Binary Search Library

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This library does binary search to find the upper and lower bounds of x corresponding to a given y. The (x, y) pairs can be specified as a monotone increasing function or as the (index, value) pairs of a sorted array.

1 Monotone functions

Definition 1. The function f is monotone increasing if for all x, y in the domain of f, $x \leq y$ implies $f(x) \leq f(y)$.

```
(infimum f <= y a b epsilon)
(supremum f <= y a b epsilon)
```

Parameters

- f: a monotone increasing function defined on real numbers
- <=: binary relation on the range of f. It should take two arguments r, s and return true if $r \leq s$.
- y: a value of the same type as returned by f
- a: real number in the domain of f
- b: real number in the domain of f such that $a < b$
- epsilon: real number > 0

Returns The values $x, f(x)$ satisfying:
For infimum, $x \in [a, b)$ such that $f(x) \leq y < f(x + \epsilon)$
For supremum, $x \in (a, b]$ such that $f(x - \epsilon) < y \leq f(x)$

Exceptions Throws OUT-OF-BOUNDS-ERROR if the infimum or supremum isn't in the domain of f.

2 Sorted arrays

Definition 2. A n-element array A is in ascending order if $f(i) = A[i]$ is a monotone increasing function for all integer $i \in [0, n)$.

```
(infimum-array arr <= y a b)
(supremum-array arr <= y a b)
```

Parameters

- arr: an array in ascending order of <=
- <=: a binary relation on the elements of arr. It should take two arguments r, s and return true iff $r \leq s$
- y: a value that can be compared with the elements of arr
- a: index of arr. Default value is 0.
- b: index of arr. Default value is the last index of arr.

a and b must satisfy $0 \leq a < b < n$ where n is the length of arr.

Returns The values $i, arr[i]$ satisfying:
For infimum-array, $i \in [a, b)$ such that $arr[i] \leq y < arr[i+1]$
For supremum-array, $i \in (a, b]$ such that $arr[i-1] < y \leq arr[i]$

Exceptions Throws OUT-OF-BOUNDS-ERROR if the infimum or supremum isn't in the array.