UNIT-1 ⇒ Differential calculus

Review of succesive differentiation.

formula :-

i, $D^n(e^{ax+b}) = a^n e^{ax+b}$

ii, $D^n(a^{mx}) = a^{mx}(\log a)^n m^n$

iii, $D^n(ax+b)^m = m.(m-1).(m-2)....(m-n+1).a^n(ax+b)^{m-n}$; $m>n$

↳ if $n>m$, then $D^n(ax+b)^m = 0$

↳ if $n=m$, then $D^n(ax+b)^n = n!a^n$

iv, $D^n\sin(ax+b) = a^n\sin(ax+b+n\pi/2)$

v, $D^n\cos(ax+b) = a^n\cos(ax+b+n\pi/2)$

vi, $D^n e^{ax}\sin(bx+c) = (\sqrt{a^2+b^2})^x e^{ax}\sin(bx+c+n\tan^{-1}b/a)$

vii, $D^n e^{ax}\cos(bx+c) = (\sqrt{a^2+b^2})^x e^{ax}\cos(bx+c+n\tan^{-1}b/a)$

ix, $D^n\log(ax+b) = \dfrac{a^n(-1)^{n-1}(n-1)!}{(ax+b)^n}$

Ex $y = \sin^2 x$ n^{th} derivative.

Sol $y = (\sin x)^2$

$y' = 2\sin x.\cos x$

$y' = \sin 2x$

$y'' = \cos 2x.2$

$y''' = -2\sin 2x.2 = -4\sin 2x$

$y_n = 2^{n-1}\cos(\cancel{\sin 2x}.\frac{n\pi}{2}+2x)$

$1-2\sin^2 x = \cos 2x$

$\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{1}{2}\cos 2x$

diff n times

$= 0 - \frac{1}{2}2^n\cos(2x+n\pi/2)$

$= -2^{n-1}\cos(2x+n\pi/2)$

Leibnitz theorem.

$D^n(u.v) = D^n(u).v + {}^nC_1.D^{n-1}(u).D(v) + {}^nC_2.D^{n-2}(u).D^2(v)$
$\dots + {}^nC_n.u.D^n(v)$.

${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

Ex- find the n^{th} differential coefficiante of $x^3\cos x$.

Sol Given, $y = x^3\cos x$

$\Rightarrow D^n(x^3\cos x) = D^n(\cos x).x^3 + {}^nC_1.D^{n-1}(\cos x).D^1(x^3)$
$+ {}^nC_2.D^{n-2}(\cos x).D^2(x^3) + {}^nC_3.D^{n-3}(\cos x).D^3(x^3)$

$\cos(n\pi/2 - \pi/2 + x)$

$\cos(\pi/2 - (n\pi/2 + x))$

$\sin(n\pi/2 + x)$

$\Rightarrow D^n(x^3\cos x) = \cos(n\pi/2+x).x^3 + n.\cos((n-1)\pi/2+x).3x^2$
$+ \frac{n(n-1)}{2}\cos((n-2)\pi/2+x).6x + \frac{n(n-1)(n-2)}{6}\cos((n-3)\pi/2+x).6$

$\Rightarrow D^n(x^3\cos x) = x^3\cos(n\pi/2+x) + 3x^2 n\sin(n\pi/2+x)$
$- 3x.n(n-1)\cos(n\pi/2+x) - n(n-1)(n-2)\sin(n\pi/2+x)$

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$D^n(x^3\cos x) = x^3[x^2 - 3x(n-1)]\cos\left(x + \frac{n\pi}{2}\right) + n[6x^2 - (n-1)(n-2)]\sin\left(x+\frac{n\pi}{2}\right)$ Ans.

Q find the n^{th} derivative of $(1-x^2)y_2 + xy_1 + y = 0$

Sol Given $(1-x^2)y_2 + xy_1 + y = 0$.

$\because D^n[(1-x^2)y_2] = D^n(y_2).(1-x^2) + {}^nC_1 D^{n-1}(y_2).D(1-x^2) + {}^nC_2 D^{n-2}(y_2).D^2(1-x^2) + 0$

$= y_{n+2}(1-x^2) + n.y_{n+1}(-2x) + \frac{n(n-1)}{2}y_n(-2)$

$= (1-x^2)y_{n+2} - 2nx\,y_{n+1} - n(n-1)y_n$

$\because D^n(x.y_1) = D^n(y_1)x + nD^{n-1}(y_1)D(x) + 0$

$= y_{n+1}.x + ny_n$

Thus, $(1-x^2)y_{n+2} - 2nx.y_{n+1} - n(n-1)y_n + xy_{n+1} + ny_n + y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - (2n-1)x\,y_{n+1} - (n(n-1) - n - 1)y_n = 0$

$\Rightarrow y_{n+2}(1-x^2) + y_{n+1}x(1-2n) + y_n(-n^2+2n+1) = 0$

$\rightarrow$ it is called recurn. Ans

$D^n(ax+b)^{-1} = \frac{(-1)^n n!\,a^n}{(ax+b)^{n+1}}$

Q find y^n, $y = \frac{1}{1-5x+6x^2}$

Sol $y = \frac{1}{6x^2-5x+1} = \frac{1}{(3x-1)(2x-1)} = \frac{A}{(3x-1)} + \frac{B}{(2x-1)}$

by P.f $A=-3$ & $= \frac{-3}{(3x-1)} + \frac{2}{(2x-1)}$

$\frac{3}{2}-\frac{1}{2}$ $\frac{3-1}{2}$ $-\frac{1}{3}$

Thus, $y^n = -\frac{(-1)^n n!\,3^{n+1}}{(3x-1)^{n+1}} + \frac{(-1)^n n!\,2^{n+1}}{(2x-1)^{n+1}}$ Ans

{Trick} P.f

$\frac{x-1}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$

i, orignal $=0$ $(x+1)^2=0 \Rightarrow x=-1$

ii, for find B put $x=-1$ in thain oution except orignal $B=-2$

iii, Put $x=0$ in every where

$\frac{-1}{1^2} = \frac{A}{1} + \frac{-2}{1^2}$

$A = 1$

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Maclaurin's Series :-

An $f(x)$ can be expanded in ascending power of x & this expansion be differentiable any no: of times then

$$\left\{ f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^n}{n!} f^n(0) \right\}$$

Q $y = \frac{e^x}{1+e^x}$, Expand by maclaurin's theorem for the term containing x^3.

Sol. $y = \frac{e^x}{1+e^x}$; $y]_0 = \frac{1}{1+1} = \frac{1}{2}$

$\Rightarrow y']_0 = \frac{e^x(1+e^x) - e^x . e^x}{(1+e^x)^2} = \frac{2-1}{(1+1)^2} = \frac{1}{4}$

$\frac{e^x}{1+e^x} - \left(\frac{e^x}{1+e^x}\right)^2 \Rightarrow y - y^2 = y'$

$\Rightarrow y'']_0 = y_1 - 2yy_1 = \frac{1}{4} - 2 \times \frac{1 \times 1}{2 \, 4} = 0$

$\Rightarrow y''']_0 = \cancel{y'' - 2y'y''} = y'' - 2[y'y' + yy'']$

$= 0 - 2\left[\frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times 0\right] = -\frac{1}{8}$

$\Rightarrow y = \frac{1}{2} + \frac{x}{4} + 0 - \frac{x^3}{6} \times \frac{1}{8} + \dots$ Ans.

Q $y = \log(\sec(x))$, Expand by maclorines theorem

Sol $y = \log(\sec x)$ $\log 1 = 0$

$\Rightarrow y_0 = \log(\sec 0) = 0$ $2\sec^2 x . \tan x$

$\Rightarrow y_1 = \frac{1}{\sec x} . \sec x . \tan x = 0$ $2[(2\sec x . \sec x . \tan x . \tan x$

$+ 2\sec^2 x \dots$

$\Rightarrow y_2 = \sec^2 x = 1$

$\Rightarrow y_3 = 2 \sec x (\sec x . \tan x) = 0$

$\Rightarrow y_4 = 2 \sec^2 x \sec^2 x + 2\tan(2\sec x)(\sec x . \tan x) = 2$

Thus, $\log(\sec x) = \frac{x^2}{2} + \frac{x^4 \times 2}{24} + \dots$

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Taylor's theorem :-

an $f(a+h)$ is a funᶜ of varible h (a=constant) then $f(a+h)$ can be expanding in ascending power of h such that

$$f(a+h) = f(a) + \frac{h}{1!}f'(a) + \frac{h^2}{2!}f'(a) + \dots \frac{h^n}{n!}f^n(a).$$

$\because x = a+h \Rightarrow h = x-a$

now, $f(x) = f(a) + \frac{x-a}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \dots \frac{(x-a)^n}{n!}f^n(a).$

Q Expand $\tan x$ in assending power of $x - \frac{\pi}{4}$

Sol Given $f(x) = \tan x$ & $h = x - \frac{\pi}{4}$, then $a = \frac{\pi}{4}$

$\Rightarrow f(a) = f(\pi/4) = \tan \pi/4 = 1$

$\Rightarrow f'(a) = \sec^2 x = 1 + \tan^2 x = 2$
$\qquad 1 + y^2$

$\Rightarrow f''(a) = 2yy_1 = 2 \times 1 \times 2 = 4$

$\Rightarrow f'''(a) = 2(y_1y_1 + yy_2) = 2(2\times 2 + \cancel{4}\times 4) = 8 + 8 = 16$

$\Rightarrow f''''(a) = 4y_1y_2 + 2(y_1y_2 + yy_3) = 80.$

By taylor series.

$$\tan x = 1 + x - \pi/4\,(2) + \frac{(x-\pi/4)^2(4)}{2} + \frac{(x-\pi/4)^3(16)}{6} + \frac{(x-\pi/4)^4(80)}{24}$$

$$\tan x = 1 + 2(x-\pi/4) + 2(x-\pi/4)^2 + \frac{8}{3}(x-\pi/4)^3 + \dots$$

Ans.

Q Expand $\log x$ in power $(x-1)$ & find $\log(1.1)$ in for decimal plau

Sol Given, $f(x) = \log x$ & $h = x - 1$
then, $a = 1$

$\Rightarrow f(1) = \log 1 = 0$

$f'(x) = \frac{1}{x} \Rightarrow f'(1) = 1$

$\Rightarrow f''(x) = -1x^{-2} \Rightarrow f''(1) = -1$

$\Rightarrow f'''(x) = -1 \times -2 \times x^{-3} = \frac{2}{x^3} \Rightarrow f'''(1) = 2$

$\Rightarrow f^4(x) = 2 \times -3\, x^{-4} \Rightarrow f^4(1) = -6$

$\Rightarrow f^5(x) = -6 \times -4\, x^{-5} \Rightarrow f^5(1) = 24$

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By taylor series.

$$\log x = 0 + \frac{x-1}{1}(1) + \frac{(x-1)^2}{2}(-1) + \frac{(x-1)^3}{6}(2) + \frac{(x-1)^4}{24}(-6) + \frac{(x-1)^5}{24\times5}(24)$$

$$\log x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5}$$

Put $x = 1.1$, then $x - 1 \Rightarrow 1.1 - 1 = 0.1$

$$\log 1.1 = (0.1) - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} - \frac{(0.1)^4}{4} - \frac{(0.1)^5}{5}$$

$$\log 1.1 = 0.09531$$ Ans.

Q Expand $\tan(x + \pi/4)$ as for as the ~~term~~ x^4 and find $\tan(46.5)$ upto four significant figure.

Sol $f(x) = \tan(x + \pi/4)$ & $h = x$

By previcious ques $a = \pi/4$

$y = 1$, $y_1 = 2$, $y_2 = 4$, $y_3 = 16$; $y_4 = 80$

By Taylor's series.

$$\tan(x+\pi/4) = y + \frac{h}{1!}y_1 + \frac{h^2}{2!}y_2 + \frac{h^3}{3!}y_3 + \frac{h^4}{4!}y_4$$

$$\tan(x+\pi/4) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{3}x^4$$

$180° = \pi$

$1° = \frac{\pi}{180}$

Put $x = 1.5° \Rightarrow 0.02618$

$$\tan(46.5°) = 1 + 2(0.02618) + 2(0.02618)^2 + 8/3(0.02618)^3 + 10/3(0.02618)^4$$

$1.5° = \frac{3}{2} \times 0.01745$

$\tan(46.5°) = 1.0538$ (approx) Ans

$1.5° = 0.02618$

MST-1 2021

Q Find the n^{th} differential coefficient of $\cos^4 x$

Sol. Let $y = \cos^4 x$

$y = \frac{1}{4} \times 4\cos^4 x$

$\cos 2x = 2\cos^2 x - 1$ $\quad y = \frac{1}{4}(2\cos^2 x)^2$

$y = 1/4(1 + \cos 2x)^2$

$y = 1/4(1 + \frac{2}{2}\cos^2 2x + 2(\cos 2x))$

$y = \frac{1}{4} + \frac{1}{8}(1 + \cos 4x) + \frac{1}{2}(\cos 2x)$

diff n time

$y_n = 0 + \frac{1}{8}(0 + 4^n \cos(n\pi/2 + 4x)) + \frac{1}{2}(2^n \cos(\frac{n\pi}{2} + 2x))$

$y_n = \frac{4^{n-1}}{2}\cos(\frac{n\pi}{2} + 4x) + 2^{n-1}\cos(\frac{n\pi}{2} + 2x)$

$y_n = 2^{2n-3}\cos(\frac{n\pi}{2} + 4x) + 2^{n-1}\cos(\frac{n\pi}{2} + 2x)$

Ans

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Θ Expand $y = (\sin^{-1}x)^2$ by Maclaurins theorem.

Sol Given $y = (\sin^{-1}x)^2$

$\Rightarrow y_0 = 0$

$\Rightarrow y' = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}}$ at 0 = 0

$\frac{d}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{-1(1-x^2)^{-3/2} \cdot -2x}{2}$

$= x(1-x^2)^{-3/2}$

$\Rightarrow y'' = 2\left(\frac{\left(\frac{1}{\sqrt{1-x^2}}\right)^2 - \sin^{-1}x \cdot x(1-x^2)^{-3/2}}{(\sqrt{1-x^2})^2}\right) =$

$y' = \frac{2\sqrt{y}}{\sqrt{1-x^2}} = 2\sqrt{\frac{y}{1-x^2}}$

$2\left(\frac{\frac{1}{1-x^2} -}{1-x^2}\right)$

$\sqrt{(1-x^2)}(1-x^2)$ $\frac{1}{2}+1$

$\Rightarrow y'' = \frac{2\times 1}{2}\frac{y}{1-x^2} \cdot \left[y'\sqrt{1-x^2} \right.$

Asymptotes :- it is a straight line which touches the curve at infinty.

jitne degree utni asymptotes

working rule -

i. Let $f(x,y) = 0$ be the curve of n^{th} degree

ii. put $y = mx + c$ in $f(x,y) = 0$ & simplify.

iii. name the coefficient of x^n, x^{n-1}, x^{n-2} is $\phi_n(m), \phi_{n-1}(m), \phi_{n-2}(m)$

iv. put $\phi_n(m) = 0$ & solve it for m

v. put $\phi_{n-1}(m) = 0$ & solve it for c, $c = g(m)$ find c for each m such that $c_1 = g(m_1)$ & $c_2 = g(m_2)$ .... etc.

vi. write asymptotes as $y = m_1x + c_1$, $y = m_2x + c_2$ & $y = m_3x + c_3$

Shorcut method -

i. simplify $f(x,y) = 0$ & put $x = 1$ & $y = m$ into highest degree term & name $\phi_n(m)$ & similarly find $\phi_{n-1}(m)$, $\phi_{n-2}(m)$.

ii. put $\phi_n(m) = 0$ & solve it for m.

iii. find $c = -\frac{\phi_{n-1}(m)}{\phi'_n(m)}$ (non repeated root) / $\phi''_n(m)c^2 + \phi'_{n-1}(m)c + \phi_{n-2}(m) = 0$ (2 repeated root)

iv. write asymptotes as $y = mx + c$

Θ write asymptotes of $y^3 - 3xy^2 - x^2y + 3x^3 - 3x^2 + 10xy - 3y^2 - 10x - 10y + 7 = 0$

Sol $f(x,y) \Rightarrow y^3 - 3xy^2 - x^2y + 3x^3 - 3x^2 + 10xy - 3y^2 - 10x - 10y + 7 = 0$

put $x = 1$ & $y = m$

$\phi_3(m) = m^3 - 3m^2 - m + 3$

$\phi_2(m) = -3 + 10m - 3m^2$

$\phi(m) = -10 - 10m$

Put $\phi_3(m) = 0$

$m^3 - 3m^2 - m + 3 = 0$

$m^2(m-3) - 1(m-3) = 0$

$(m-3)(m^2-1) = 0$

$(m-3)(m+1)(m-1) = 0$

$m = 3, -1, 1$

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now, $c = \frac{-(-3m^2+10m-3)}{(3m^2-6m-1)} = \frac{3m^2-10m+3}{3m^2-6m-1}$

Put $m_1 = 1$

$c_1 = \frac{3-10+3}{3-6-1} = \frac{-4}{-4} = 1$ ∴ $y = x+1$

Put $m_2 = -1$

$c_2 = \frac{3(-1)^2-10(-1)+3}{3(-1)^2-6(-1)-1} = \frac{16}{8} = 2$ ∴ $y = -x+2$

put $m = 3$

$c_3 = \frac{3(3)^2-10(3)+3}{3(3)^2-6(3)-1} = \frac{27-30+3}{27-18-1} = \frac{0}{8} = 0$

∴ $y = 3x$

Thus, asymptotes is $y-x-1=0$, $y+x-2=0$ & $y-3x=0$ Ans

2019.

Q find the asymptote to the given curve:

$(x^2-y^2)(y^2-4x^2) - 6x^3 + 5x^2y + 3xy^2 - 2y^3 - x^2 + 3xy - 1 = 0$

Sol. Given, $x^2y^2 - 4x^4 - y^4 + 4x^2y^2 - 6x^3 + 5x^2y + 3xy^2 - 2y^3 - x^2 + 3xy - 1 = 0$

$-4x^4 - y^4 + 5x^2y^2 - 6x^3 + 5x^2y + 3xy^2 - 2y^3 - x^2 + 3xy - 1 = 0$

put $y = m$ & $x = 1$ in 4th, 3rd, 2nd degree term.

$\phi_4(m) = -4 - m^4 + 5m^2$

$\phi_3(m) = -6 + 5m + 3m^2 - 2m^3$

$\phi_2(m) = -1 + 3m$

Put $\phi_4(m) = 0$

$m^4 - 5m^2 + 4 = 0$

~~$m^3(m^2-5)+4=0$~~

$m^4 - 4m^2 - m^2 + 4$

$m^2(m^2-4) - 1(m^2-4) = 0$

$(m^2-1)(m^2-4) = 0$

$m = \pm 1, \pm 4$

Thus, asymptotes is

$y = x$, $y = -x+1$

$y = 4x + \frac{33}{103}$, $y = -4x + \frac{75}{103}$

Ans.

now, $c = \frac{2m^3 - 3m^2 - 5m + 6}{10m - 4m^3}$

Put $m = +1$

$c_1 = \frac{2-3-5+6}{10-4} = 0$

put $m = -1$

$c_2 = \frac{-2-3+5+6}{-10+4} = 1$

put $m = +4$

$c_3 = \frac{2(64) - 3(16) - 5(4) + 6}{10(4) - 4(64)}$

$c_3 = \frac{128 - 48 - 20 + 6}{40 - 246} = \frac{\cancel{66}\ 33}{\cancel{206}\ 103}$

Put $m = -4$ $c_4 = \frac{-128 - 48 + 20 + 6}{-40 + 246} = \frac{150}{206} = \frac{75}{103}$

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Radius of curvature :- R.O.C $\propto$ 1/radius

$\kappa \propto 1/\rho$

i) Cartesian formula for Radius of curvature - if given curve is a cartesian plan in xr plane then ROC will be

$$\rho = \frac{\{1 + (dy/dx)^2\}^{3/2}}{d^2y/dx^2}$$

ii) Polar formula for ROC - if $r = f(\theta)$ & $f(r,\theta) = 0$ then roc is

$$\rho = \frac{\{r^2 + (dr/d\theta)^2\}^{3/2}}{r^2 + 2(dr/d\theta)^2 - r\,d^2r/d\theta^2}$$

Q find R.O.C of a curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $(1/4, 1/4)$.

Sol $\sqrt{y} = \sqrt{a} - \sqrt{x}$

now, $\frac{1}{2} y^{-1/2} \frac{dy}{dx} = -\frac{1}{2} x^{-1/2}$

$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$ at $(1/4, 1/4) \Rightarrow -1$

again, $\frac{d^2y}{dx^2} = -\left[\frac{\frac{1}{2} y^{-1/2} dy/dx \cdot \sqrt{x} - \frac{1}{2} x^{-1/2} y^{1/2}}{x}\right]$

$\frac{d^2y}{dx^2} = \left[-\frac{1}{2} \times \frac{1}{\sqrt{y}} \times \frac{-\sqrt{y}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{x} + \frac{1}{2} \frac{\sqrt{y}}{\sqrt{x} \cdot x}\right]$

$\left.\frac{d^2y}{dx^2}\right]_{(1/4, 1/4)} = \frac{1}{2\sqrt{x} \cdot x} + \frac{1}{2} \sqrt{\frac{y}{x \cdot x}} = \cancel{\frac{1 + \sqrt{y}}{2\sqrt{x}}}$ $4 + 4 = 8$

Thus, $\rho = \frac{\{1 + (-1)^2\}^{3/2}}{8} = \frac{2^{3/2}}{8} = \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$ Ans

Q Prove that Radius of circle is constant :- $x^2 + y^2 = a^2$

Sol $x = a\cos t$ & $y = a\sin t$

$\frac{dx}{dt} = -a\sin t$ & $\frac{dy}{dt} = a\cos t$

$\therefore \frac{dy}{dx} = -\cot t$ again, $\frac{d^2y}{dx^2} = \operatorname{cosec}^2 t \frac{dt}{dx}$

$= \operatorname{cosec}^2 t \left(\frac{-1}{a\sin t}\right)$

$= -\frac{\operatorname{cosec}^3 t}{a}$

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$$\rho = \frac{\{1+(dy/dx)^2\}^{3/2}}{d^2y/dx^2} = \frac{\{1+(-\cot t)^2\}^{3/2}}{-\operatorname{cosec}^3 t/a} = -a\frac{(\operatorname{cosec}^2 t)^{3/2}}{\operatorname{cosec}^3 t}$$

$$\rho = -a$$

Hence, R.O.C of circle is constant.

Q. ROC $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\rho = \frac{a^2b^2}{p^3}$ where p be the length of the of the ⊥ from the centre ~~the~~ upon the tangent.

Sol^n :-

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{---} 1,$$

now, $$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{2y}{b^2}\frac{dy}{dx} = \frac{-2x}{a^2}$$

$$\frac{dy}{dx} = \frac{-b^2x}{a^2y}$$

therefore, eq^n of tangent at poin (x,y).

$$Y-y_1 = \frac{dy}{dx}(X-x_1)$$

$$Y-y = \frac{-b^2x}{a^2y}(X-x)$$

$$(Y-y)a^2y = -b^2x(X-x)$$

$$\frac{Yy}{b^2}-\frac{y^2}{b^2} = -\frac{Xx}{a^2}+\frac{x^2}{a^2}$$

$$\frac{Xx}{a^2}+\frac{Yy}{b^2} = \frac{x^2}{a^2}+\frac{y^2}{b^2}$$

$$\frac{Xx}{a^2}+\frac{Yy}{b^2} = 1$$

Put $x = a\cos t$ & $y = b\sin t$

$$\frac{a\cos t\, X}{a^2}+\frac{b\sin t\, Y}{b^2} = 1$$

$$(b\cos t)X + (a\sin t)Y = ab$$

if P is length of ⊥ from the centre upon the tangent.

$$P = \frac{|ax_1+by_1+c_1|}{\sqrt{a^2+b^2}}$$

$$P = \frac{ab}{\sqrt{(a\sin t)^2+(b\cos t)^2}}$$

$$P^2 = \frac{a^2b^2}{a^2\sin^2 t+b^2\cos^2 t}$$

$$a^2\sin^2 t+b^2\cos^2 t = \frac{a^2b^2}{P^2}$$

from eq 1, $x^2/a^2+y^2/b^2=1$

$\because \frac{dy}{dx} = \frac{-b^2x}{a^2y}$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2}\left[\frac{y - \frac{dy}{dx}\cdot x}{y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{-b^2}{a^2}\left[\frac{y}{y^2} - \frac{b^2x^2}{a^2y\cdot y^2}\right]$$

$$\frac{d^2y}{dx^2} = \frac{b^2}{a^2}\left[\frac{b^2x^2}{a^2y^3}-\frac{1}{y}\right]$$

$$\frac{d^2y}{dx^2} = \frac{b^4}{a^4}\frac{x^2}{y^3}-\frac{b^2}{a^2}\frac{1}{y}$$

now, $$\rho = \frac{\{1+b^4x^2/a^4y^2\}^{3/2}}{\frac{b^4x^2}{a^4y^3}-\frac{b^2}{a^2y}}$$

Q find the ROC of $r^n = a^n \cos n\theta$

Sol. $r^n = a^n \cos n\theta$

taking log

$$\log(r^n) = \log(a^n \cos n\theta)$$

$$n \log r = n \log a + \log \cos n\theta$$

diff w r t x

$$\cancel{n} \cdot \frac{1}{r} \cdot \frac{dr}{dx} = 0 + \frac{1}{\cos n\theta} \cdot -\sin n\theta \cdot \cancel{n}$$

$$\frac{dr}{dx} = -r \frac{\sin n\theta}{\cos n\theta} = -r \tan n\theta.$$

again, diff w r t x

$$\frac{d^2 r}{dx^2} = -\left[\frac{dr}{dx} \tan n\theta + r \sec^2 n\theta . n\right]$$

$$= r \tan^2 n\theta - nr \sec^2 n\theta$$

$= \cancel{r}\,[$

now,

$$\rho = \frac{\{r^2 + (dr/d\theta)^2\}^{3/2}}{r^2 + 2(dr/d\theta)^2 - r\, d^2r/d\theta^2}$$

$$\rho = \frac{\{r^2 + (-r \tan n\theta)^2\}^{3/2}}{r^2 + 2(-r \tan n\theta)^2 - r(r \tan^2 n\theta - nr \sec^2 n\theta)}$$

$$\rho = \frac{[r^2 + r^2 \tan^2 n\theta]^{3/2}}{r^2 + 2r^2 \tan^2 n\theta - r^2 \tan^2 n\theta - nr^2 \sec^2 n\theta}$$

$$\rho = \frac{r^3 [1 + \tan^2 n\theta]^{3/2}}{\cancel{r^2}(1 + \tan^2 n\theta - n \sec^2 n\theta)}$$

$$\rho = \frac{r \sec^3 n\theta}{\sec^2 n\theta - n \sec^2 n\theta}$$

$$\rho = \frac{r \sec n\theta}{(1-n)}$$

$$\rho = \frac{r}{(1-n) \cos n\theta} \qquad \left[\because \cos n\theta = \frac{r^n}{a^n}\right]$$

$$\rho = \frac{a^n}{(1-n) r^{n-1}} \qquad \text{Ans.}$$