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Appendix A. Peer-reviewed articles discussed in this survey

Table 1: Table containing the published articles reviewed in this survey ordered by year of publication. Each row corresponds to an article. The first column contains the title; the second column contains a one-line summary; the third column contains the categories introduced in Section 1 associated to each article; and the last column contains the applications introduced in Section 2.4. The four possible categories are 1. Structure of the neural network; 2. Input and output spaces; 3. Internal representations and activations; 4. Training dynamics and loss functions. The seven possible applications are 1. Regularization; 2. Pruning of neural networks; 3. Detection of adversarial, out-of-distribution and shifted examples; 4. Detection of trojaned networks; 5. Model selection; 6. Prediction of accuracy; 7. Quality assessment of generative models.

Title	**Summary**	**Cat.**	**Apps.**
On the complexity of neural network classifiers: A comparison between shallow and deep architectures (Bianchini and Scarselli, 2014)	Bounds on the Betti numbers of the positive decision region generated by binary classification neural networks with Pfaffian activations	(2)	-
Topological approaches to deep learning (Carlsson and Brüel Gabrielsson, 2020)	Topological analysis of the weights of convolutional neural networks and generalization of convolutional neural networks	(3)	-
Topological data analysis of decision boundaries with application to model selection (Ramamurthy et al., 2019)	Study and approximation of the topology of network decision boundaries	(2)	(5)
A topological regularizer for classifiers via persistent homology (Chen et al., 2019)	Regularization of neural networks by modifying their decision regions using differentiable persistent homology	(2)	(1)
Geometry Score: A method for comparing generative adversarial networks (Khrulkov and Oseledets, 2018)	Measurement of GAN quality using persistent homology	(2)	(7)
What does it mean to learn in deep networks? And, how does one detect adversarial attacks? (Corneanu et al., 2019)	Study of generalization in terms of the persistent homology of neuron activations and their correlations	(3)	(1, 3)
On connected sublevel sets in deep learning (Nguyen, 2019)	Study of the connectivity, boundedness, and local minima of sublevel sets of convex losses for overparameterised neural networks with piecewise linear activation functions	(4)	-

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