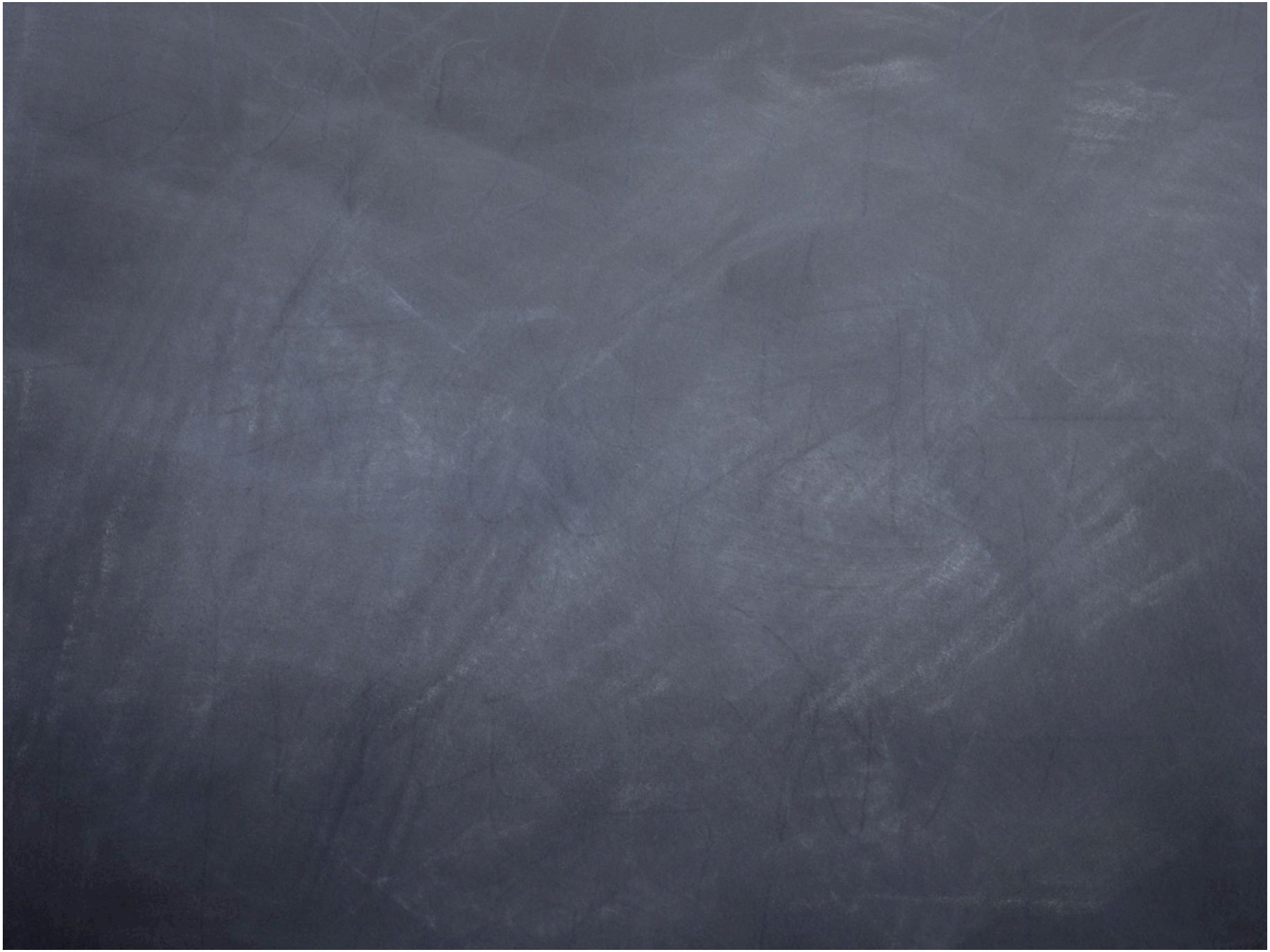
lemma rev [a, b] = [b, a]

```
proof neg_clausify
  assume rev [a, b] ≠ [b, a]
  hence ∃x3 x2. [x2, x3] ≠ rev [x3, x2]
    by metis
  hence ∃x3 x2. [x2] @ [x3] ≠ rev [x3, x2]
    by (metis Cons_eq_appendI eq_Nil_appendI)
  thus False
    by (metis rev.simps(2) rev_singleton_conv)
qed
```

lemma rev [a, b] = [b, a]

```
proof -
  have ∀x3 x2. [x2] @ [x3] = rev [x3, x2]
    by (metis rev.simps(2) rev_singleton_conv)
  hence ∀x3 x2. [x2, x3] = rev [x3, x2]
    by (metis Cons_eq_appendI eq_Nil_appendI)
  thus rev [a, b] = [b, a]
    by metis
qed
```



conjecture (DNF): $1 \vee 2 \vee 3$

conjecture (DNF): $1 \vee 2 \vee 3$

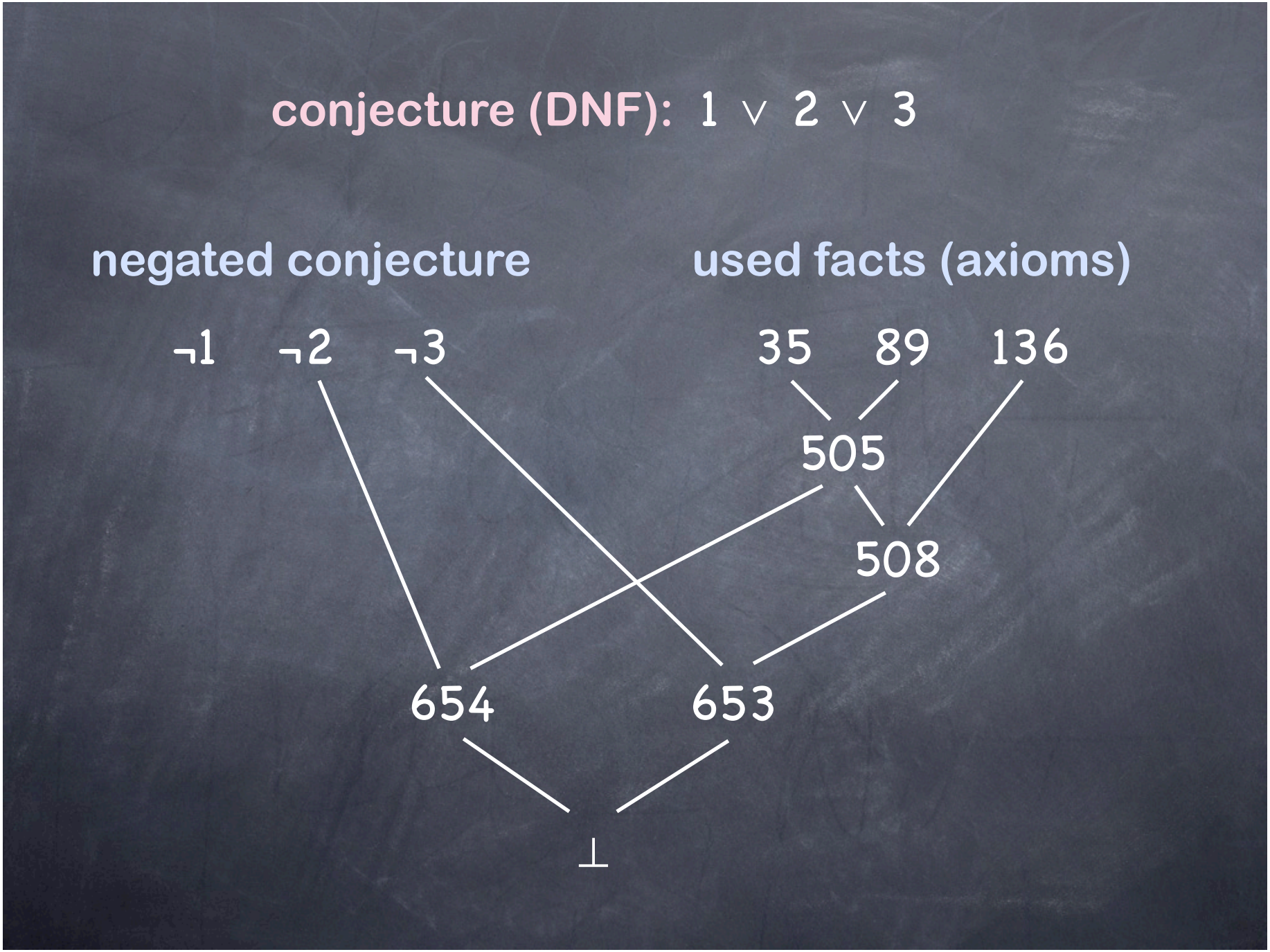
negated conjecture

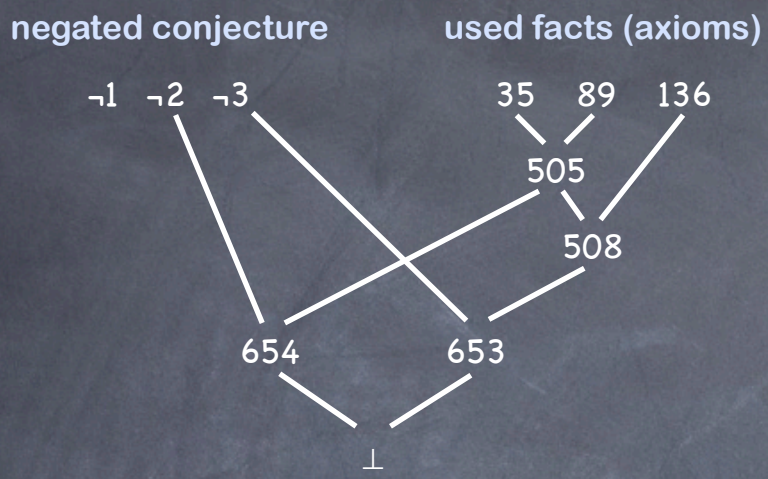
$\neg 1 \quad \neg 2 \quad \neg 3$

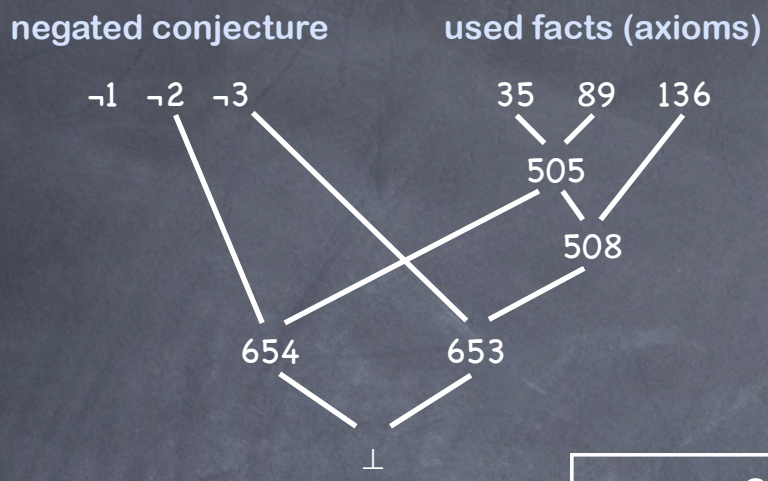
conjecture (DNF): $1 \vee 2 \vee 3$

negated conjecture	used facts (axioms)
$\neg 1$ $\neg 2$ $\neg 3$	35 89 136

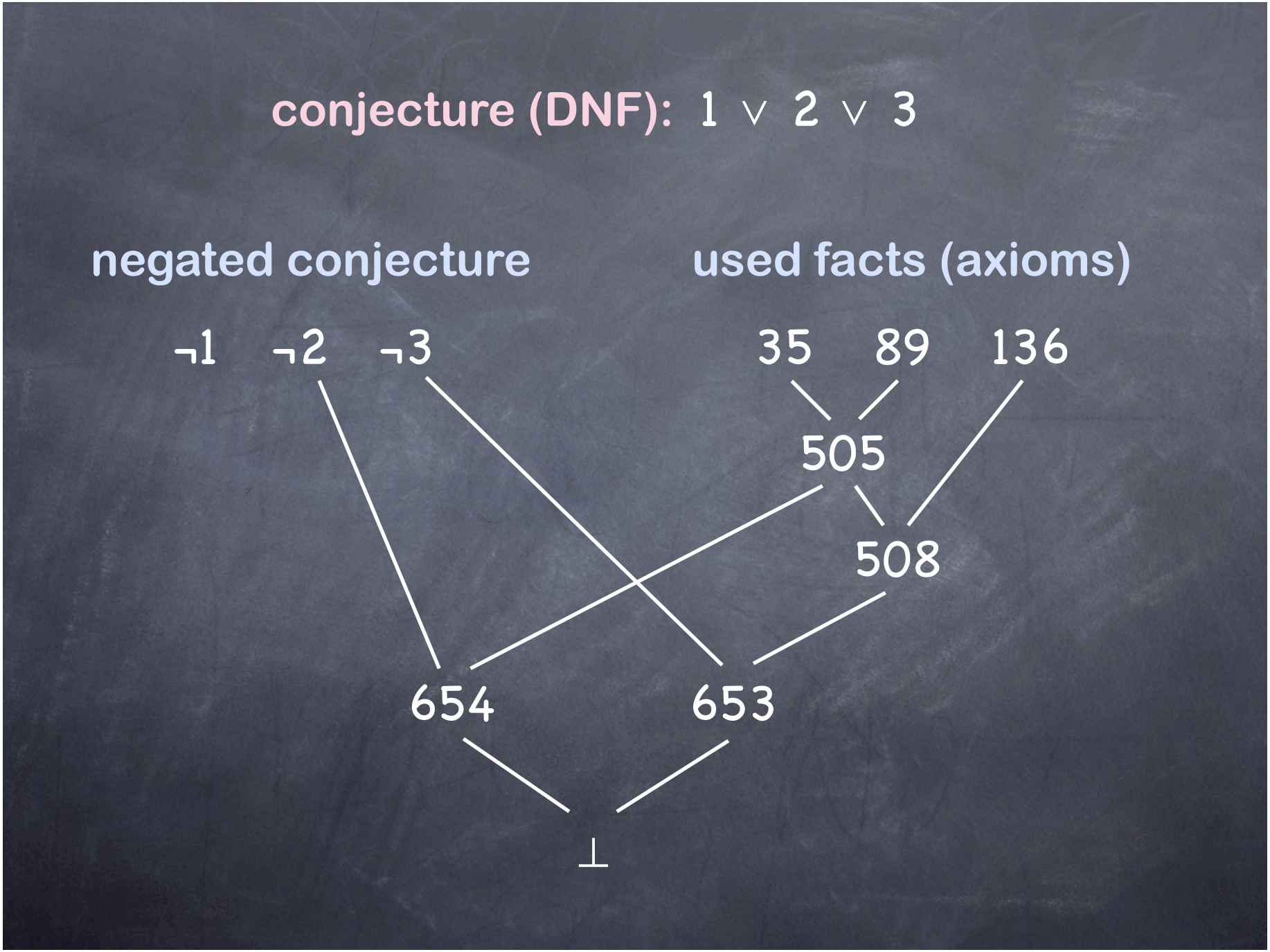
conjecture (DNF): $1 \vee 2 \vee 3$

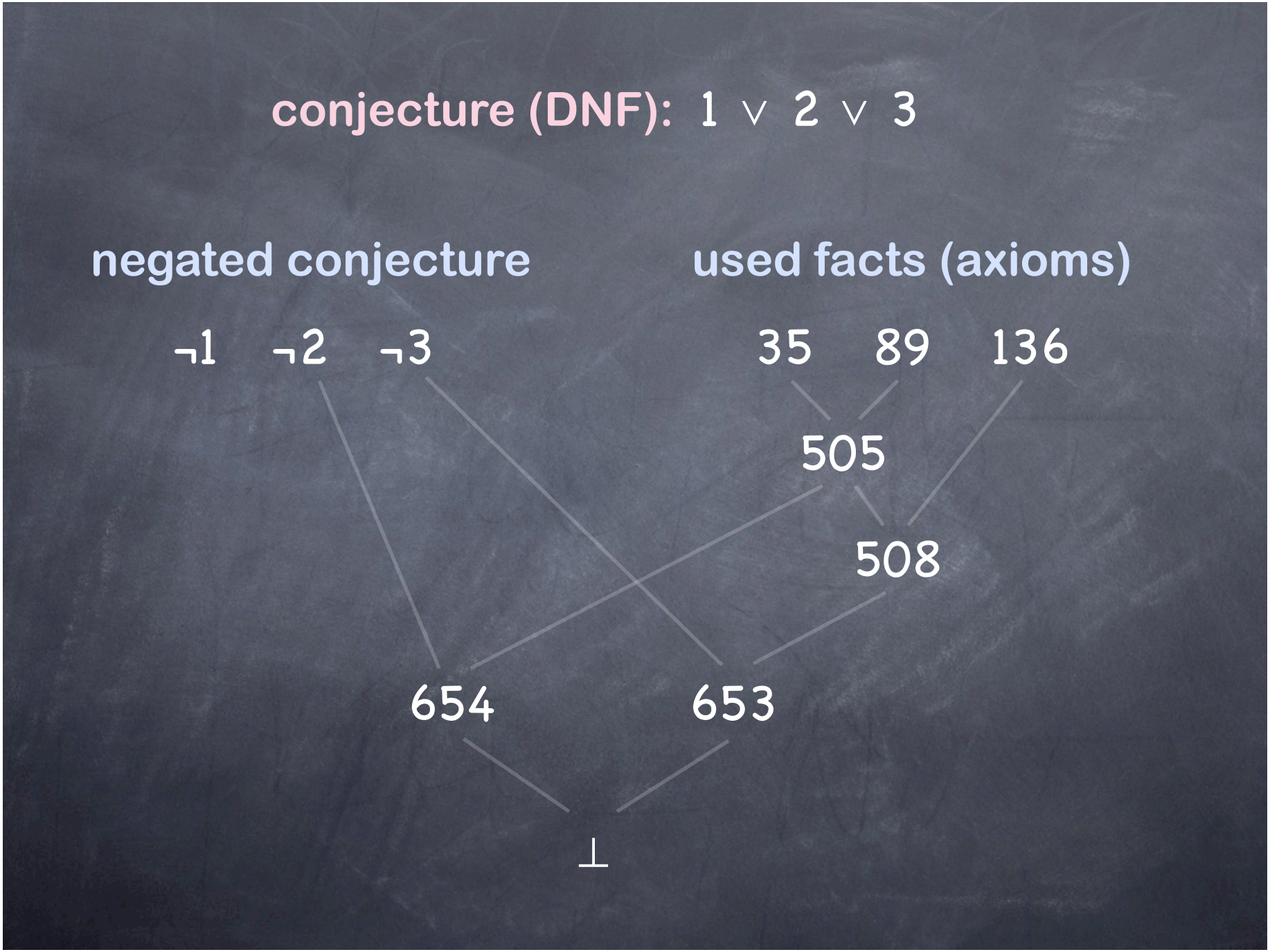


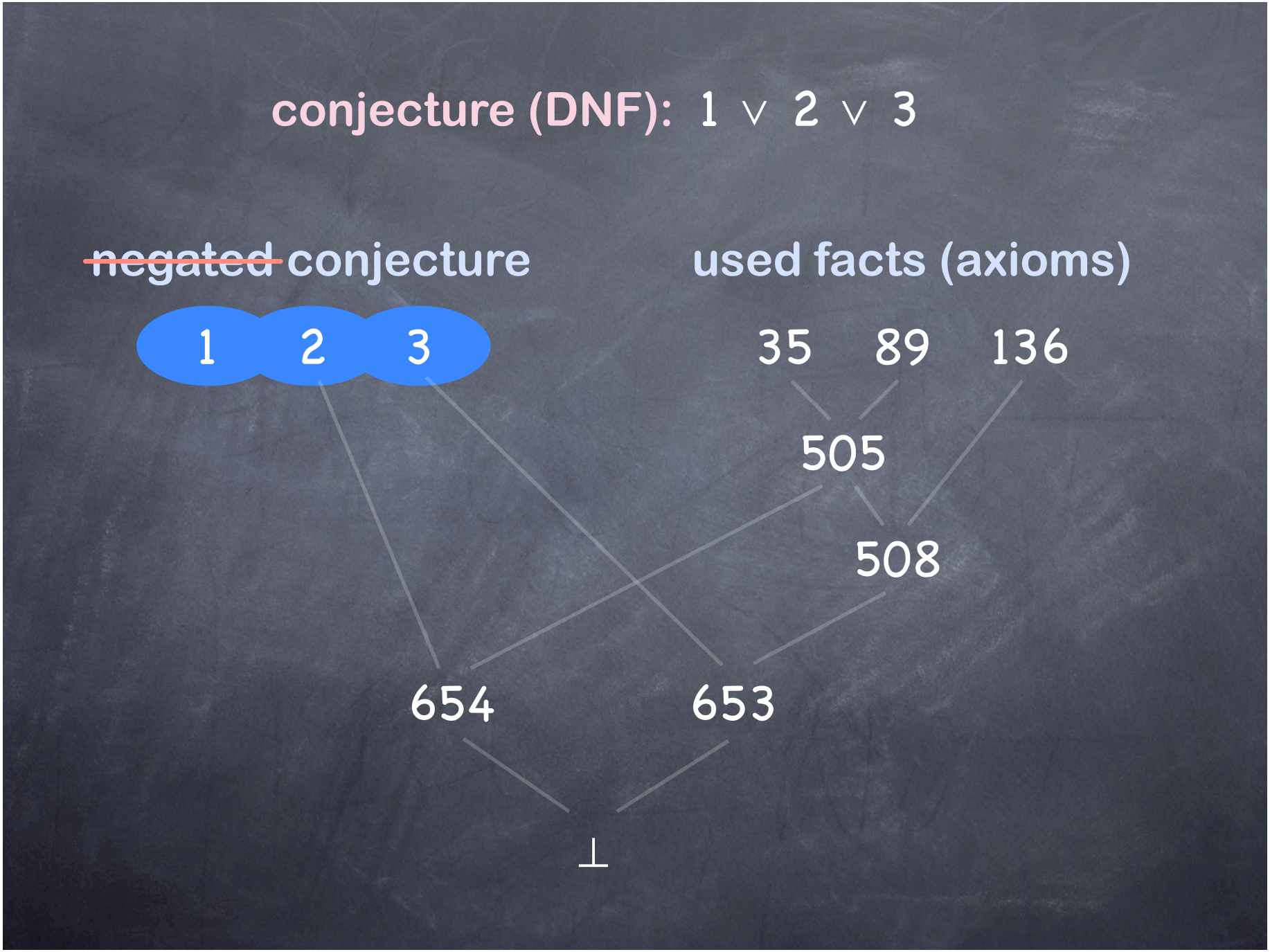

negated conjecture
used facts (axioms)
¬1
¬2
¬3
35
89
136
505
508
654
653
⊥

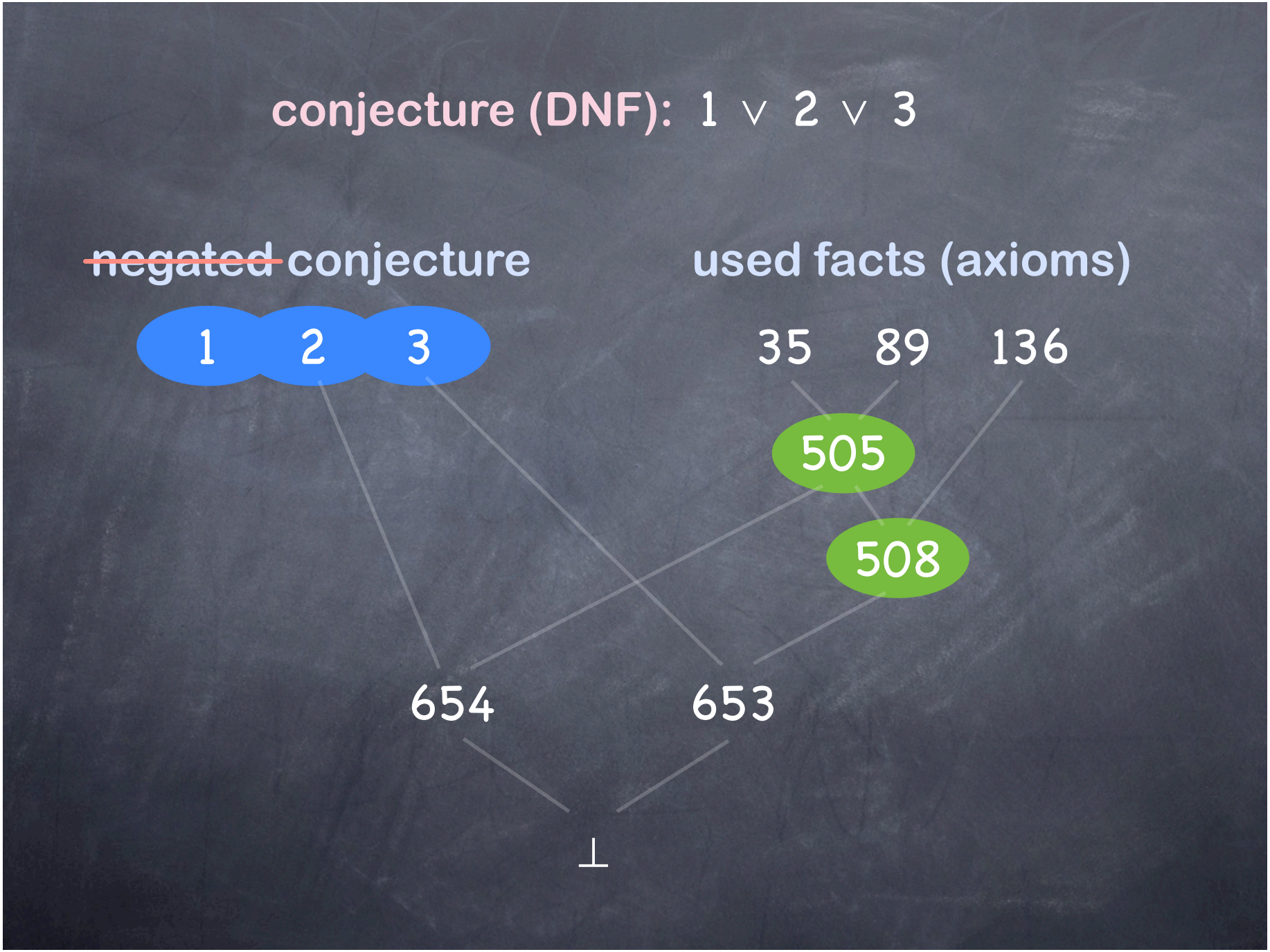


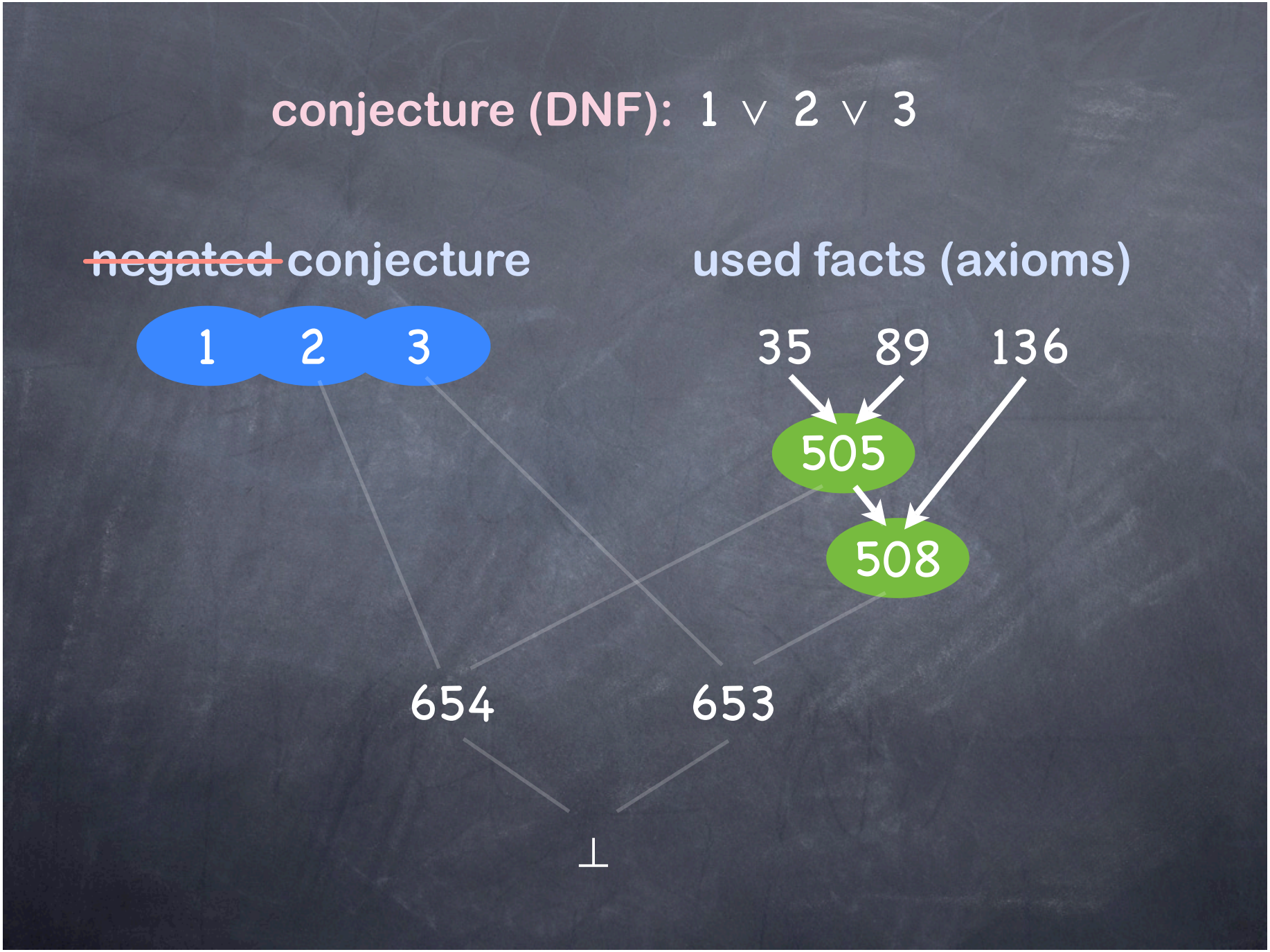

```
proof neg_clausify
  assume ¬2
  assume ¬3
  have 505 by (metis 35 89)
  have 508 by (metis 505 136)
  have 653 by (metis ¬3 508)
  have 654 by (metis ¬2 505)
  show False by (metis 654 653)
qed
```

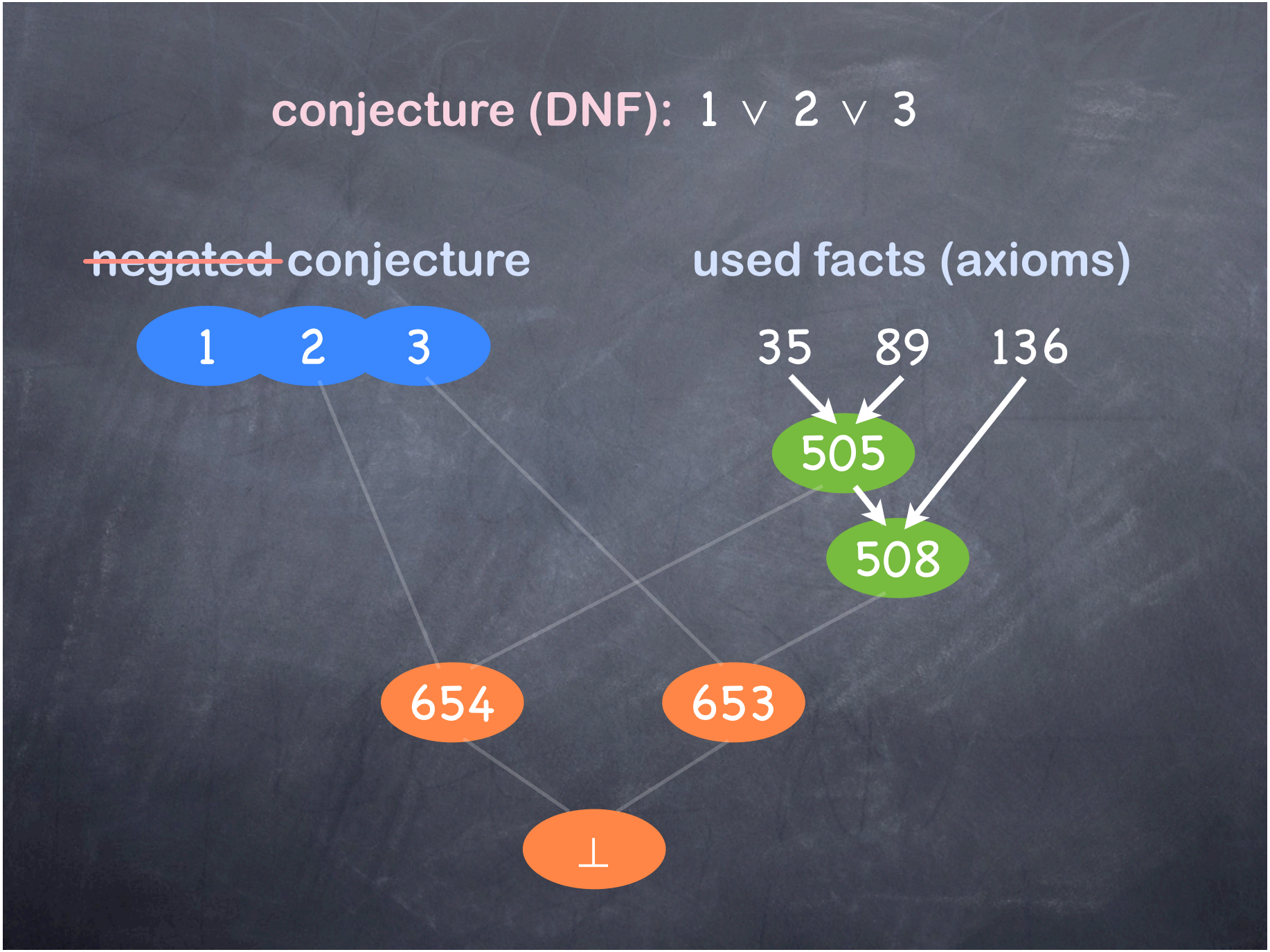

conjecture (DNF): 1 ∨ 2 ∨ 3
negated conjecture
used facts (axioms)
¬1
¬2
¬3
35
89
136
505
508
654
653
⊥

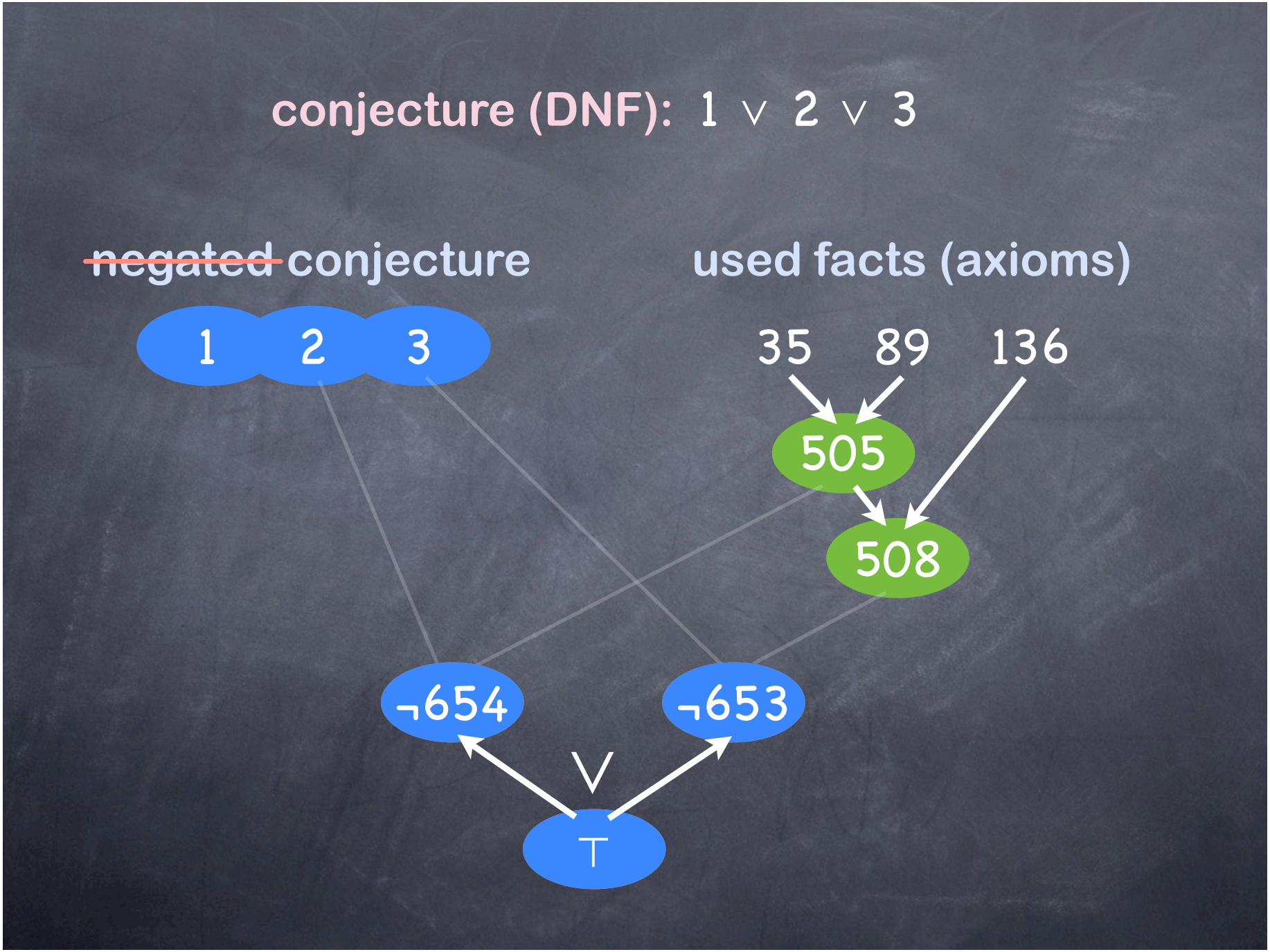

conjecture (DNF): 1 ∨ 2 ∨ 3
negated conjecture
used facts (axioms)
¬1
¬2
¬3
35
89
136
505
508
654
653
⊥


conjecture (DNF): 1 ∨ 2 ∨ 3
~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
654
653
⊥

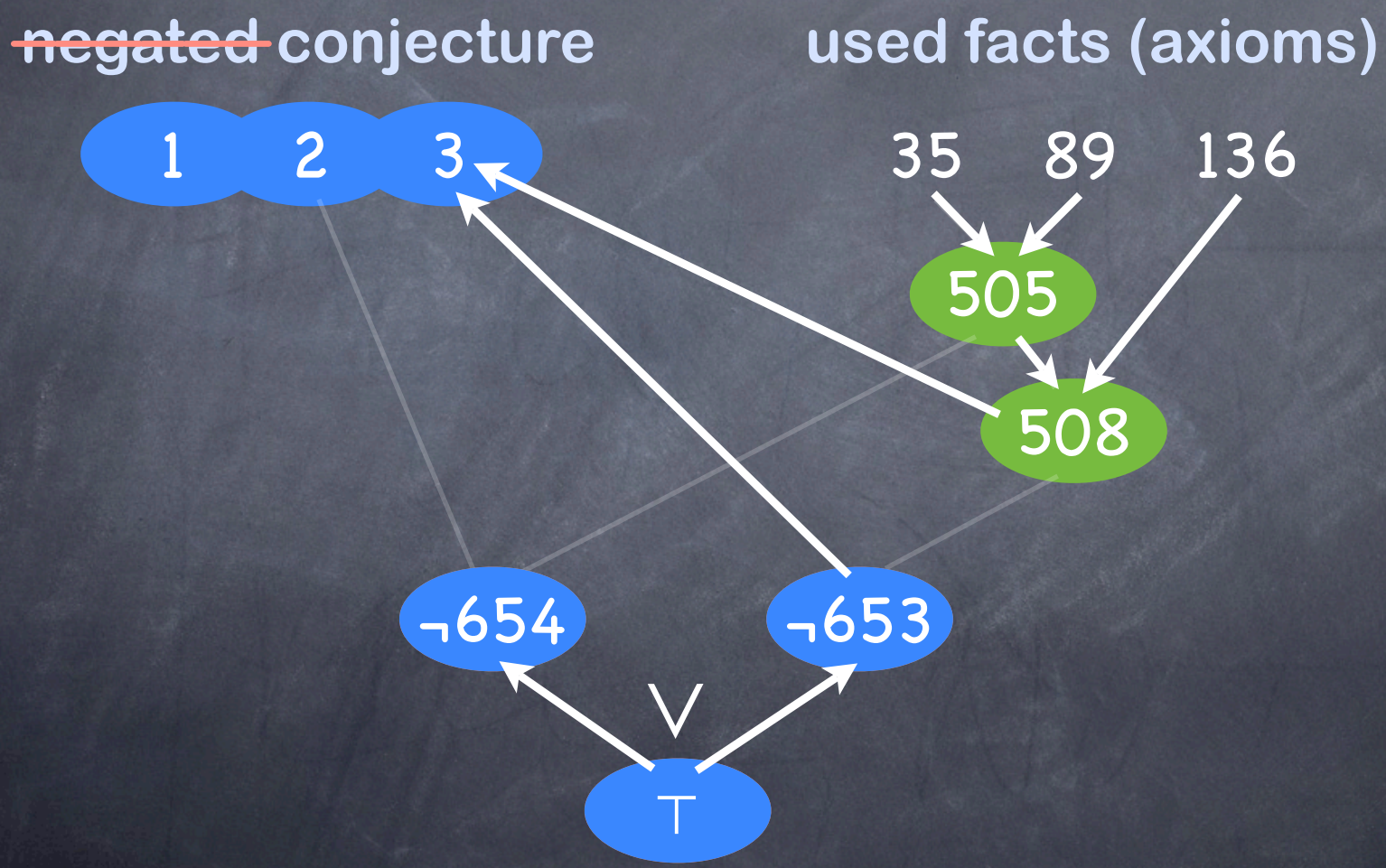

conjecture (DNF): 1 ∨ 2 ∨ 3
~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
654
653
⊥


conjecture (DNF): 1 ∨ 2 ∨ 3
~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
654
653
⊥

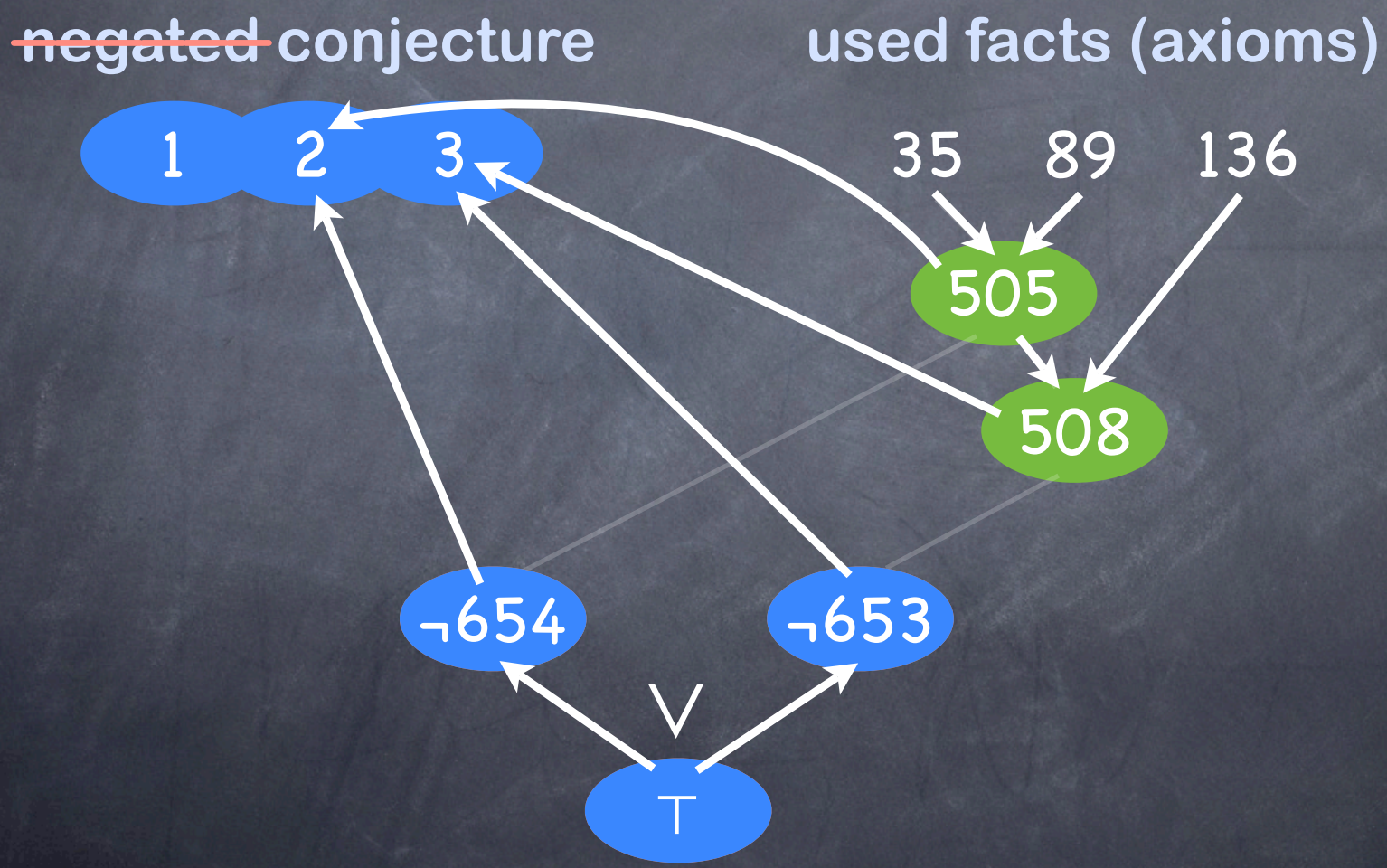

conjecture (DNF): 1 ∨ 2 ∨ 3
~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
654
653
⊥

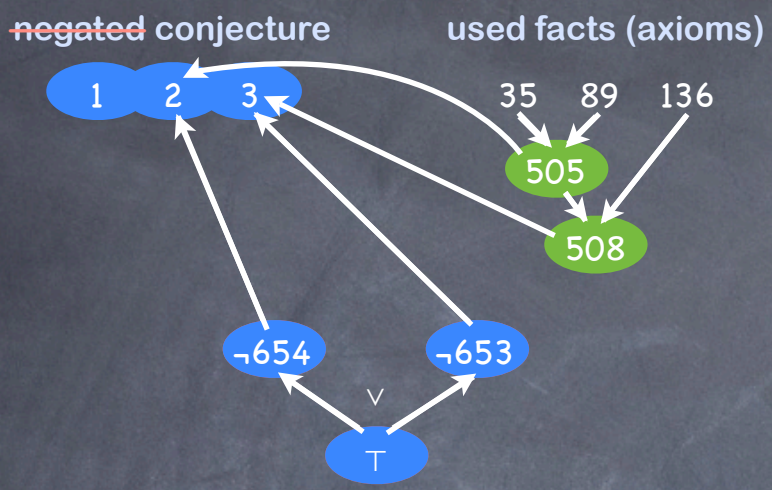

conjecture (DNF): 1 ∨ 2 ∨ 3
~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
¬654
¬653
∨
⊤

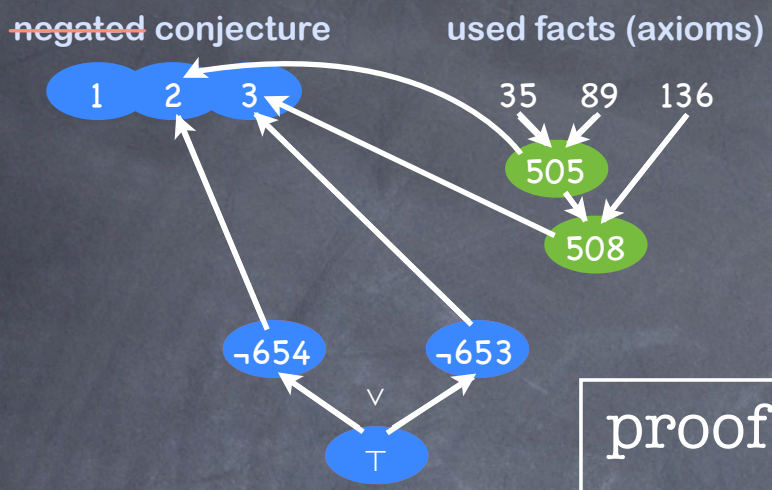
conjecture (DNF): 1 ∨ 2 ∨ 3



conjecture (DNF): 1 ∨ 2 ∨ 3

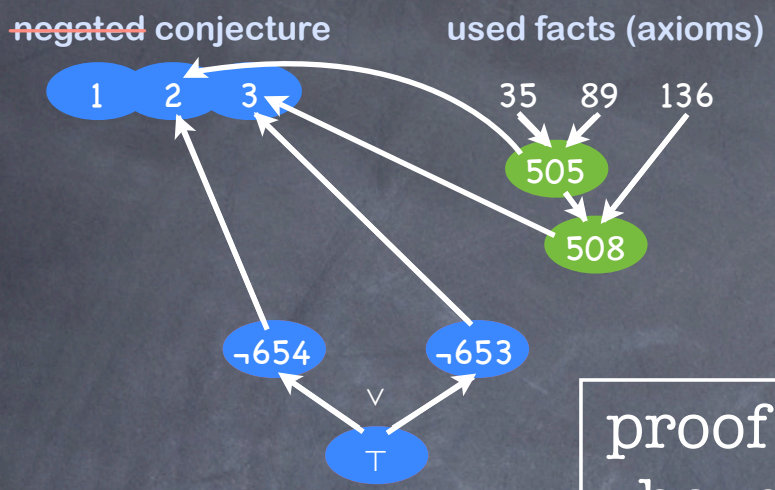



~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
¬654
¬653
∨
⊤


~~negated~~ conjecture
used facts (axioms)
1
2
3
35
89
136
505
508
¬654
¬653
∨
⊤

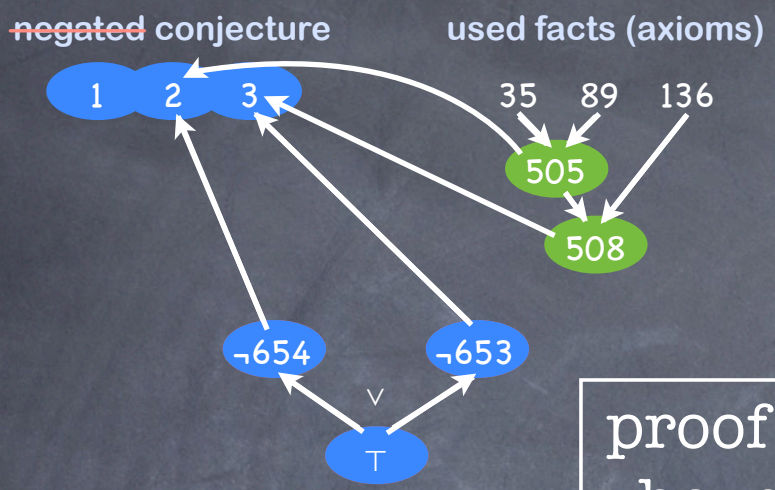
proof -

qed




```
proof -
  have 505 by (metis 35 89)
  have 508 by (metis 505 136)

qed
```




```
proof -
  have 505 by (metis 35 89)
  have 508 by (metis 505 136)
  { assume ¬653
    have 3 by (metis ¬653 508) }
  moreover
  { assume ¬654
    have 2 by (metis ¬654 505) }
  ultimately show thesis by metis
qed
```