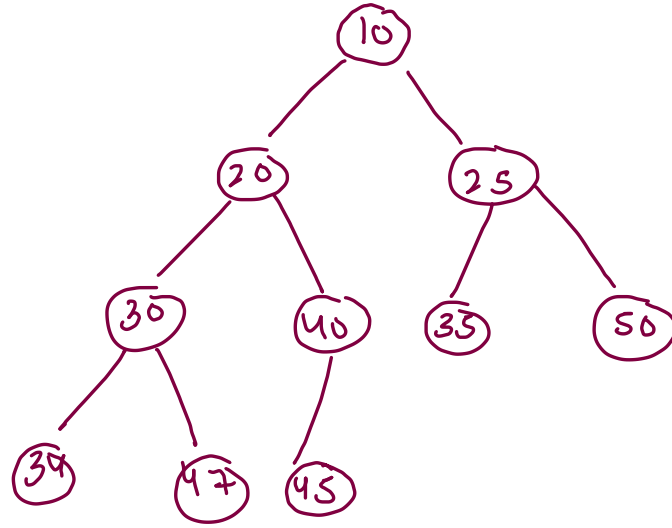


DS \rightarrow heap

add $\rightarrow \log n$

remove $\rightarrow \log n$

peek $\rightarrow O(1)$



Smaller value has higher priority

(i) heap order property.
 $\text{priority}(\text{parent}) > \text{priority}(\text{lc, rc})$

(ii) complete binary tree property.

0 to $h-1$ levels

are completely filled and last level is filled

from left to right.

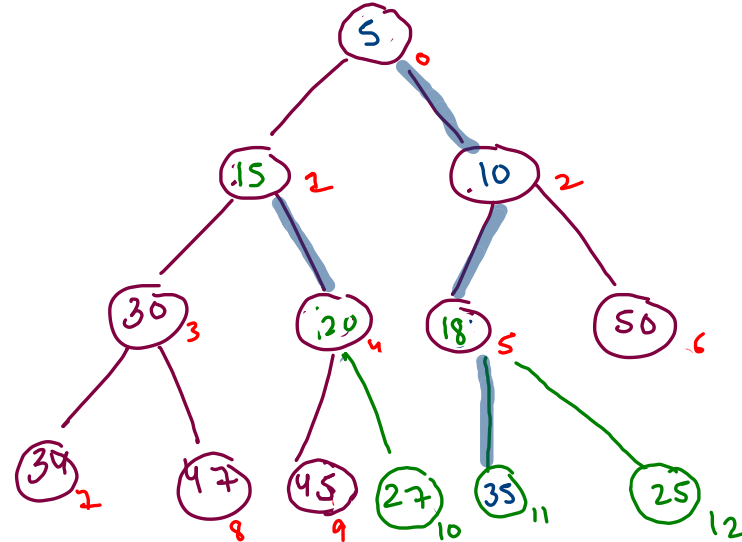
10_0 $\frac{15}{20}$ 25_2 30_3 $\frac{15}{27}$ 35_5 50_6 39_7 47_8 45_9 $\frac{27}{15}$ 5_{11} 18_{12}

add $\rightarrow \log n$

remove $\rightarrow \log n$

peek $\rightarrow O(1)$

(or \rightarrow) p_i to $c_i \rightarrow \log n$
 c_i to p_i



parent to child

$$lci = 2p_i + 1$$

$$rci = 2p_i + 2$$

child to parent

$$p_i = \frac{c_i - 1}{2}$$

```

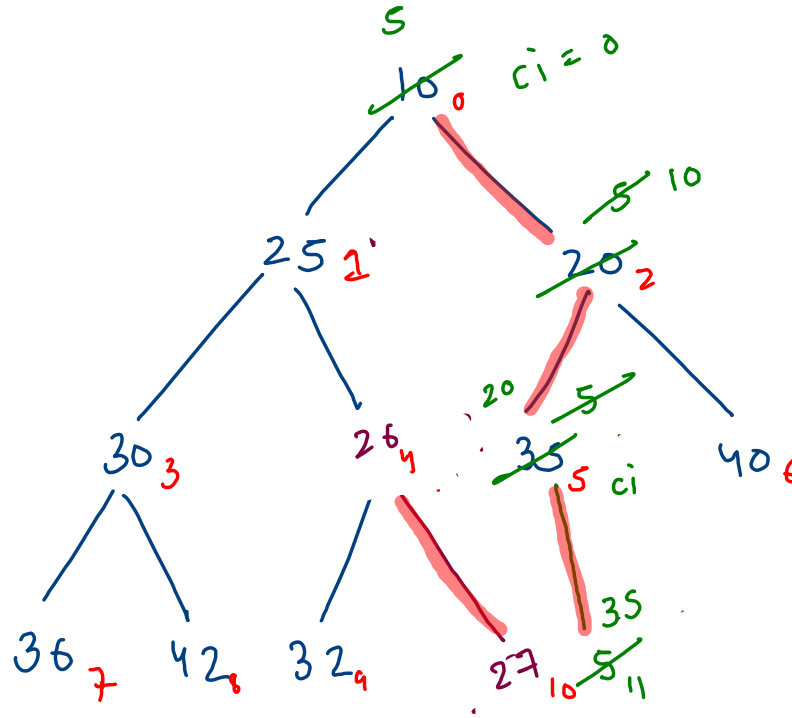
public void add(int val) {
    data.add(val); //may be hop is not honoured
    upheapify(data.size()-1); //to honour hop
}

private void upheapify(int ci) {
    if(ci == 0) {
        return;
    }

    int pi = (ci - 1) / 2;

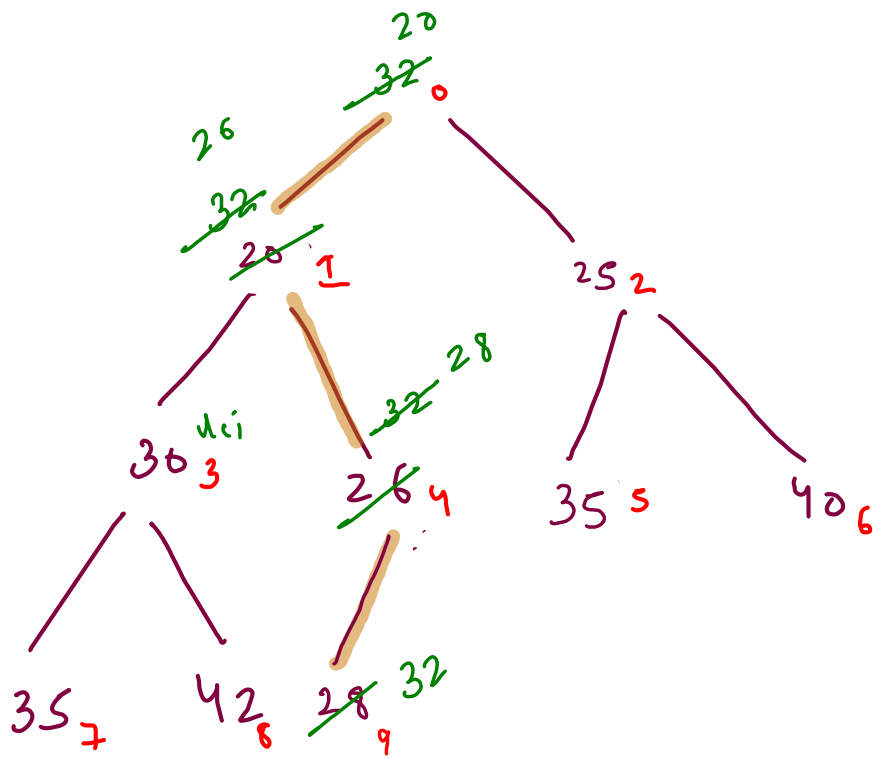
    if(list.get(ci) < list.get(pi) {
        swap(ci,pi);
        upheapify(pi);
    }
}

```



add(26)

add(5)



```

public int remove() {
    if(data.size() == 0) {
        System.out.println("Underflow");
        return -1;
    }
    else {
        int peek = data.get(0);
        swap(0, data.size()-1); //swap first and last element
        data.remove(data.size()-1);

        downheapify(0);
        return peek;
    }
}

```

```

private void downheapify(int pi) {
    int min_idx = pi; //min idx
    int lci = 2*pi + 1;
    int rci = 2*pi + 2;

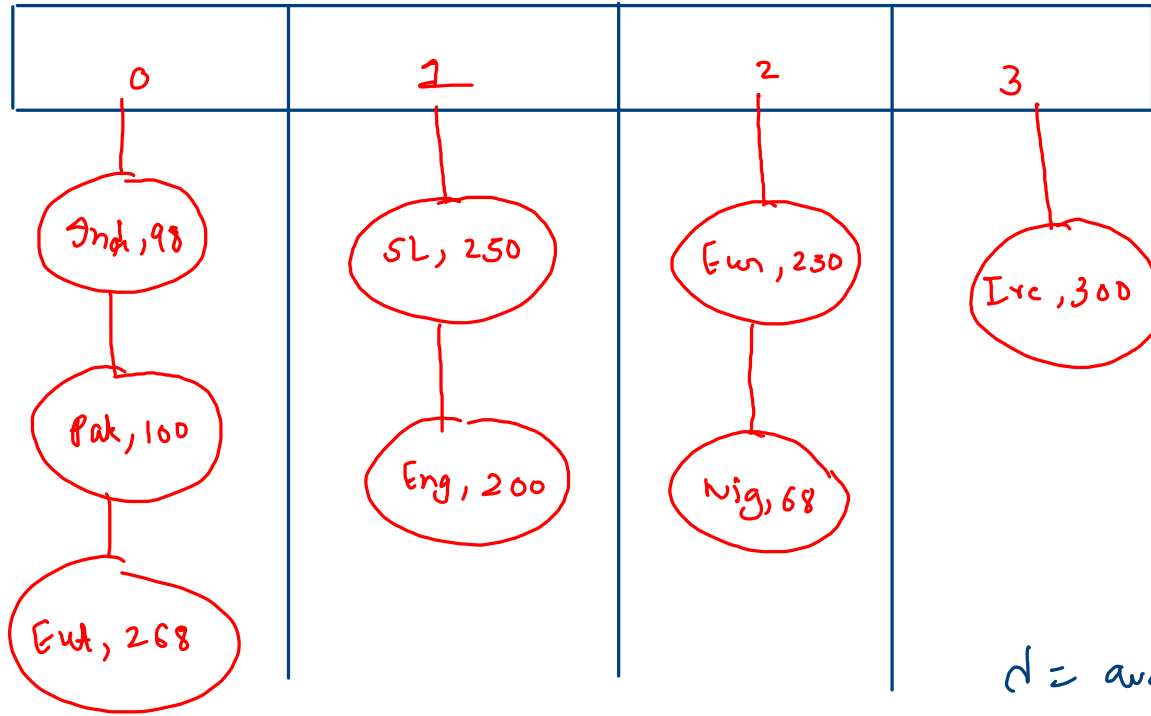
    if(lci < data.size() && data.get(lci) < data.get(min_idx)) {
        min_idx = lci;
    }

    if(rci < data.size() && data.get(rci) < data.get(min_idx)) {
        min_idx = rci;
    }

    if(min_idx != pi) {
        swap(min_idx, pi);
        downheapify(min_idx);
    }
}

```

bucket



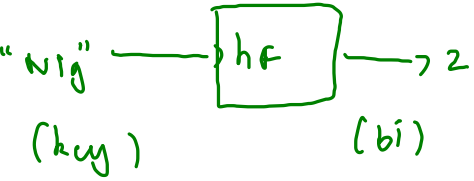
$N \rightarrow 4$
 $n \rightarrow 7$

$d = \text{avg number of nodes in a bucket}$

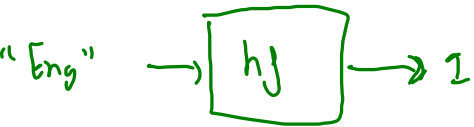
$O(d)$

$$d = \frac{n}{N}$$

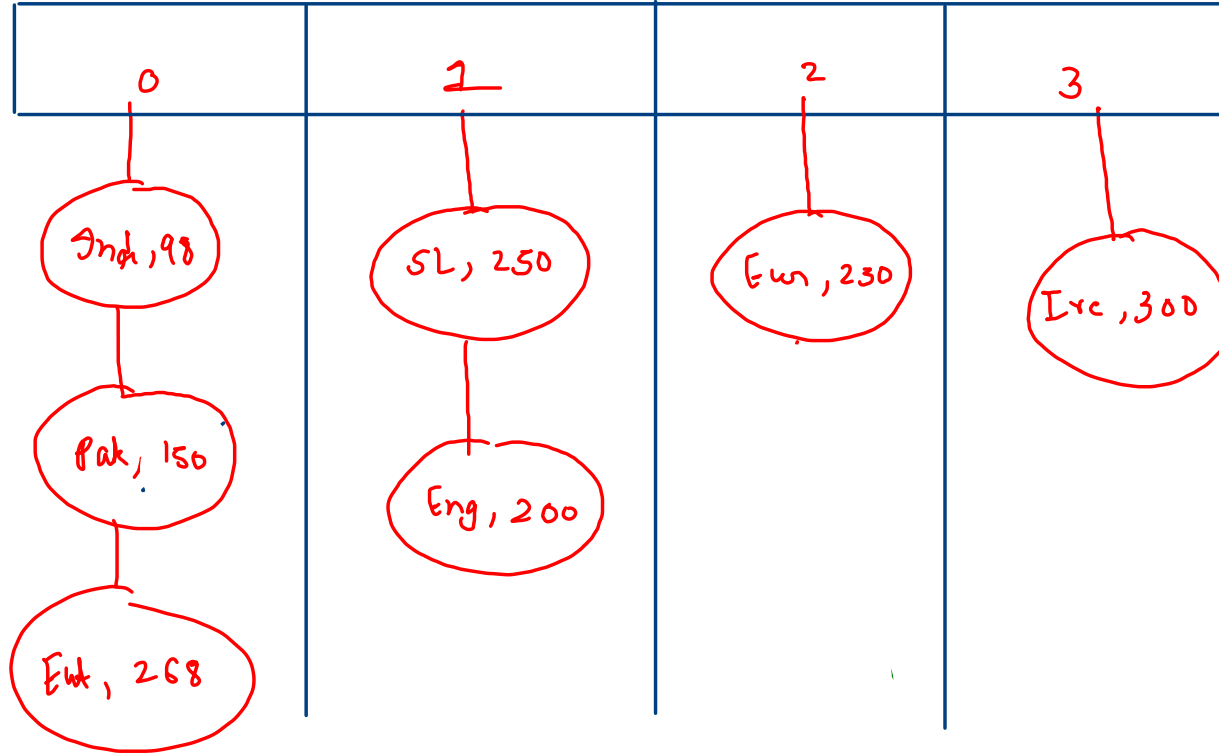
① $hm.put("Nig", 68)$



② $hm.put("Eng", 200)$



bucket



$n = \text{total nodes}$

$N = \text{buckets} \cdot \text{length}$

$d = \text{avg. no. of nodes per bucket.}$

$$d = \frac{n}{N} = \frac{7}{4} = 1.75$$

$$d \leq 2$$

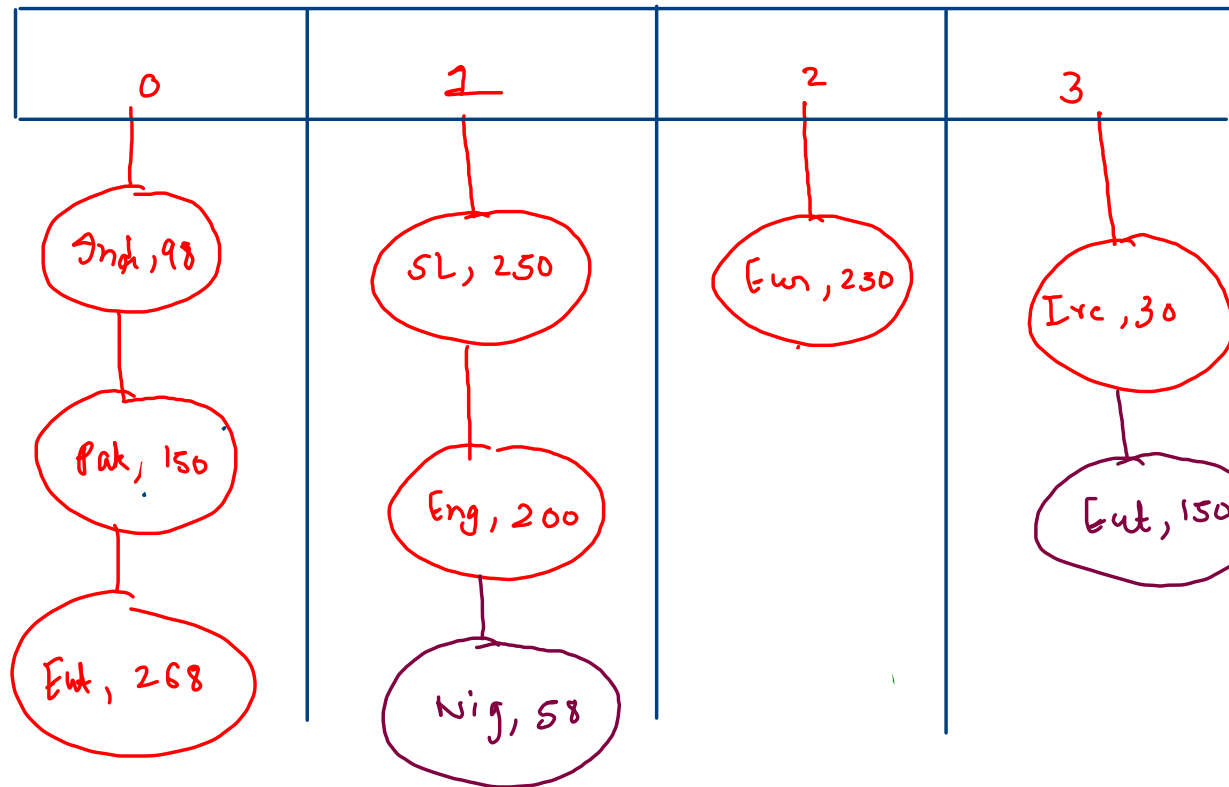
$O(d)$

$n \uparrow, d \uparrow$

bucket

$$d \leq 2$$

$$O(d)$$



$$d = \frac{1}{2}$$

$$d = \frac{9}{4} = 2.25$$

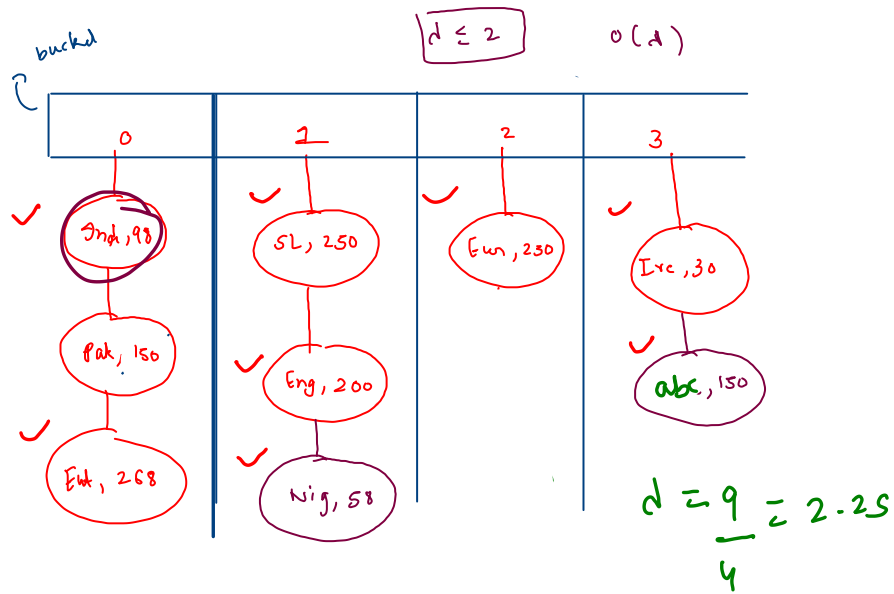
$$\text{Ind} \xrightarrow{hc} 44 \xrightarrow{bi} 0$$

$$\text{SL} \xrightarrow{hc} 57 \xrightarrow{bi} 1$$

$$\text{Eng} \xrightarrow{hc} 61 \xrightarrow{bi} 1$$

$$\text{Nig} \xrightarrow{hc} 65 \xrightarrow{bi} 1$$

$$\text{Eur} \xrightarrow{hc} 83 \xrightarrow{bi} 3$$



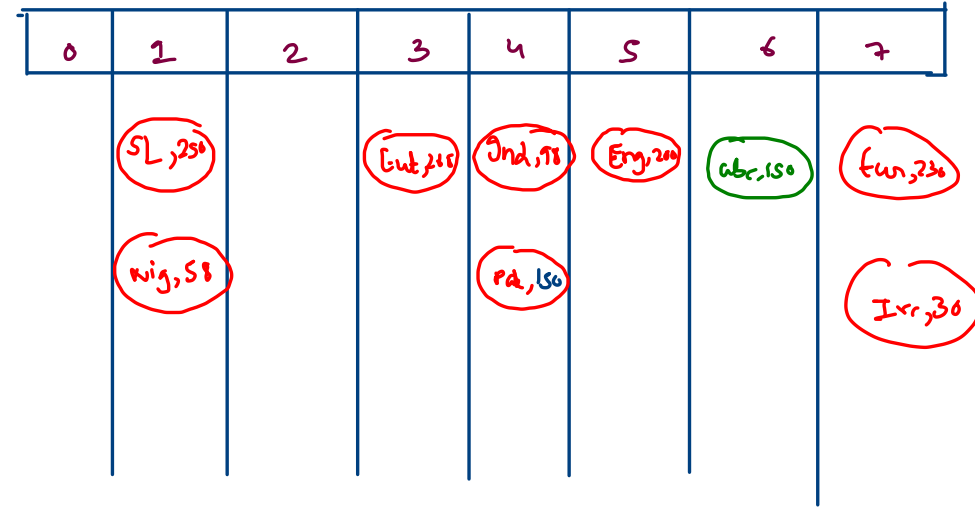
$$\text{Ind} \xrightarrow{hc} 44 \xrightarrow{bi} 0 \quad (44 \cdot 1 \cdot 4 = 20)$$

$$\text{SL} \xrightarrow{hc} 57 \xrightarrow{bi} 1$$

$$\text{Eng} \xrightarrow{hc} 61 \xrightarrow{bi} 1$$

$$\text{Nig} \xrightarrow{hc} 65 \xrightarrow{bi} 1$$

$$\text{Eut} \xrightarrow{hc} 83 \xrightarrow{bi} 3$$



$$\text{Ind} \xrightarrow{hc} 44 \xrightarrow{bi} 44 \cdot 1 \cdot 8 = 4$$

$$\text{SL} \xrightarrow{hc} 57 \xrightarrow{bi} 57 \cdot 1 \cdot 8 = 1$$

$$\text{Eng} \xrightarrow{hc} 61 \xrightarrow{bi} 61 \cdot 1 \cdot 8 = 5$$

$$\text{Nig} \xrightarrow{hc} 65 \xrightarrow{bi} 65 \cdot 1 \cdot 8 = 1$$

$$\text{Eut} \xrightarrow{hc} 83 \xrightarrow{bi} 3$$

$$d = \frac{9}{8} = 1.125$$