda$$

$$2\mu t \cos r + \frac{\lambda}{2} = \pm m\lambda$$

$$\boxed{2\mu t \cos r = \pm (2m-1)\frac{\lambda}{2}}$$

(II) Minima:
(Dark Fringe)

$$\Delta = \pm (2m+1)\frac{\lambda}{2}$$

$$\cancel{\Delta =}\; 2\mu t \cos r + \frac{\lambda}{2} = \pm (2m+1)\frac{\lambda}{2}$$

In Interference, we use extended source

$$\boxed{2\mu t \cos r = \pm m\lambda}$$

For <u>Transmitted Light</u>-

Optical Path Diff. = $2\mu t \cos r$

(Denser to Rarer)

(I) Maxima (Bright Fringe)-

$$\Delta = \pm m\lambda$$

$$\boxed{2\mu t \cos r = \pm m\lambda}$$

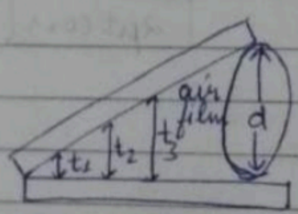
(II) Minima (Dark Fringe)-

$$\Delta = \pm (2m+1)\frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = \pm (2m+1)\frac{\lambda}{2}}$$

Fringes: Fringes are locus of equal thickness

<u>Wedge Shaped</u>:

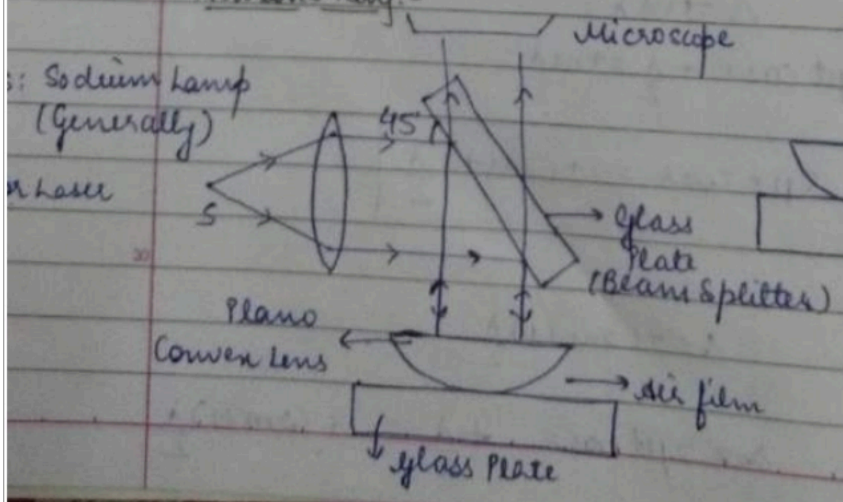


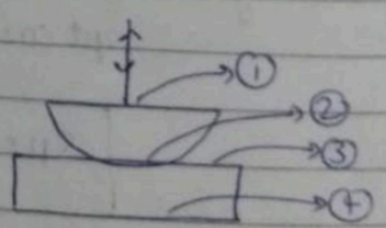
$t_1 < t_2 < t_3 < \ldots$

Straight line pattern will be obtained.

(cross secn of air film is straight line)

<u>Newton's Ring</u>:-





All ①, ②, ③, ④ will interfere but only ② & ③ will produce Newton's Fringes.

[air film cross-section is consecutive circles and along this circle air film thickness is constant]

Other interference pattern are not that much intense / visible.

① Condition for maxima / minima:-

(Bright) (Dark)

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2} \quad — ①$$

Normal Incidence , $r = 0$

Air film, $\mu = 1$

$$2t + \frac{\lambda}{2} \quad — ②$$

(i) Maxima:- $2t + \frac{\lambda}{2} = \pm m\lambda$

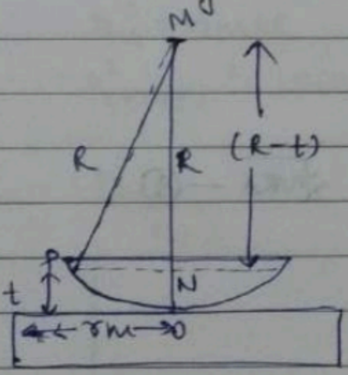
$$\boxed{2t = (2m-1)\frac{\lambda}{2}} \quad — ③$$

(ii) Dark :- $2t + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}$

$$\boxed{2t = m\lambda} \quad — ④$$

② Circular Fringes -

Ro



③ (Radii of Dark Fringes) -

Dark

Δ MNP :-

$$PM^2 = MN^2 + PN^2$$

$$R^2 = (R-t)^2 + r_m^2$$

$$r_m^2 = R^2 - R^2 - t^2 + 2Rt$$

$$r_m^2 = 2Rt - t^2 \quad — ⑤$$

$\because$ Air film thickness is very small

$\therefore R >> t^2$

$$\Rightarrow \boxed{r_m^2 = 2Rt} \quad — ⑥$$

$$\Rightarrow r_m^2 = m\lambda R$$

$$\Rightarrow \boxed{r_m = \sqrt{m\lambda R}} \quad — ⑦$$

[air film cross-section consecutive is circles and along this circle air film thickness is constant]

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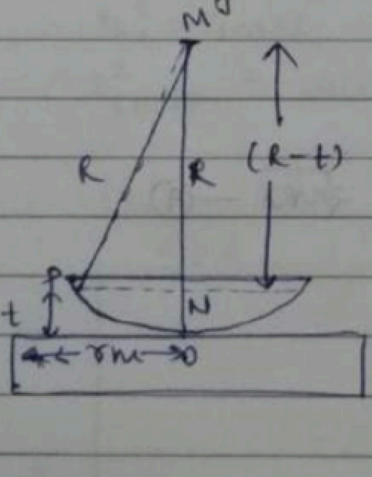

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∵ Air film thickness is very small

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$$\Rightarrow \boxed{r_m^2 = 2Rt} \quad —⑥ $$

$$\Rightarrow r_m^2 = m\lambda R$$

$$\Rightarrow \boxed{r_m = \sqrt{m\lambda R}} \quad —⑦ $$

(4) Spacing:-

$m = 1, 2, 3, ---$

$r_2 - r_1 = \sqrt{2\lambda R} - \sqrt{\lambda R} = 0.414\sqrt{\lambda R}$

$r_3 - r_2 = \sqrt{3\lambda R} - \sqrt{2\lambda R} = 0.318\sqrt{\lambda R}$

$r_4 - r_3 = \sqrt{4\lambda R} - \sqrt{3\lambda R} = 0.268\sqrt{\lambda R}$

Area of m^{th} circle - Area of $(m-1)^{th}$ circle

[Spacing b/w fringes decreases but area remains constant.]

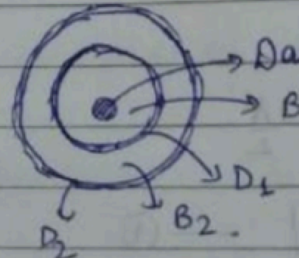
$\pi r_m^2 - \pi r_{m-1}^2$

$\pi\,[m\lambda R - (m-1)\lambda R]$

$\pi\lambda R\,[m - m + 1]$

$\boxed{\pi\lambda R}$ — (8)

(5) central Dark ~~fringe~~ spot -



(6) Fringes of Equal Thickness -

(7) Diameter -

$$D_m = \sqrt{4mR\lambda} = 2\sqrt{mR\lambda} \quad \text{— (9)}$$

Bright -

$r_m^2 = \cancel{2}R\,(2m-1)\frac{\lambda}{\cancel{4}\,2}$

$$r_m = \sqrt{\frac{(2m-1)\lambda R}{2}}$$

$$D = 2\sqrt{\frac{(2m-1)\lambda R}{2}}$$

$$D = \sqrt{2(2m-1)\lambda R}$$

Dark fringes are clearly confined on bright fringe. Thats why we see dark.

<u>Transmitted</u> :- $\Delta = 2\mu t\cos r$

$\mu = 1, \quad r = 0$

$\Delta = 2t$

<u>Bright</u> : $2t = n\lambda$

<u>Dark</u> :- $2t = (2n+1)\frac{\lambda}{2}$

⑧ Newton's Fringes are called Localised Fringes. Because they are formed on top of convex lens.

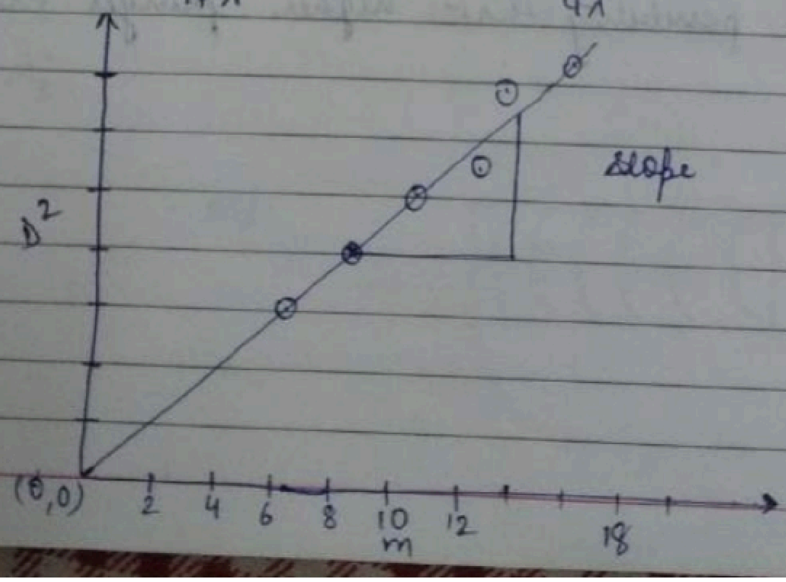
18 − 8

$A - B = D \approx 1$

$D_m^2 = 4m\lambda R$

n=10 ⇒ $D_{(m+P)}^2 = 4(m+P)\lambda R$

$$\frac{D_{(m+P)}^2 - D_m^2}{4P\lambda} = R = \frac{\text{slope}}{4\lambda}$$



Q- How Newton Rings (Fringes) are formed?
② & ③ interference

$$\rightarrow \quad \mu \frac{[D_{m+p}^2 - D_m^2]_{liq}}{4p\lambda} = R$$

$$\frac{[D_{m+p}^2 - D_m^2]_{air}}{4p\lambda} = R$$

$$\Rightarrow \quad \mu = \frac{[D_{m+p}^2 - D_m^2]_{air}}{[D_{m+p}^2 - D_m^2]_{liq}}$$

(We can find out the refractive index of liquid by performing the exp. two times - 1) for air 2) for liquid by placing liquid in place of air).

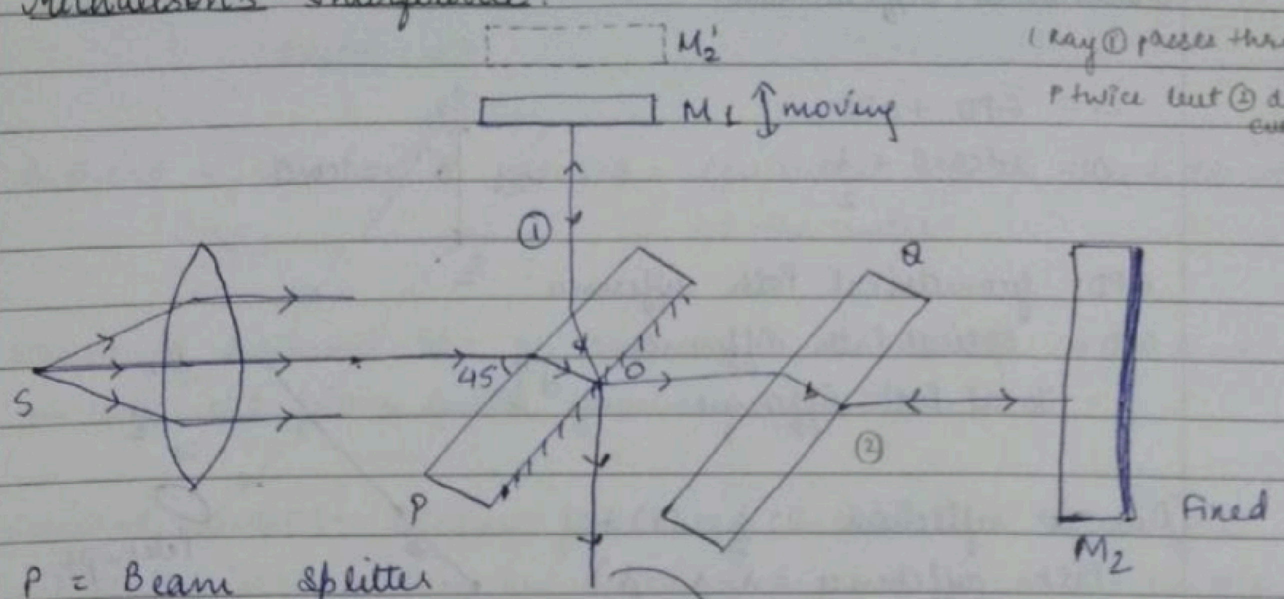
- Newton's Ring ~~...~~ exp is type of division of amplitude

- We can not use mirror instead of glass plate because due to mirror reflection brightness will be very large & dark fringes will get dull. (not appear clearly)

- If we use white light in place of monochromatic light (sodium lamp), then centre will be bright and then coloured fringes will be formed of those colours ~~whos which~~ whose 'λ' satisfy maxima condⁿ. There is a possibility that higher fringes may not appear.

Michaelson's Interference :-

(Ray ① passes through P twice but ② does not) ~~even once~~

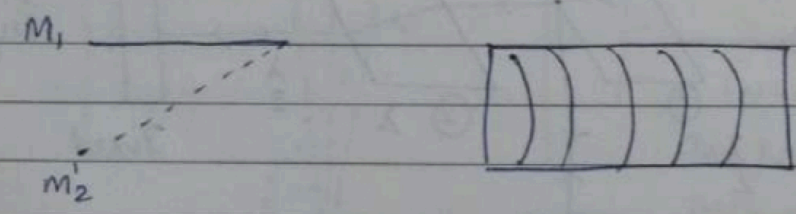


P = Beam splitter
50% silvered
Q = Compensating glass plate
M_2' = Image of M_2

This ray appears to come from M_2' but not really comes from M_2'.

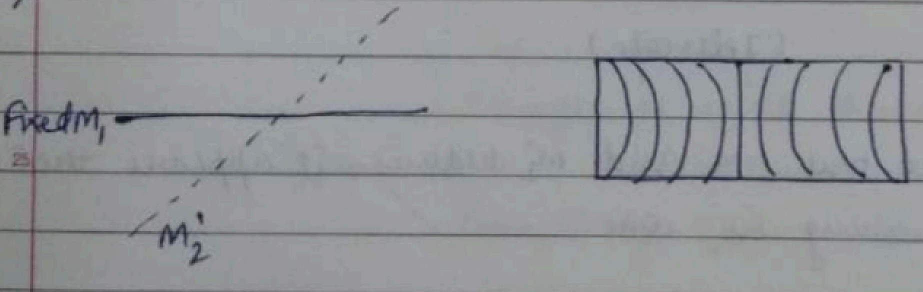
Types of Fringes:-

1)



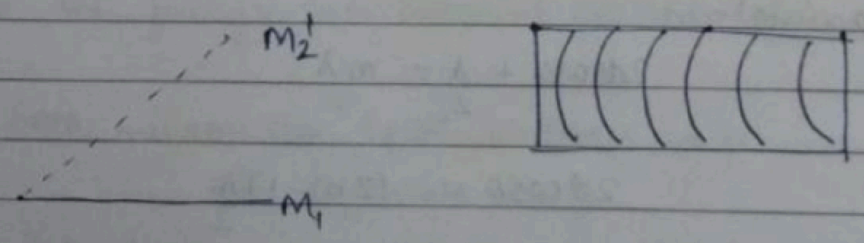
Curved fringes with ~~circular fringes~~ convexity towards thin edge of the wedge

2)



~~Curvature~~ straight fringes

3)

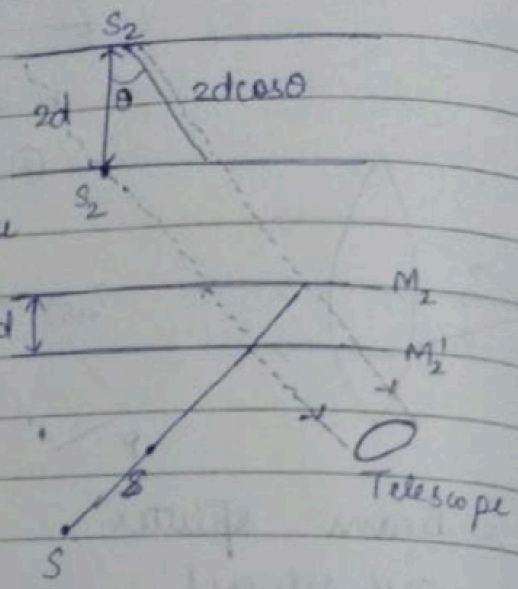


$M_2' =$ Image of ~~com~~ M_1 (just ...

Michelson's Interferometer:-

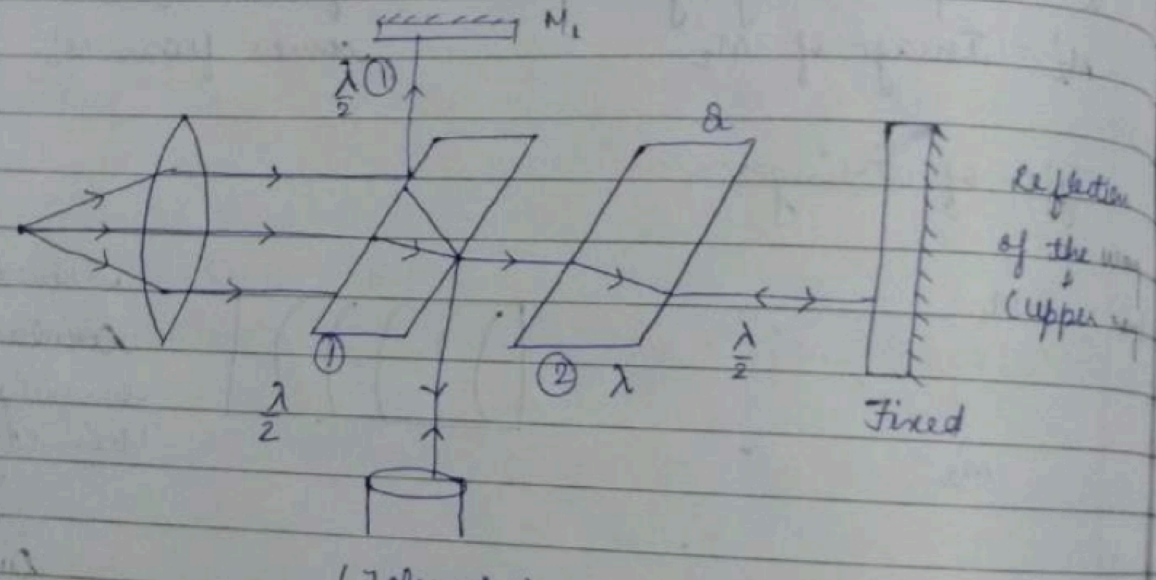
$\Delta =$ GPD + OPD

$$\Delta = 2d\cos\theta + \frac{\lambda}{2}$$



GPD = Geometrical Path difference
OPD = Optical Path difference
Δ = Total Path difference

(Due to reflection (1) $\frac{\lambda}{2}$ (2) λ
Path difference $= \lambda - \frac{\lambda}{2} = \frac{\lambda}{2}$)



(Telescope)

If the distance b/w M_1 and M_2' reduces, it appears that circles are vanishing in cone.

(1) Bright (Maxima)-

$$2d\cos\theta + \frac{\lambda}{2} = m\lambda$$

$$2d\cos\theta = (2m-1)\frac{\lambda}{2}$$

use of compensating plate ?

(2) Minima -

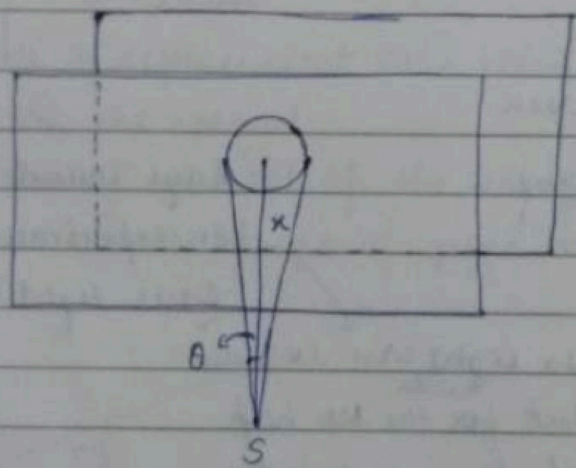
$$2d\cos\theta = n\lambda$$

- As M_1 and M_2' distance is reduced = concentric circles tends to vanish at the centre

- Whereas if distance b/w M_1 and M_2 is increased : circles tend (appear to come out from centre growing outwards.

- CIRCULAR FRINGES - Because for every inclination (θ) we get different fringe. so they are called fringes of equal inclination.

Fringes of equal inclination - (Haidinger's Fringes)



No. of fringes vanished is given by

$$x = x_1 - x_2 = \frac{N\lambda}{2}$$

(N = No of fringes disappear in centre)

For particular angle ray path difference is equal to λ.

∴ Circular rings are obtained.

Types
- Fringes of equal thickness
- Fringes of equal inclination

- Here, the fringes are obtained on telescope.

- If both mirrors are at equal distance $d = 0$

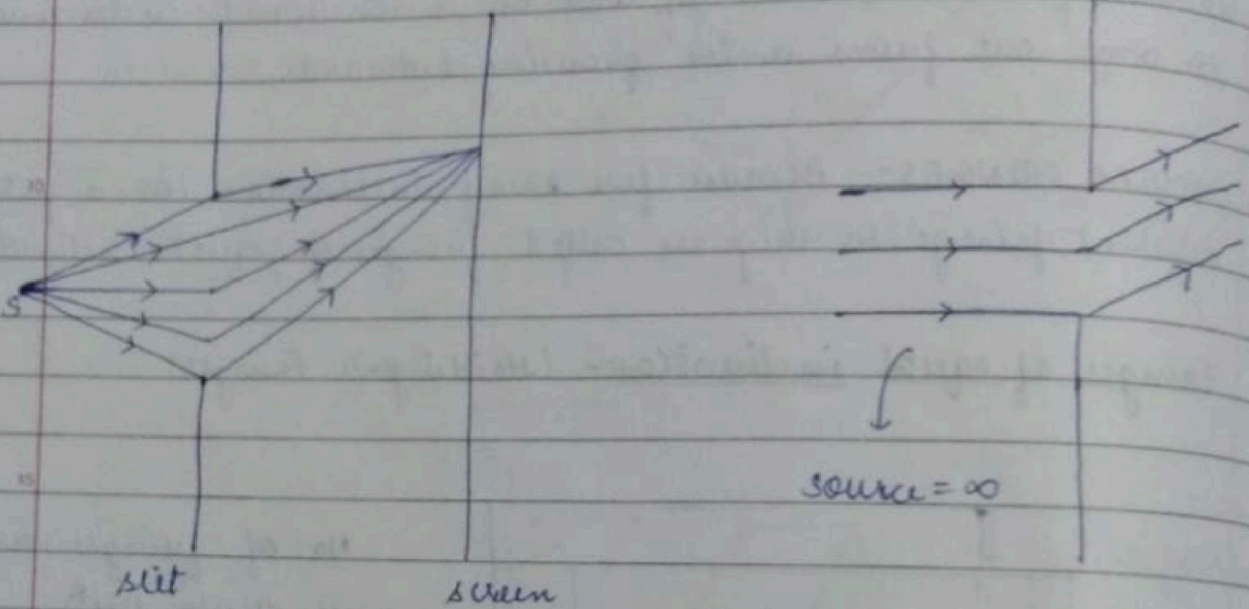
∴ Path difference $\Delta = \frac{\lambda}{2}$

∴ Only dark light will be seen throughout the plane.

Diffraction:-

Bending of light

Diffraction
- Fresnel's
- Fraunhoffer



Rays travel making angles

Rays travel parallel (In experiment we use laser light)

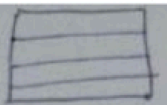
No light can be used but with the help of lens combination

→ Fresnel's has drawback

Fresnel's half period zone method:-

Theoritical concept

$\frac{\lambda}{2}$ assumption path difference b/w Top and bottom rays $\Rightarrow \frac{\lambda}{2}$

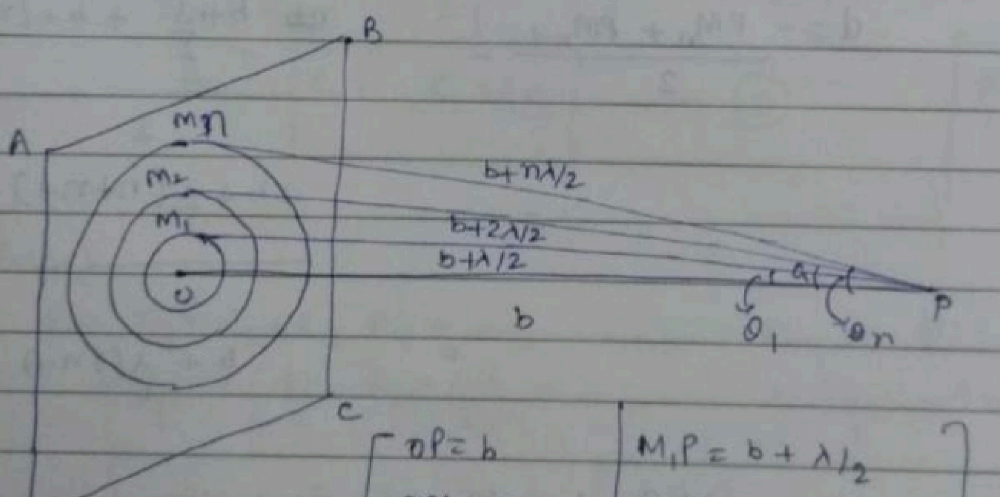


Fresnel gave 1/2 period zone method to find intensity of light at a point due to wavefront. This method is based on following assumptions:-

(1) Given wavefront is divided into small region - each region is called a zone. These zones are such that there is a phase difference of π (path difference $= \lambda/2$, time difference $= t/2$) b/w the light rays reaching the point P from the extreme of the zone. Hence, these are called as 1/2 period zone. Each 1/2 period zone behaves like a source of secondary wavelets.

(2) Displacement of a point on the screen due to entire wavefront = algebraic sum of the displacements at that point due to secondary wavelets emitted from each zones.

(3) Amplitude of displacement at a point due to a zone depends -
(i) Angle b/w the normal
(ii) Line joining the pole of the wavefront to that point.
(iii) On the distance of point from the zone.



(1) —
$$\left.\begin{aligned} OP &= b \\ OM_1 &= r_1 \\ OM_2 &= r_2 \\ &\vdots \\ OM_n &= r_n \end{aligned}\right. \qquad \left.\begin{aligned} M_1P &= b + \lambda/2 \\ M_2P &= b + 2\lambda/2 \\ M_nP &= b + n\lambda/2 \end{aligned}\right] \; -(2)$$

Date

1) Area of half period zone-

$$OM_nP \rightarrow r_n^2 = \frac{n^2\lambda^2}{4} + nb\lambda \quad \text{--- (3)}$$

$$\lambda^2 <<<$$

$$\boxed{r_n^2 = nb\lambda} \quad \text{--- (4)}$$

Area of n^{th} zone = Area of n^{th} circle − Area of $(n-1)^{th}$ circle

$$= \pi nb\lambda - \pi(n-1)b\lambda$$

$$= \pi b\lambda[n-n+1]$$

$$\boxed{\text{Area of } n^{th} \text{ zone} = \pi b\lambda} \quad \text{--- (5)}$$

2) Separation between the zones-

Width of zone $= r_n - r_{n-1}$ —— (6)

$$r_2 - r_1 = \sqrt{2b\lambda} - \sqrt{b\lambda} = 0.414\sqrt{b\lambda}$$

$$r_3 - r_2 = \sqrt{3b\lambda} - \sqrt{2b\lambda} = 0.318\sqrt{b\lambda}$$

$$r_4 - r_3 = \sqrt{4b\lambda} - \sqrt{3b\lambda} = (2-1.732)\sqrt{b\lambda} = 0.268\sqrt{b\lambda}$$

$$\begin{array}{r} 1.732 \\ 1.414 \\ \hline 318 \end{array}$$

3) Average distance of n^{th} zone from point P:-

$$d = \frac{PM_n + PM_{n-1}}{2} = \frac{\cancel{nb}\; b + \frac{n\lambda}{2} + b + (n-1)\frac{\lambda}{2}}{2}$$

$$= \frac{2b + \frac{\lambda}{2}[n+n-1]}{2}$$

$$= b + \frac{\lambda}{4}(2n-1) \quad \text{--- (7)}$$

4) Amplitude at point P due to one zone :-

Amplitude of the displacement reaching the pt. P from zone depends on the following factors -

Date / /

1) Area of half period zone -

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$$= b + \frac{\lambda}{4}(2n-1) \quad \text{--- (7)}$$

4) Amplitude at point P due to one zone :-

Amplitude of the displacement reaching the pt. P from zone depends on the following factors -

(i) Amplitude is directly proportional to the area of the zone.

$$R_n \propto \pi\left[b\lambda + (2n-1)\frac{\lambda^2}{4}\right]$$

(ii) Amplitude is directly proportional to the $\cos\theta$.

θ = Angle of inclination of zone.

$$R_n \propto (1+\cos\theta_n)$$

(iii) Amplitude is inversely proportional to the avg. distance of point P from zone.

$$R_n \propto \frac{1}{b+(2n-1)\frac{\lambda}{4}}$$

$$R_n \approx \pi\lambda\,[1+\cos\theta_n] \quad \text{—— (8)}$$

(iii) 5) Resultant amplitude at the point of observation due to entire wavefront ABCD:-

$$R = R_1 - R_2 + R_3 - R_4 + \cdots\cdots + R_n \quad \text{—— (9)}$$

(Odd)

$$= \frac{R_1}{2} + \left[\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right] + \left[\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right] - \cdots\cdots + \frac{R_n}{2}$$

$$\boxed{R = \frac{R_1+R_n}{2}} \quad \text{—— (10)}$$

$R_2 = \frac{R_1+R_3}{2}$

↓

Assumption

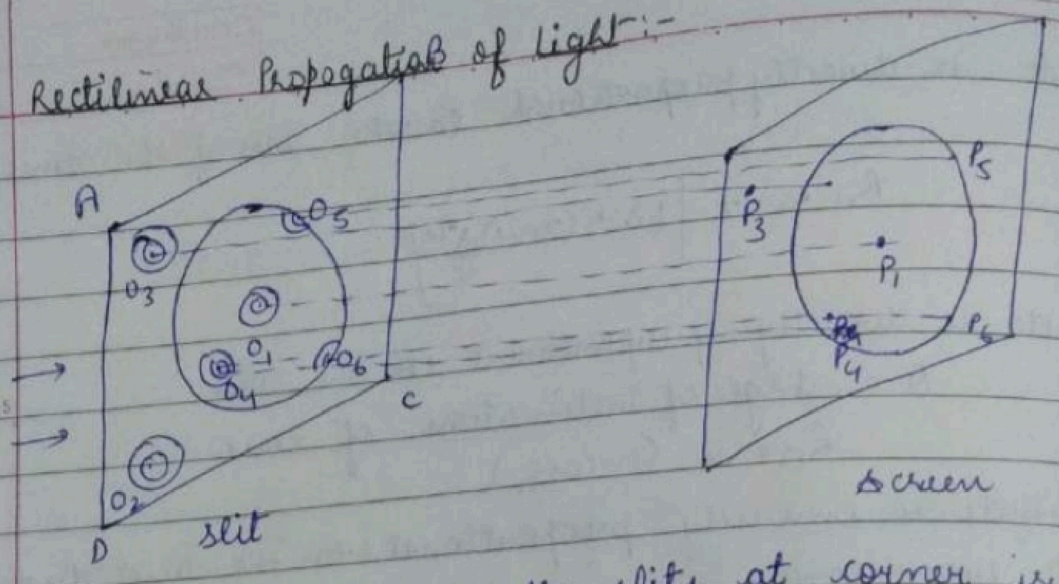
(Even)

$$R = R_1 - R_2 + R_3 - R_4 + \cdots\cdots - R_n \quad \text{—— (11)}$$

$$= \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \cdots\cdots + \frac{R_{n-1}}{2} - R_n \quad \text{—— (12)}$$

$$\boxed{R = \frac{R_1+R_{n-1}}{2} - R_n} \quad \text{—— (13)}$$

Rectilinear Propogation of Light :-



The intensity through the slits at corner is much different from theoritical data.

Zone Plate :-

- When we have to deal with X-ray, zone plate is used in Fresnel's Diffraction.
- Dark → opaque
  Blank → Transparent

Central Zone
Transparent → +ve
Opaque → −ve

$$r \propto \sqrt{n}$$

$$SO + OP = a + b \quad \text{--- (1)}$$

$$SM_1 + M_1P = SO + OP + \frac{\lambda}{2} = a + b + \frac{\lambda}{2}$$

$$\vdots$$

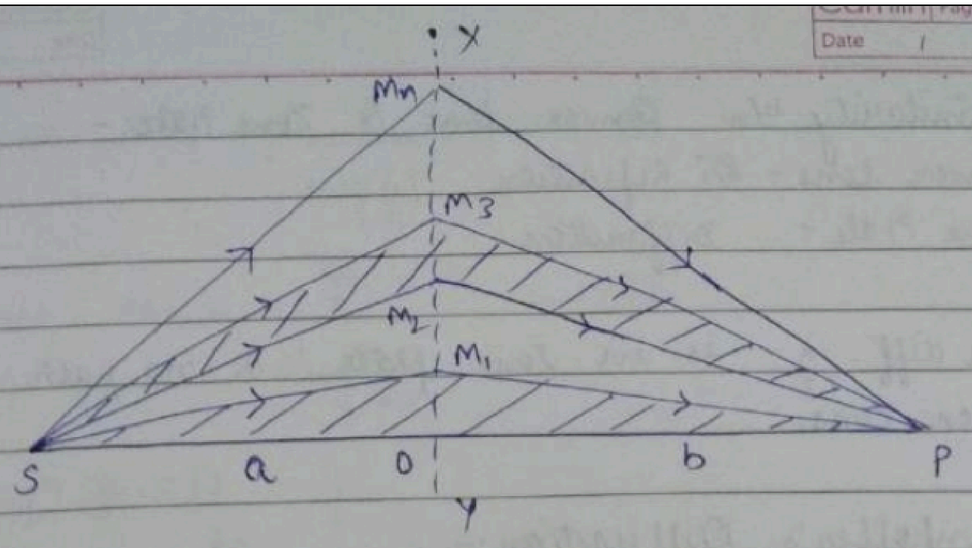
$$SM_n + M_nP = a + b + \frac{n\lambda}{2} \quad \text{(2)}$$

$$\Delta SM_nO \rightarrow \quad SM_n^2 = a^2 + r_n^2 \quad \text{--- (3)}$$

$$SM_n = (a^2 + r_n^2)^{1/2}$$

$$= a\left[1 + \frac{r_n^2}{a^2}\right]^{1/2} = a\left[1 + \frac{r_n^2}{2a^2}\right]$$

$$SM_n = a + \frac{r_n^2}{2a} \quad \text{--- (4)}$$



$$\Delta PM_nO \rightarrow \quad PM_n^2 = b^2 + r_n^2$$

$$PM_n = b\left[b + \frac{r_n^2}{2b}\right] \quad \text{— (5)}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r_n^2} \quad \text{— (6)}$$

$$SM_n + SM_nP = a + b + \frac{n\lambda}{2}$$

$$\cancel{a} + \cancel{b} + \frac{n\lambda}{\cancel{2}} = \cancel{a} + \cancel{b} + \frac{r_n^2}{\cancel{2}a} + \frac{r_n^2}{\cancel{2}b}$$

$$\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r_n^2}$$

$$\boxed{-\frac{1}{a} + \frac{1}{b} = \frac{1}{f_n}} \quad \text{— (7)}$$

$$f_n = \frac{r_n^2}{n\lambda} \quad \text{— (8)}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{— (9)}$$

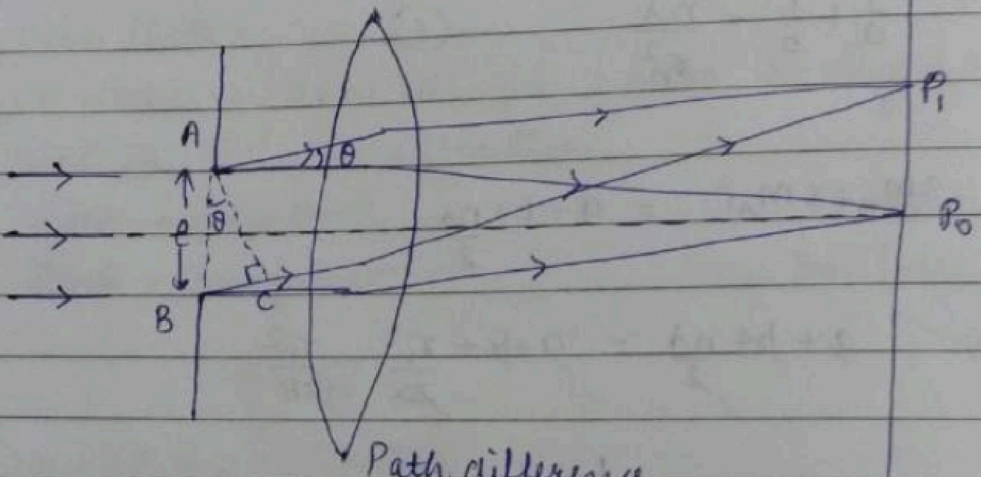
- Like a convex lens, ~~it~~ zone plate focusses light at pt. wi same intensity.

- It also possess chromatic abberation & focus light of different wavelength at different pts.

- Dissimilarity b/w Convex lens & Zone Plate:-
  Convex lens - Refraction
  Zone Plate - Diffraction

- Path diff. $\frac{\lambda}{2}$ is in Zone plate & no path diff in Convex lens.

Fraunhoffer's Diffraction:-
Single slit Diffraction:-



Path difference

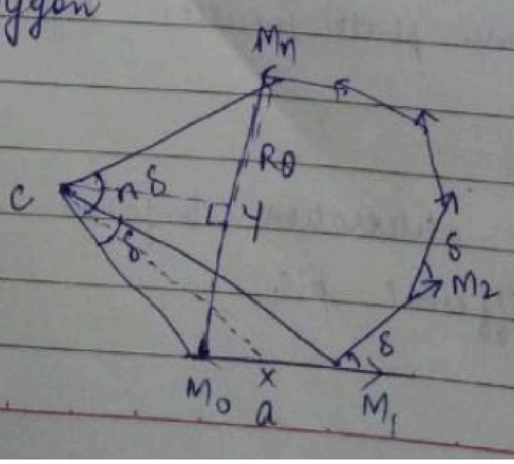
$AB = e \quad , BC = e\sin\theta$ — ①

Phase difference $= \frac{2\pi}{\lambda} e\sin\theta$ — ②

n-part

$\delta = \frac{1}{n} \cdot \frac{2\pi}{\lambda} e\sin\theta$ — ③

Polygon



$M_0X = \frac{a}{2}$

$M_0Y = \frac{R_\theta}{2}$

$\angle M_0CX = \frac{\delta}{2}$

$\Delta CM_0X \rightarrow \quad CM_0 = \dfrac{\frac{a}{2}}{\sin\left(\frac{\delta}{2}\right)}$ — (4)

$\Delta CM_0Y \rightarrow \quad CM_0 = \dfrac{\frac{R_\theta}{2}}{\sin\left(\frac{n\delta}{2}\right)}$ — (5)

eq^n (5) ÷ (4)

$$\frac{\frac{R_\theta}{2}}{\sin\left(\frac{n\delta}{2}\right)} = \frac{\frac{a}{2}}{\sin\left(\frac{\delta}{2}\right)}$$

$$R_\theta = \frac{a\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \quad \text{— (6)}$$

$$R_\theta = \frac{a\sin\left[\frac{\pi e\sin\theta}{\lambda}\right]}{\sin\left[\frac{\pi e\sin\theta}{n\lambda}\right]} \quad \text{— (7)}$$

$$R_\theta = \frac{a\sin p}{\sin\left(\frac{p}{n}\right)} \quad \text{— (8)}$$

$$R_\theta = an\left[\frac{\sin p}{p}\right] \quad \text{— (9)}$$

$p = \frac{\pi e\sin\theta}{\lambda}$

$n >>>$, $\sin\left(\frac{p}{n}\right) \approx \frac{p}{n}$

If $\theta = 0$, $p = 0$, $\delta = 0$

$$R_0 = an$$

$$R_\theta = R_0\left[\frac{\sin p}{p}\right] \quad \text{— (10)}$$

P_1

$$I \propto R_\theta^2 \Rightarrow I = kR_0^2\left[\frac{\sin p}{p}\right]^2 \quad \text{— (11)}$$

P_0

$$I_0 = kR_0^2 = ka^2n^2$$

$$I = I_0 \left[\frac{\sin p}{p}\right]^2 \quad \text{——— (12)}$$

I) Principal Maxima :-

$$p = 0$$

$$I = I_0$$

$$\frac{\pi e \sin\theta}{\lambda} = 0$$

$$e \sin\theta = 0$$

$$\boxed{\theta = n\pi}$$

Principal Maxima : $\theta = 0$

II) Minima :-

$$\sin p = 0 \quad , \; p \neq 0$$

$$p = \pm m\pi$$

$$\frac{\cancel{\pi} e \sin\theta}{\lambda} = \pm m \cancel{\pi}$$

$$\boxed{e \sin\theta = \pm m\lambda} \quad \text{——— (13)}$$

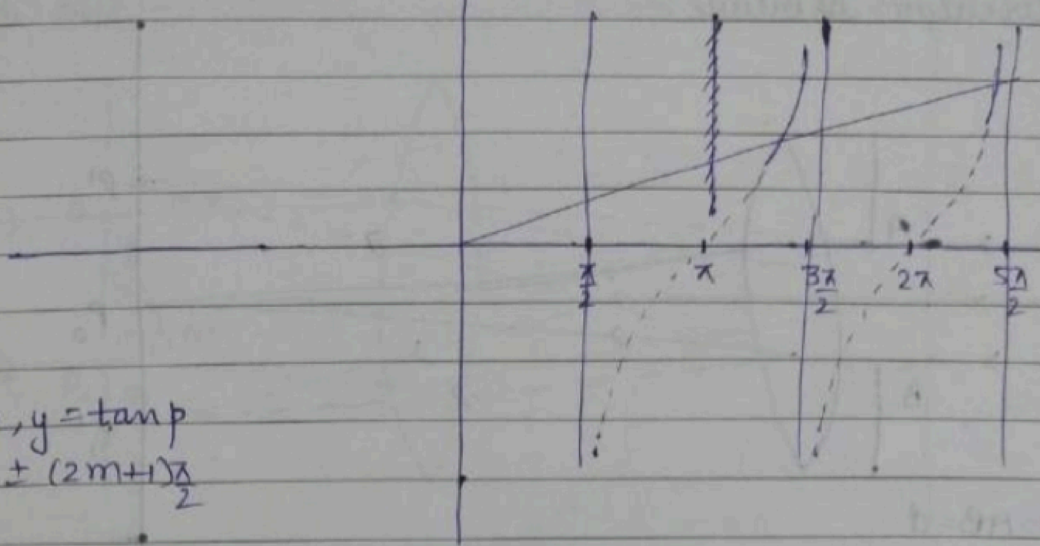
e = slit width

Eqn of Minima.

III) Secondary Maxima :-

$$\frac{dI}{dp} = 0$$

$\sin p = 0$ ✗ or $p = \tan p$ ✓ → condition for secondary Maxima



$y = p,\ y = \tan p$

$p = \pm (2m+1)\frac{\pi}{2}$

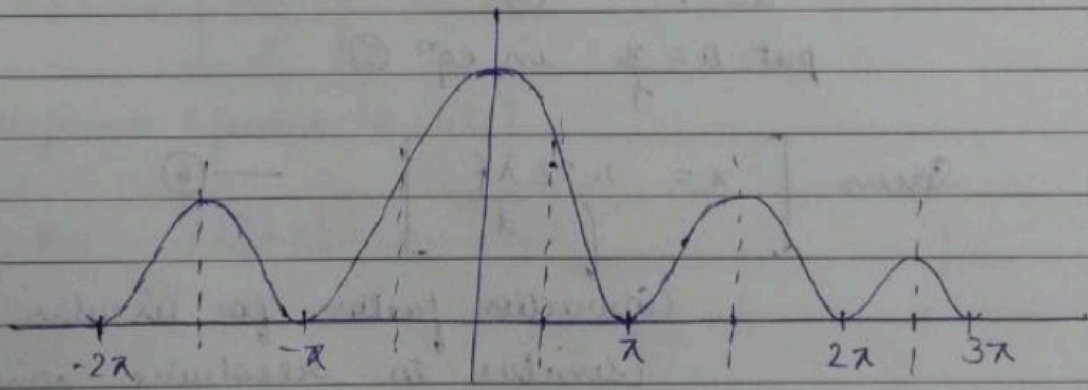
$p = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \ldots$

1^{st} order Secondary Maxima $\leftarrow \frac{3\pi}{2} \Rightarrow I_1 = \frac{I_0}{22}$

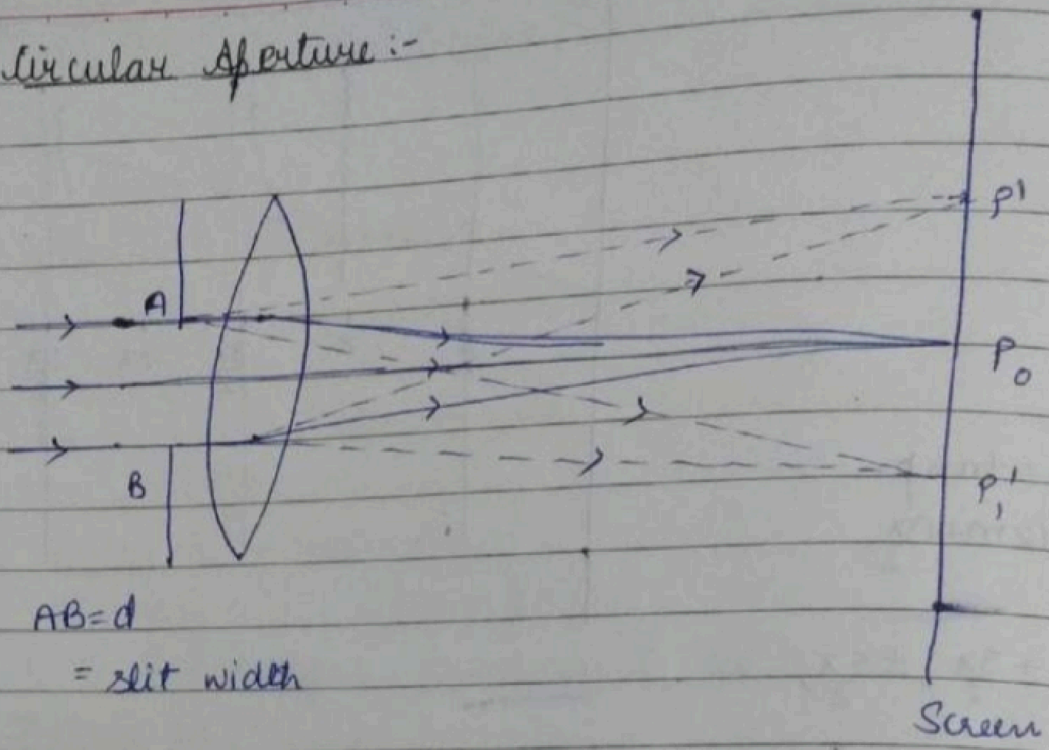
2^{nd} Order Secondary Maxima $\leftarrow \frac{5\pi}{2} \Rightarrow I_2 = \frac{I_0}{61}$

3^{rd} order Secondary Maxima $\leftarrow \frac{7\pi}{2} \Rightarrow I_3 = \frac{I_0}{121}$

Intensity goes on decreasing



Circular Aperture :-



$AB = d$
= slit width

Dark: $d\sin\theta = m\lambda$ — (1)

Bright: $d\sin\theta = (2m-1)\dfrac{\lambda}{2}$ — (2)

$m = 1$

$d\sin\theta = \lambda$ — (3)

$\sin\theta \approx \theta \approx \dfrac{x}{f}$ — (4)

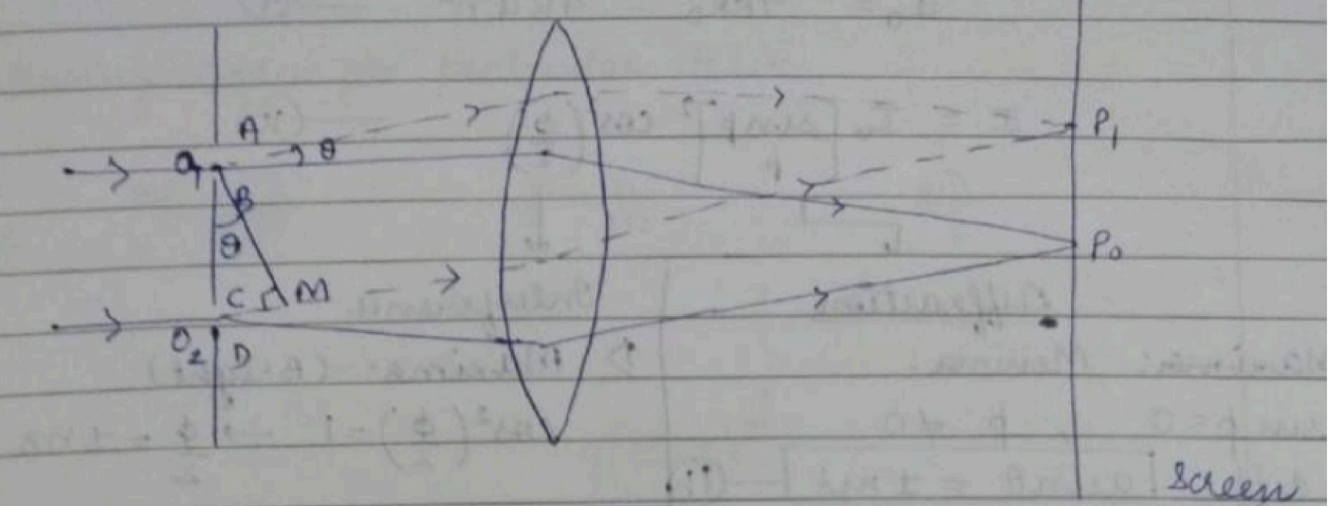
$d\theta = \lambda$ — (5)

put $\theta = \dfrac{x}{f}$ in eqn (5)

~~Screen~~ $\boxed{x = 1.22\dfrac{\lambda f}{d}}$ — (6)

(Correction factor for circular aperture [similar to resolving power of telescope]

Double slit :-

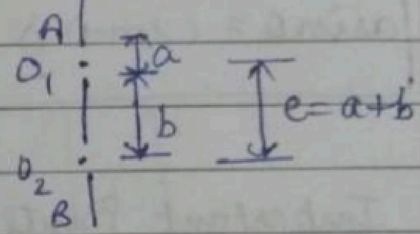


Path difference, $\boxed{DM = e\sin\theta}$ —— ①

e = distance between centre of two slits

Phase difference, $\boxed{\phi = \frac{2\pi}{\lambda} e\sin\theta}$ —— ②

$\boxed{p = \frac{\pi a\sin\theta}{\lambda}}$ —— ④



Single slit:

$$R_\theta = R_0\left[\frac{\sin p}{p}\right] \text{ — ③}$$

Interference (from 2 slits)

$$R = \sqrt{R_\theta^2 + R_\theta^2 + 2R_\theta R_\theta\cos\phi} \text{ —— ⑤}$$

$$\left[\because A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}\right]$$

$$R = 2R_\theta\cos\left(\frac{\phi}{2}\right) \text{ —— ⑥}$$

$$I \propto R^2$$

$$I = KR^2 = 4KR_\theta^2\cos^2\left(\frac{\phi}{2}\right) \text{ — ⑦}$$

$$I = 4KR_0^2\left[\frac{\sin p}{p}\right]^2\cos^2\left(\frac{\phi}{2}\right) \text{ — ⑧}$$

If $\theta = 0, p = 0, \phi = 0$

$$I_0 = 4KR_0^2 = 4Ka^2n^2 \quad \text{— (9)}$$

$$I = I_0 \left[\frac{\sin p}{p}\right]^2 \cos^2\left(\frac{\phi}{2}\right) \quad \text{— (10)}$$

Diffraction

1) ~~Maxima~~: Minima:

$\sin p = 0 \quad , \quad p \neq 0$

Path diff. $\boxed{a\sin\theta = \pm m\lambda}$ — (11)

2) Second. Maxima:

$p = \pm \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$\boxed{a\sin\theta = (2m+1)\frac{\lambda}{2}}$ — (12)

Interference

1) Maxima: (Bright)

$\cos^2\left(\frac{\phi}{2}\right) = 1 \rightarrow \frac{\phi}{2} = \pm n\pi$

Path diff. $\boxed{e\sin\theta = n\lambda}$ — (13)

$n = 0, 1, 2, 3, \ldots$

2) Minima (Dark):

$\cos^2\left(\frac{\phi}{2}\right) = 0 \rightarrow \frac{\phi}{2} = (2n+1)\frac{\pi}{2}$

$\boxed{e\sin\theta = (2n+1)\frac{\lambda}{2}}$ $\rightarrow n = 0, 1, 2, 3, \ldots$ — (14)

Important Points:-

1) Envelope of fringe pattern depends on the width 'a' of each slit and wavelength of incident light 'λ'.

2) On the other hand the fringe width and no. of interference maxima within the central maxima depends on the opaque separation 'b', b/w the slit.

3) If increase the width of each slit, central peak becomes sharp.

4) If increase the 'λ', envelopes becomes wide and fringe spread.

5) If we increase the 'b' the no. of interference maxima within the central maxima increases but separation

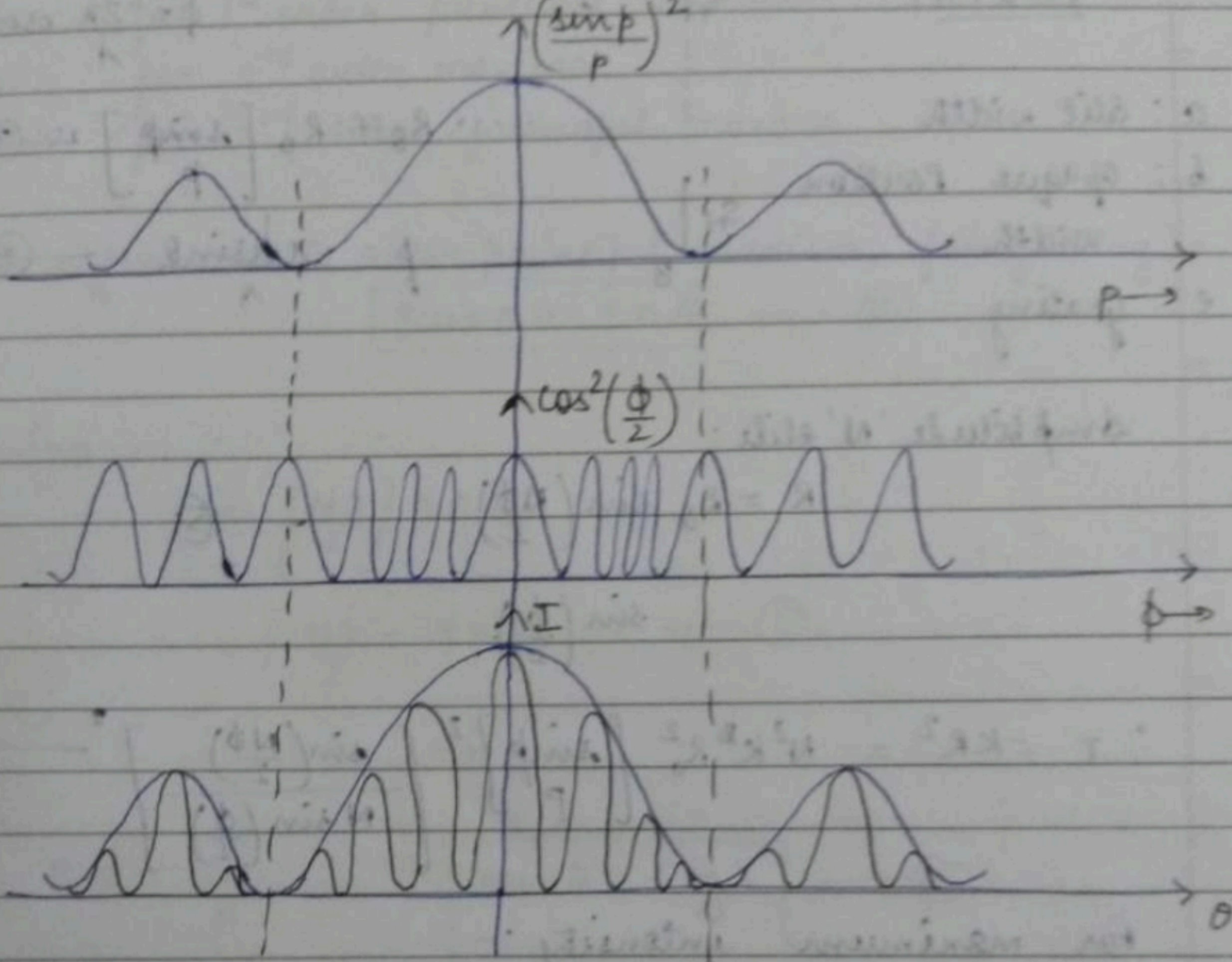
b/w interference maxima decreases.

→ Missing order for particular [θ] :-

$eq^n \frac{(13)}{(11)} \Rightarrow \frac{e}{a} = \frac{n}{m}$ —— (15)

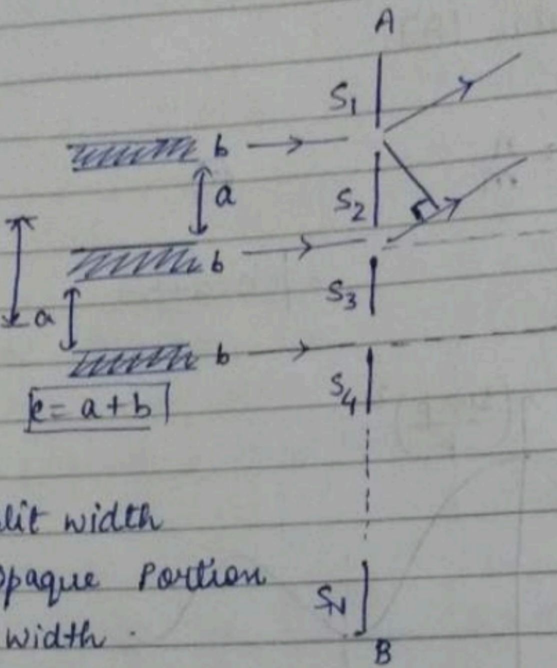
$b = 3a$ | $e = a + b$

$n = 4m$



Plane Transmission Diffraction Grating -

N - No. of slits



Path difference,

$\Delta = e\sin\theta$ — (1)

Phase difference,

$\phi = \frac{2\pi}{\lambda} \cdot e\sin\theta$ — (2)

$R_\theta = R_0 \left[\frac{\sin p}{p}\right]$ — (3)

$p = \frac{\pi a \sin\theta}{\lambda}$ — (4)

a : slit width

b : opaque portion width.

e : grating

Amplitude 'N' slits :

$$R = R_\theta \frac{\sin\left(\frac{N\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)} \quad \text{— (5)}$$

$$\therefore I = KR^2 = N^2 K R_0^2 \left[\frac{\sin p}{p}\right]^2 \left[\frac{\sin\left(\frac{N\phi}{2}\right)}{N\sin\left(\frac{\phi}{2}\right)}\right]^2 \quad \text{— (6)}$$

for maximum intensity :-

$I_0 = ?, \quad \theta = 0, \quad p \to 0, \quad \phi \to 0$

$\therefore \boxed{I_0 = KN^2 R_0^2}$

$$\therefore I = I_0 \left[\frac{\sin p}{p}\right]^2 \cdot \left[\frac{\sin\left(\frac{N\phi}{2}\right)}{N\sin\left(\frac{\phi}{2}\right)}\right]^2 \quad \text{— (7)}$$

(I) Principal Maxima-

$$\sin\left(\frac{\phi}{2}\right) = 0$$

$$\therefore I = I_0\left[\frac{\sin p}{p}\right]^2 \quad \text{——— (8)}$$

$$\frac{\phi}{2} = \pm n\lambda \quad \text{——— (9)}$$

$n = 1, 2, 3, \ldots$ [$n \neq 0$ because $n = 0$ is for central maxima]

$n=1$, for 1^{st} order principal maxima
$n=2$, for 2^{nd} order maxima
$n=3$, for 3^{rd} order principal maxima

Path difference is (for principal maxima for grating)-

$$\boxed{e\sin\theta = \pm n\lambda} \quad \text{——— (10)}$$

(II) Minima:-

$$\sin\left[\frac{N\phi}{2}\right] = 0$$

$$\frac{N\phi}{2} = \pm m\lambda \quad \text{——— (11)}$$

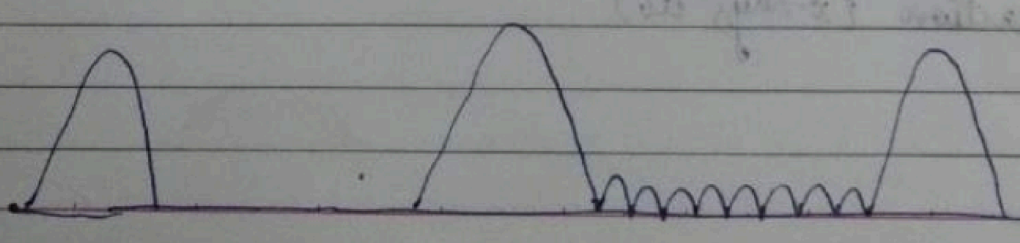
$$\frac{\phi}{2} = \pm\frac{m\lambda}{N}$$

Path difference : $\boxed{e\sin\theta = \pm\frac{m}{N}\lambda}$ ——— (12)

$m = 1, 2, 3, \ldots\ldots$, $m \neq 0, N, 2N, 3N, \ldots$

$N \rightarrow (N-1)$ No. of minima
$(N-2)$ No. of secondary maxima

N=10



1st order principal max.

(III) Secondary Maxima:-

$$\frac{dI}{d\phi} = 0$$

$$\tan\left[\frac{N\phi}{2}\right] = N\tan\left[\frac{\phi}{2}\right] \quad \text{—— (13)}$$

$$\sin\left[\frac{N\phi}{2}\right] = \frac{N\tan\left[\frac{\phi}{2}\right]}{\sec\left[\frac{N\phi}{2}\right]}$$

$$\frac{\sin^2\left[\frac{N\phi}{2}\right]}{\sin^2\left[\frac{\phi}{2}\right]} = \frac{N^2}{1+(N^2-1)\sin^2\frac{\phi}{2}} \quad \text{—— (14)}$$

$$\frac{I_{\text{at secondary max.}}}{I_{\text{at principal max.}}} = \frac{1}{1+(N^2-1)\sin^2\left(\frac{\phi}{2}\right)} \quad \text{—— (15)}$$

Conclusion -

1) As $N \uparrow$, intensity of secondary maxima decreases.

2) There are large no. of slits in a grating thats why secondary maximas are not visible and there is darkness b/w consecutive principal maximas.

- We can find out crystal structure of ~~an atom~~ a material by using diffraction (x-rays etc.)