Physics 1C Notes

Kyle Chui

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Contents

1 Lecture 1

1.1 Electromagnetism

In Physics 1B, we discussed how

$$\text{charge} \to \text{electric fields} \begin{cases} \text{Coulomb's Law} \\ \text{Gauss' Law} \end{cases}.$$

Furthermore, we know that moving charge → magnetic field (Ampere's Law).

$$\text{Gauss' Law: } \oint_{\text{surface}} \vec{E} \cdot \mathrm{d}\vec{A} = \frac{q}{\varepsilon_0} \qquad \text{Ampere's Law: } \oint_{\text{loop}} \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I.$$

There are two other main laws for electromagnetism, and all of these laws give us magnets, motors, power generators, etc.

1.2 Electromagnetic Waves

These four laws also lead to electromagnetic waves, examples of which are radio waves, visible light, etc.

Definition. *Speed of Light*
We define the speed of light c to be

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \cdot 10^8 \text{m/s}.$$

1.3 Light and Optics

Light and matter gives us the ideas of reflection and refraction, which lead to prisms, lens, and mirrors. In 1905, Einstein noticed that space and time depend on the observer, and that energy and momentum are different from what we thought.

2 Lecture 2

Definition. *Magnetic field*
The *magnetic field* is denoted by $\vec{B}$, and its units are given in Tesla (which is a large unit!).

2.1 Demonstrations

- A current through a magnetic field experiences a force (the jumping wire). The force $\vec{F}$ was perpendicular to the magnetic field $\vec{B}$, which was perpendicular to the current $\vec{I}$.
- The lodestone is a magnet.
- Opposite poles attract, same poles repel (bar magnets).
- We visualised the magnetic field by looking at the alignment of iron filings in a viscous fluid.

2.2 Equations

1. The force due to a magnetic field is given by
$$\vec{F}_m = q\vec{v} \times \vec{B}.$$
This force is a part of the electromagnetic force,
$$\vec{F}_{EM} = q\vec{E} + q\vec{v} \times \vec{B}.$$

Note (Cross Products). Suppose we have two vectors $\vec{a}$ and $\vec{b}$. Then $\vec{a} \times \vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$. If θ is the angle between the two vectors, then
$$\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \left|\vec{b}\right| \sin\theta = ab\sin\theta.$$
The direction of the vector is given by the "right hand rule".

3 Lecture 3

3.1 Gauss' Law for Magnetic Fields

In Physics 1B, we learned Gauss' Law for electric fields:

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}.$$

We have a similar equation for magnetic fields:

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0.$$

The reason why this is zero is because there are "no magnetic monopoles", i.e. every magnet has two poles.

> **Note.** This can also be interpreted to mean that for a given surface, the number of magnetic field lines coming out must equal the number of magnetic field lines going in.

3.2 The Motion of Charged Particles in a Magnetic Field

There are [some] cases:

i) When the particle is at rest, we have
$$\vec{F} = q\vec{v} \times \vec{B} = 0.$$

ii) For a particle in 2D motion with a constant magnetic field, we have

$$\vec{F} = q\vec{v} \times \vec{B},$$

which is perpendicular to both $\vec{v}$ and $\vec{B}$ (by the properties of the cross product). We use the right hand rule to get the actual direction of the force (when the charge is positive).

> **Note.** Since the magnetic force is perpendicular to the motion of the particle, we have uniform circular motion.

To get the radius of the motion, we solve:

$$\begin{aligned} F_C &= F_B \\ \frac{mv^2}{r} &= qvB \\ r &= \frac{mv^2}{qvB} \\ \Aboxed{r &= \frac{mv}{qB} = \frac{p}{qB}.} \end{aligned}$$

iii) For a particle in 3D motion with a constant magnetic field, $\vec{v}_0$ is not necessarily perpendicular to $\vec{B}$. Thus we first decompose $\vec{v}_0$ into $\vec{v}_0 = \vec{v}_{0_\parallel} + \vec{v}_{0_\perp}$. Then we have

$$\begin{aligned} \vec{F} &= q\left(\vec{v}_{0_\parallel} + \vec{v}_{0_\perp}\right) \times \vec{B} \\ &= qv_{0_\parallel} \times B + \vec{v}_{0_\perp} \times \vec{B} \\ &= \vec{v}_{0_\perp} \times \vec{B}. \end{aligned}$$

Note. The movement of the particle is circular relative to the perpendicular plane to the magnetic field, but the velocity of the particle parallel to the magnetic field is constant. Thus the particle moves in a helix (or spiral) pattern.

3.3 Force Equation for a Current-Carrying Wire

We define $\vec{\ell}$ to be the length of the wire in the direction of $\vec{v}$. From earlier, we know that

$$\vec{F} = N \cdot q\vec{v} \times \vec{B}.$$

Furthermore, we have

$$I = \frac{Q}{t} = \frac{Nq}{t} = \frac{Nq}{\frac{\ell}{v}} = \frac{Nqv}{\ell}.$$

Thus we have

$$\begin{aligned} \vec{F} &= N \cdot q\vec{v} \times \vec{B} \\ &= N \cdot qv \cdot \frac{\vec{\ell}}{\ell} \times \vec{B} \\ &= I\vec{\ell} \times \vec{B}. \end{aligned}$$

Alternatively, if things are always changing, then

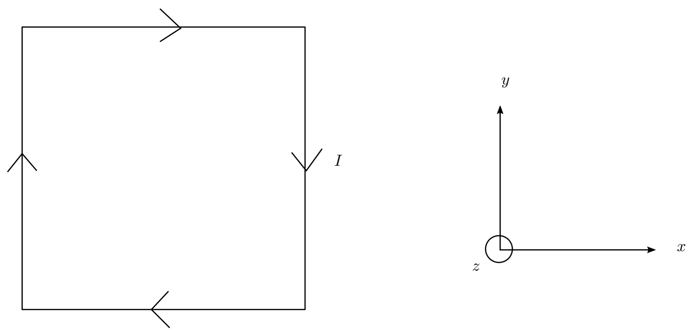
$$\vec{F} = \int_{\text{wire}} I \, \mathrm{d}\vec{\ell} \times \vec{B}.$$

4 Lecture 4

4.1 Current Loops in Magnetic fields: Force and Torque

4.1.1 Uniform Magnetic Field Perpendicular to the Loop

Consider a fictional square loop of wire with side L, where the current flowing through the loop is I. We denote $\odot$ to be out of the page, and $\otimes$ to be into the page. We will use the equations $\vec{F} = I\vec{L} \times \vec{B}$ and $\vec{\tau} = \vec{r} \times \vec{F}$, and assume that we have a constant, uniform $\vec{B}$ field.



$\vec{B}$	$\vec{F}_{\text{left}}$	$\vec{F}_{\text{top}}$	$\vec{F}_{\text{right}}$	$\vec{F}_{\text{bottom}}$	$\vec{F}_{\text{net}}$	$\vec{\tau}_{\text{net}}$
$\otimes$	$\leftarrow$	$\uparrow$	$\rightarrow$	$\downarrow$	0	0
$\uparrow$	0	$\odot$	0	$\otimes$	0	$\hat{\imath}\left(\frac{L}{2}BIL \cdot 2\right) = (BIL^2)\hat{\imath}$

The magnetic moment μ is given by $\mu = IA$, where A is the area of the loop (in this case, $L = A^2$). We use this μ to find the torque, and we have $\tau = \mu B$ or $\vec{\tau} = \vec{\mu} \times \vec{B}$. The magnetic moment is defined as perpendicular to the surface of the loop.

For the example above, the magnetic moment must be *into the page*. We can find this by using the right hand rule, where you point your thumb in the direction of the current and curl your fingers around to find the direction of the moment.

4.1.2 Generalisation to a Loop of Any Shape

We can approximate an irregularly shaped loop of wire by breaking it into smaller, rectangular loops of wire.

> **Note.** If the currents of the rectangular loops are all travelling in the same direction, the currents inside of the outer loop will cancel, leaving only the current around the perimeter of the outside loop.

5 Lecture 5

Last week, we studied the force due to a $\vec{B}$. This week, we will study how to produce a $\vec{B}$.

5.1 Biot-Savart Law

Point charge observations:

- When you have a point charge, you have an electric field $\vec{E}$ that radiates outwards from the point charge.
- This is slightly different from the magnetic field, which is generated by a *moving* point charge.
  - The closer you are to the point charge, the larger the magnetic field.

Theorem — *Coulomb's Law*
Given the charge and distance from a point charge, we may find the electric field:

$$\vec{E} = \frac{q\hat{r}}{4\pi\varepsilon_0 r^2}.$$

Biot-Savart Law precursor:

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v} \times \hat{r}}{r^2}.$$

Note. In the equation for the magnetic field, we see that it is both orthogonal to the velocity vector and $\hat{r}$ (due to the cross product).

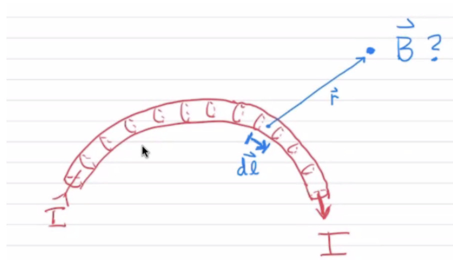
Points:

- Both are inversely proportional to the square of the distance.
- $\mu_0 \coloneqq 4\pi \cdot 10^{-7}$ Tm/A, and is used in the definition of the ampere (and so the Coulomb).

Note. This means that the $\vec{E}$ is very large, but $\vec{B}$ is quite small.

Theorem — *Biot-Savart Law*
Given the current through a wire, and the displacement from a point to the wire, we can find the magnetic field:

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,\mathrm{d}\vec{\ell} \times \hat{r}}{r^2}.$$

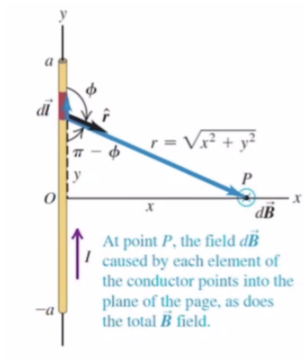


In the above diagram, you can see how the cross product of $\mathrm{d}\vec{\ell}$ and $\vec{r}$ is in the direction of the magnetic field. Furthermore, we can integrate the Biot-Savart Law to get

$$\vec{B} = \int_{\text{wire}} \mathrm{d}\vec{B}$$

5.2 Magnetic Field of a Straight Current-Carrying Wire

For a long, straight current-carrying wire, we try to find the field on some point P from locations $-a$ to a on the wire.



We can see that $r = (x^2 + y^2)^{\frac{1}{2}}$, and

$$\hat{r} - \frac{x\hat{\imath} - y\hat{\jmath}}{(x^2 + y^2)^{\frac{1}{2}}}. \qquad \text{(Scale down } \vec{r}\text{)}$$

Furthermore, $\mathrm{d}\vec{\ell} = \mathrm{d}y\hat{\jmath}$. The hard part is calculating the following:

$$\begin{aligned} \frac{\mathrm{d}\vec{\ell} \times \hat{r}}{r^2} &= \frac{\mathrm{d}y\hat{\jmath} \times (x\hat{\imath} - y\hat{\jmath})}{(x^2 + y^2)^{\frac{3}{2}}} \\ &= \frac{-\hat{k}x\,\mathrm{d}y}{(x^2 + y^2)^{\frac{3}{2}}}. \end{aligned}$$

We then integrate over the length of the wire:

$$\vec{B} = -\hat{k}\frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x\,\mathrm{d}y}{(x^2 + y^2)^{\frac{3}{2}}}$$

Letting $y = x\tan\theta$, and going through some algebra, we get

$$= -\hat{k}\frac{\mu_0 I}{4\pi}\left(\frac{2a}{x\sqrt{a^2 + x^2}}\right).$$

For an infinitely long wire, we see what happens when $a \to \infty$. In this case, we have

$$\vec{B} = -\hat{k}\frac{\mu_0 I}{4\pi}\left(\frac{2}{x}\right) = -\hat{k}\frac{\mu_0 I}{2\pi x}.$$

We started with a special point P on the wire, which is on the perpendicular bisector of the wire. However, when $a \to \infty$, we have an infinite amount of wire on either side, so the y becomes irrelevant and all we care about is the distance x to the wire. Thus to find the magnitude of the wire, we have

$$B = \frac{\mu_0 I}{2\pi r},$$

where r is the distance to the wire. If we were to look at the wire from the top (the xz-plane), we would have that $\vec{B}$ circles the wire.

> **Note.** Another right-hand rule is to put your thumb in the direction of the current, and your thumbs will curl to form the magnetic field.

6 Lecture 6

The force between long parallel wires defines the ampere. As a reminder, last class we found the formula

$$B = \frac{\mu_0 I}{2\pi r}.$$

Consider two parallel, infinitely-long wires that are carrying current. If the currents are going in the same direction, then we use the right-hand rule to find that the wires have an attractive force between them. We find that the force acting on wire 2 is

$$\begin{aligned}\vec{F} &= I_2\vec{L} \times \vec{B}_1\\ &= I_2 L\frac{\mu_0 I_1}{2\pi r} \text{ to the left,}\end{aligned}$$

so we have

$$F_2 = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2 L}{r}.$$

Rearranging some more, we get

$$\frac{F_2}{L} = \left(2 \cdot 10^{-7}\right)\frac{I_1 I_2}{r} \text{ Newtons.}$$

What happens when we ave $I_1 = I_2$? Well, we get that

$$\frac{F}{L} = \left(2 \cdot 10^{-7}\right)\frac{I^2}{r},$$

which we can rearrange to get

$$I = \underbrace{\left(r \cdot \frac{F}{L} \cdot \frac{1}{2 \cdot 10^{-7}}\right)^{\frac{1}{2}}}_{\text{defines the Ampere}}.$$

6.1 Ampere's Law

Just like Coulomb's Law leads to Gauss' Law (which is an integral), the Biot-Savart Law leads to Ampere's Law (also an integral). Remember the Biot-Savart Law from before:

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\vec{\ell} \times \hat{r}}{r^2}.$$

Theorem — *Ampere's Law*
The amount of magnetic field in a loop can give you the current, given by

$$\oint_{\text{loop}} \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_{\text{enc}}.$$

The analogue to the Gaussian surface is what we call an *Amperium loop*. There is no radial component to $\vec{B}$ because there are no monopoles. Thus we have that $\vec{B} \parallel \mathrm{d}\vec{\ell}$, and $\vec{B} \cdot \mathrm{d}\vec{\ell} = B\,\mathrm{d}\ell$. Then

$$\begin{aligned}\oint B\,\mathrm{d}\ell &= B\oint \mathrm{d}\ell\\ &= B \cdot 2\pi r\\ &= \mu_0 I,\end{aligned}$$

so

$$B = \frac{\mu_0 I}{2\pi r}.$$

The differential form of Ampere's Law is given by:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J},$$

where $\vec{J}$ is the current density.

6.1.1 Ampere's Law Example—Long Wire, Thin but Finite Thickness

Consider a wire with radius R that has uniform current density J. We define current density to be the current per unit area, in other words

$$J = \frac{I}{\pi R^2}, \text{ a constant.}$$

We have a few cases here:

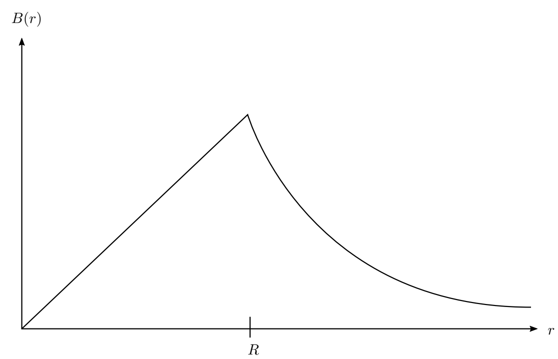
(a) Outside the wire, i.e. $r > R$.

$$\begin{aligned}\oint \vec{B} \cdot \mathrm{d}\vec{\ell} &= \mu_0 I_{\text{encl}} \\ 2\pi r B &= \mu_0 I \\ \Aboxed{B &= \frac{\mu_0 I}{2\pi r}.}\end{aligned}$$

(b) Inside the wire, i.e. $r < R$.

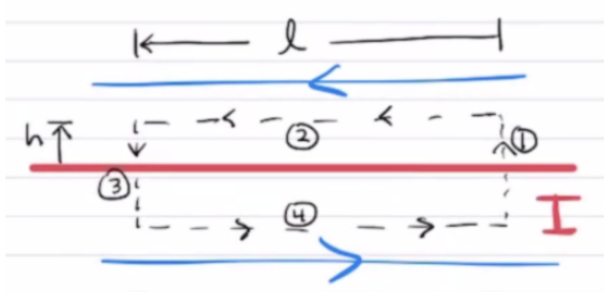
$$\begin{aligned}\oint \vec{B} \cdot \mathrm{d}\vec{\ell} &= \mu_0 I_{\text{encl}} \\ 2\pi r B &= \mu_0 I \cdot \frac{\pi r^2}{\pi R^2} \\ 2\pi r B &= \mu_0 I \cdot \frac{r^2}{R^2} \\ \Aboxed{B &= \frac{\mu_0 I r}{2\pi R^2}.}\end{aligned}$$

Pictorially, we can draw a graph for the magnetic field as a function of the distance to the centre of the wire as follows:



6.1.2 Ampere's Law Example—Plane of Current

If we have two parallel wires next to each other with the currents moving in the same direction, by superposition of field lines we generate a space with zero field between the wires. Extending this idea to more than just two wires (read: an infinite number, creating a plane of current), we create a planar magnetic field.



> **Definition.** *Winding Density*
> We define the *winding density* n to be the number of wires per unit length, or
> $$n = \frac{N}{\ell}.$$

Applying Ampere's Law, we have

$$\begin{aligned}
\oint \vec{B} \cdot \mathrm{d}\vec{\ell} &= \mu_0 I_{\text{encl}} \\
\int_1 + \int_2 + \int_3 + \int_4 &= \mu_0 N I \\
2 \int_4 \vec{B} \cdot \mathrm{d}\vec{\ell} &= \mu_0 N I \\
2B\ell &= \mu_0 N I \\
B &= \frac{\mu_0 n I}{2}.
\end{aligned}$$