2023019

B.E. I EXAMINATION FEB.' 2023
Computer Engineering
1AMRC1 Applied Mathematics-I

Time: 3 Hrs.] [**Max. Marks:** 60

Note: Attempt any **TWO** parts from each Question. Each carries equal marks.

Q.1. (a) Expand $f(x) = \frac{e^x}{e^x - 1}$ as far as x^4 (06)

(b) Find the asymptotes for the curve: $r = a\sec\theta + b\tan\theta$. (06)

(c) Show that the radius of curvature at a point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a\sin\theta\cos\theta$. (06)

Q.2. (a) If $x^x y^y z^z = C$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(y\log ey)^{-1}$. (06)

(b) If $x = u(1 - v)$, $y = uv$, evaluate $J = \frac{\partial(x,y)}{\partial(u,v)}$ and $J' = \frac{\partial(u,v)}{\partial(x,y)}$. Hence verify that $JJ' = 1$. (06)

(c) Find the extreme values of $x^2 + y^2 + z^2$, having given $ax + by + cz = p$. (06)

Q.3. (a) Prove that : (06)

$$\int_0^1 \frac{x^2}{\sqrt{1 - x^4}} \cdot \int_0^1 \frac{dx}{\sqrt{1 + x^4}} = \frac{\pi}{4\sqrt{2}}$$

(b) Trace the curve: $r = a\cos 2\theta$. Write steps properly. (06)

(c) Find the volume of the solid generated by revolving the lemniscate $r^2 = a^2\cos 2\theta$ about the line $\theta = \frac{\pi}{2}$. (06)

Q.4. (a) (06)

Evaluate: $\int_0^{\pi/2}\int_0^{a\sin\theta}\int_0^{\frac{a^2 - r^2}{a}} r\, dr\, d\theta\, dz$.

(b) Change the order of integration in $I = \int_0^{\infty}\int_0^{x} x e^{-\frac{x^2}{y}}\, dy\, dx$. Hence evaluate it. Also mention the appropriate reasons. (06)

(c) Find the centre of gravity of the area of the cardioid $r = a(1 + \cos\theta)$, by double integration. (06)

Q.5. (a) Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{-1/2}$ at the point P(3, 1, 2) in the direction of vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$. (06)

(b) (i) Prove that the vector $f(r)\,\vec{r}$ is irrotational. (03)

(ii) Prove that: $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (03)

(c) Verify Divergence theorem for the function $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (06)

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u 9+1+4 $-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}$

Roll no. ____________

2021238

B.E. I EXAMINATION FEB.' 2022
Computer Engineering
AMRICI: Applied Mathematics-I

Duration: 3 Hrs] **[Max. Marks: 60**

Note: Attempt any **TWO** parts from each question. Each carries equal marks.

Q:1 (a) Using Leibnitz theorem, if $y = x^2 \log x$, find y_n ; $n > 3$. (06)

(b) Find the asymptotes of the curve: (06)
$x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0$

(c) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$ where the parameter a & b are connected by the relation: (i) $a^2 + b^2 = c^2$ (ii) $ab = c^2$. (06)

Q2. (a) If $x^x y^y z^z = c$, show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$. (06)

(b) Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$. (06)

(c) (i) A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01m and length by 0.05m, find the % change in the volume of balloon. (03)

(ii) If $u = \tan x$, $v = \sin(x^2 + y^2)$ & $w = e^{xyz}$, then prove that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 2xy_{\wedge}^{2} \sqrt{1 - v^2}\, w$. (03)

$(1+u^2)$

Q3. (a) Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma m + n}$ (06)

(b) Trace the curve (write proper steps): $x^{2/3} + y^{2/3} = a^{2/3}$. (06)

(c) Find the surfaces of the solid formed by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. (06)

Q4. (a) Evaluate : $\int_0^{\frac{\pi}{2}} \int_0^{a \sin\theta} \int_0^{\frac{a^2 - r^2}{a}} r \, dz \, dr \, d\theta$. (06)

(b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx \, dy$ by changing to polar co-ordinate. (06)

(c) Prove that the volume enclosed by cylinders $x^2 + y^2 = 2ax$ & $z^2 = 2ax$ is $\frac{128}{15} a^3$. (06)

Q5. (a) If $\bar{a} = \sin\theta \hat{i} + \cos\theta \hat{j} + \theta \hat{k}$, $\bar{b} = \cos\theta \hat{i} - \sin\theta \hat{j} - 3\hat{k}$ & $\bar{c} = 2\hat{i} + 3\hat{j} - \hat{k}$ (06)
find ~~$\frac{d}{d\theta}(\bar{a} \times \bar{b}) = \bar{c} \times (\bar{a} \times \bar{b})$ at $\theta = 0$~~.

$\frac{d}{d\theta} \{ \vec{a} \times (\vec{b} \times \vec{c}) \}$ at $\theta = 0$

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B.E. I EXAMINATION July' 2022
Electronics & Telecommunication Engg. (A & B)
AMRICI: Applied Mathematics-I

2022262

Duration: 3 Hrs] [Max. Marks: 60

Note: Attempt any TWO parts from each question. Each carries equal marks.

(a) If $y = x^2 \log x$, then prove that $y_n = \frac{2(-1)^{n-1}(n-3)!}{x^{n-2}}$.

Show that the Radius of Curvature at a point (a ... $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a \sin\theta \cos\theta$

... and $f(x) = \log(1 + e^x)$ as far as the ...

... function of x and y

(b) Prove that : (i) $\nabla r^m = m r^{m-2} \bar{r}$. (06)
(ii) $\mathrm{div}(\mathrm{grad}\, r^m) = m(m+1) r^{m-2}$.

(c) Evaluate $\int_S \bar{f} . \hat{n} ds$ where $\bar{f} = 2x^2 y \hat{i} - y^2 \hat{j} + 4xz^2 \hat{k}$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and the plane $x = 0$, $x = 2$, y ... $z = 0$. (06)

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