



Figure 6: Quantifying the failure modes of PINNs. Here we show additional experimental results for PINN models solving increasingly difficult convection problems (Eq. 3 for $\beta \in [1..10]$) using a fixed random seed ($seed = 0$). We show our proposed TDA-based metrics, including the number of saddle points in the merge tree and average persistence, along with more traditional ML-based metrics, including the Absolute Error, Top-1 Hessian Eigenvalue, and Hessian Trace. These plots provide additional insights beyond the qualitative changes in the loss landscapes we observed in Fig. 2, confirming that the number of saddle points in the merge tree (left column) increases, along with the average persistence (right column), as the value of β increases. In each row, we overlay our TDA-based metrics with a different ML-based metric. We observe similar trends in the Absolute Error, Top-1 Hessian Eigenvalue, and Hessian Trace as β increases. Together, these results reaffirm previous findings and provide new insights into the failure modes of PINNs, suggesting that the topology of the loss landscape becomes significantly more complex and difficult to optimize.