MATH 210 Exam 1

February 11, 2016

INSTRUCTIONS

- ◇ Create a new Jupyter notebook, set the kernel to Python 3, present your solutions in the notebook and clearly label the solutions
- ◇ This is **an open book exam** and you may consult any online resources (such as `python.org`), notes from class and past assignments, but **the only rule is that you may NOT communicate with others in the class** (via email, text, Snapchat, Slack, Facebook, etc.)
- ◇ Your solutions should include clear explanations (including proper use of markdown language and LaTeX) and your functions should include comments
- ◇ There are 7 questions and 30 total points: each question is worth 4 points and 2 points will be awarded for the overall presentation of your notebook
- ◇ Submit the completed `.ipynb` file to Connect **by 7:30pm**, sign your name in the space below and submit this page to the instructor

Name:

Student Number: **Signature:**

QUESTIONS

1. Write LaTeX code in a markdown cell to display the triple integral formula (in spherical coordinates) for the volume of a sphere:

$$\frac{4\pi r^3}{3} = \int_0^{2\pi} \int_0^{\pi} \int_0^{r} \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta$$

(Hint: `\rho`, `\phi`, `\theta`, `\frac`)

2. Define a function called `fun` which takes three positive integers m, n and d (in that order `fun(m,n,d)`) and returns the remainder of mn divided by d.

3. Define a function called `divide_either` which takes two positive integers m and n and returns a Python list of positive integers (in increasing order) which divide either m or n. For example, `divide_either(18,15)` returns `[1,2,3,5,6,9,15,18]` since the divisors of 18 are 1, 2, 3, 6, 9, and 18, and the divisors of 15 are 1, 3, 5, and 15.

4. Define a function called `a_sequence` which takes a nonzero number a and a positive integer N (in that order `a_sequence(a,N)`) and returns the Nth term x_N of the recursive sequence

$$x_1 = a$$
$$x_{n+1} = x_n + \frac{2}{x_n}$$

For example, `a_sequence(1,2)` returns `3.0` and `a_sequence(8,4)` returns `8.727928256697213`.

5. (a) Write LaTeX code in a markdown cell to display the Taylor series of arctan:

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

(b) Define a function called `arctan_taylor` which takes a number x (in the closed interval $[-1,1]$) and an integer N (in that order `arctan_taylor(x,N)`) and returns the Nth partial sum of the Taylor series evaluated at x:

$$\sum_{n=0}^{N} \frac{(-1)^n}{2n+1} x^{2n+1}$$

If the input x is outside the interval $[-1,1]$, the function should print an error message and return `None`.

6. (a) Write LaTeX code in a markdown cell to display the following definition:

The **Fourier series** of the **sawtooth wave** is the infinite series:

$$y(t) = \frac{1}{2} - \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2\pi k t)}{k}$$

(b) Plot the following partial sum of the Fourier series of the sawtooth wave:

$$y(t) = \frac{1}{2} - \frac{1}{\pi} \left(\sin(2\pi t) + \frac{\sin(4\pi t)}{2} + \frac{\sin(6\pi t)}{3} \right) \quad \text{for } t \in [0,3] \ .$$

7. An elliptic curve is a curve of the form $y^2 = Ax^3 + Bx + C$ for some coefficients A, B and C. Elliptic curves are very important in number theory and cryptography and, when the coefficients are integers, finding all integer solutions of the elliptic curve is a deep and beautiful problem.

Define a function called `elliptic` which takes integers A, B, and C and a list `x_range` (in that order `elliptic(A,B,C,x_range)`) and returns the list of integer solutions (x,y) of the elliptic curve $y^2 = Ax^3 + Bx + C$ with x contained in the closed interval `x_range`.

For example, `elliptic(1,0,1,[-1,2])` would return `[[-1,0],[0,1],[0,-1],[2,3],[2,-3]]` since the integer solutions of the elliptic curve $y^2 = x^3 + 1$ with $x \in [-1,2]$ are $(-1,0)$, $(0,1)$, $(0,-1)$, $(2,3)$ and $(2,-3)$.