C. Additional Experimental results

C.1. Generative behavior of LLMs: Qualitative inspection

Prompt	stage1-step1000-5B	stage1-step920000-3895B
Sunlight filtered through dense canopy, dappling the forest floor	The first thing you can see is that the first thing you see in the first place is that you can see the first thing you see in the first place. The second thing you see in the first place is that you can see the first thing you see in the first place. The second thing you see in the second place is that you can see the first thing you see	The air was still and warm, and the only sound was the occasional chirp of a bird. The forest was quiet, and the only movement was the slow, steady sway of the trees.\n\nA small, furry creature scurried through the underbrush, its eyes darting from side to side. It was a squirrel, and it was looking for food. It had
An old, weathered map lay spread across the wooden table.	The table was a large, large, and large, and the table was a large, large, and large, and the table was a large, and the table was a large, and the table was a large, and the table was a large, and the table was a large, and the table was a large, and the table	The map was of the area around the town of Harken, and it was marked with a number of locations. The map was covered with notes, and the notes were covered with a number of different colored markers. The markers were of different shapes and sizes, and they were all labeled with names and numbers. The map was a mess, and it was obvious that it
A lone ship sailed silently towards the crimson horizon	The sky was a dark, dark	The ship was a small, two-masted vessel, with a single mast on the stern. The ship was painted a dark blue, with a white stripe running along the hull. The ship was a pirate ship, and the captain was a pirate. The captain was a pirate, and the ship was a pirate ship. The captain was a pirate, and the ship was
Intricate carvings covered the surface of the ancient stone tablet	The first part of the 19th century was the first part of the 19th century. The first part of the 19th century was the first part of the 19th century. The first part of the 19th century was the first part of the 19th century. The first part of the 19th century was the first part of the	The carvings were so intricate that they were almost impossible to decipher. The tablet was covered in a thick layer of dust and dirt. The dust and dirt had been there for so long that it had become a part of the stone. It was as if the dust and dirt had been there since the beginning of time.\n\nThe tablet was covered in a thick layer of dust
A faint melody drifted from an open window down the cobblestone street	The two cars are the most expensive and the most expensive and the most expensive. The cars are the most expensive and the most expensive and the most expensive and the most expensive and the most expensive and the most expensive and the most expensive and the most expensive and the most expensive and the most expensive	The music was soft and sweet, and it was enough to make the young man smile. He was a tall, handsome man with dark hair and eyes. He was dressed in a fine suit, and he carried a violin case in his hand. He walked with a confident stride, and he seemed to be in no hurry. He stopped in front of a small house and knocked
An unspoken tension filled the crowded room, heavy and palpable	The room was a large, heavy, and very large, very	The air was thick with the scent of fear and the stench of death. The room was silent, save for the sound of the wind howling outside. The only light came from the flickering flames of the fire. The room was dark and cold, and the only warmth came from the bodies of the men and women huddled together in the corner. The smell
A single drop of rain landed on the dusty, parched earth	The earth was a great, strong	It was the first rain in months. The earth drank it up greedily, and the thirsty plants and trees drank it up too. The thirsty animals drank it up. The thirsty people drank it up. The rain was a blessing. It was a gift from God. It was a gift of life.\n\nThe rain continued to fall. The earth drank it up. The plants

Figure 7: Early checkpoints exhibit echolalia. Generated text from early checkpoints (step-1000) of OLMo-2-7B models shows repetitive, non-contextual patterns characteristic of the "warmup" phase, contrasting sharply with coherent outputs from later checkpoints (step-920000).

C.2. Computing spectral metrics, RankMe and αReQ

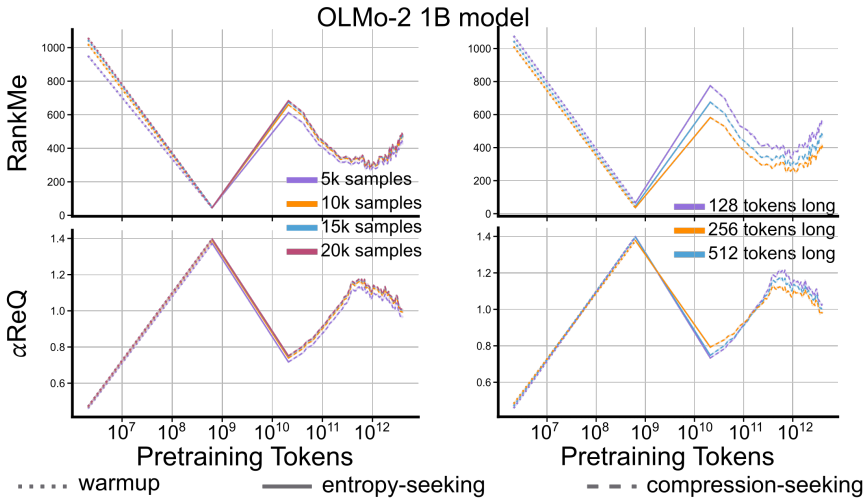

Figure 8: **Spectral metrics are robust to sample count and sequence length.** RankMe and α_{ReQ} computed for intermediate checkpoints of the OLMo-2-1B model using **(Left)** different number of samples, and **(Right)** sequence length. The three-phase pattern remains consistent across these methodological choices. Shaded error bars indicate standard error about mean.

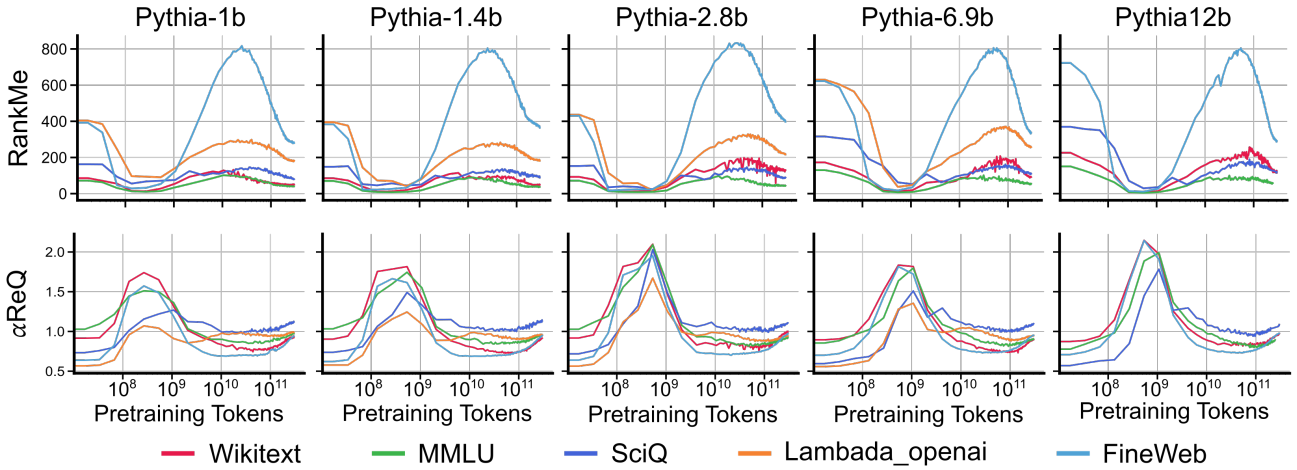

Figure 9: **Pythia spectral metrics are robust across datasets.** RankMe and α_{ReQ} computed for intermediate checkpoints of models from the Pythia family (1B-12B) on different datasets, showing consistent phase patterns across evaluation data.

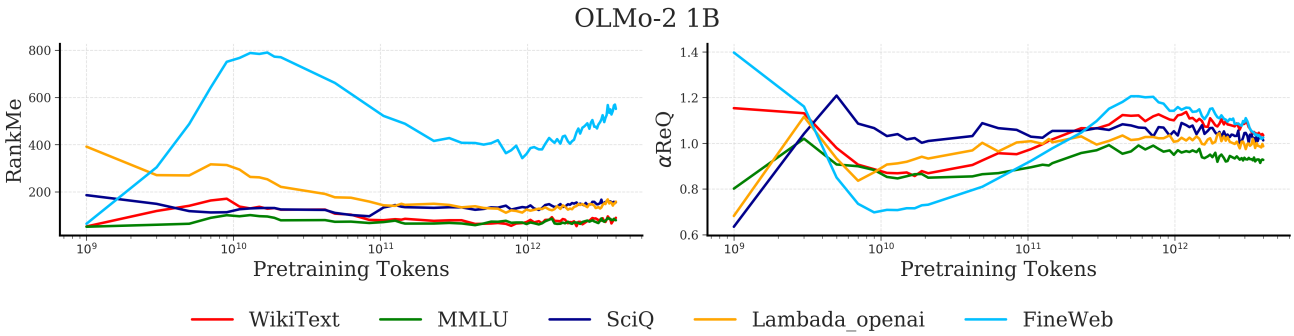

Figure 10: **OLMo spectral metrics are robust across datasets.** RankMe and α_{ReQ} computed for intermediate checkpoints of OLMo-2 1B model on different datasets, showing consistent phase patterns across evaluation data.

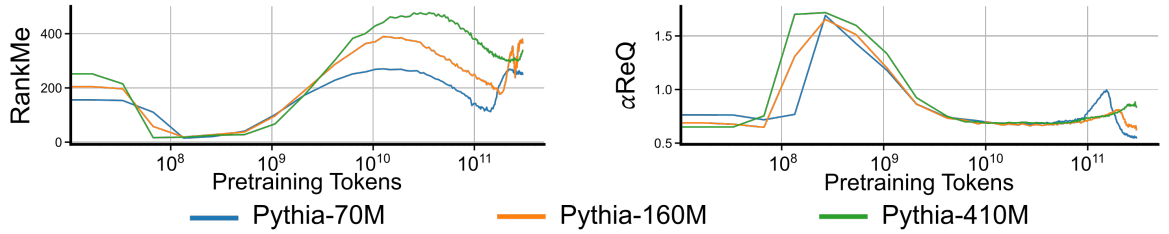

Figure 11: `RankMe` and αReQ computed for intermediate checkpoints of smaller models ($<$ 1B) from the Pythia family on the FineWeb dataset.

Model	α_{ReQ} **(p-value)**	RankMe **(p-value)**
Pythia-1B	0.810 (1.50e-5)	-0.759 (1.04e-4)
Pythia-1.4B	0.668 (1.29e-3)	-0.713 (4.18e-4)
Pythia-2.8B	0.694 (6.88e-4)	-0.635 (2.63e-3)
Pythia-6.9B	0.837 (4.20e-6)	-0.885 (2.19e-7)
Pythia-12B	0.836 (4.42e-6)	-0.839 (3.79e-6)
OLMo2-1B	0.540 (4.920e-7)	-0.616 (3.201e-9)

Table 9: **SciQ accuracy correlates with spectral geometry.** Positive correlation with α_{ReQ} (compactness) and negative correlation with RankMe (effective dimensionality) across Pythia (1–12B) and OLMo2-1B models. The p-values in parentheses indicate high statistical significance for all correlations.