MATH 210 Practice Midterm Exam

INSTRUCTIONS

- Create a new Jupyter notebook, set the kernel to Python 3, present your solutions in the notebook and clearly label the solutions
- This is **an open book exam** and you may consult any online resources (such as `python.org`), notes from class and past assignments, but **the only rule is that you may NOT communicate with others in the class** (via email, text, Snapchat, Slack, Facebook, etc.)
- Your solutions should include clear explanations (including proper use of markdown language and LaTeX) and your functions should include comments
- There are 7 questions and each is worth 5 points for 35 total points
- Submit the completed `.ipynb` file to Connect, sign your name in the space below and submit this page to the instructor (*Note: This is a practice exam and is not to be handed in. This instruction is simply what will appear on the actual midterm exam.*)

Name: **Signature:**

QUESTIONS

1. Write LaTeX code in a markdown cell to display the Cauchy stress tensor:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

(Hint: `\boldsymbol`, `\sigma`, `\tau`, `\begin{bmatrix}` ... `\end{bmatrix}`)

2. (a) Write a list comprehension to create the Python list:

`[100,81,64,49,36,25,16,9,4,1,0,1,4,9,16,25,36,49,64,81,100]`

(b) Write a list comprehension to create the Python list of all positive integers $n < 150$ (in increasing order) which are multiples of either 13 or 17:

`[13,17,26,34,39,51,52,65,68,78,85,91,102,104,117,119,130,136,143]`

3. Define a function called `fun` which takes three positive integers n, k and d (in that order `fun(n,k,d)`) and returns the remainder of n^k divided by d.

4. Given a positive integer a, consider the recursive sequence of integers given by $x_1 = a$ and

$$x_{n+1} = \begin{cases} \dfrac{x_n}{2} & \text{if } x_n \text{ is even} \\ 5x_n - 1 & \text{if } x_n \text{ is odd} \end{cases}$$

Define a function called `a_sequence` which takes positive integers a and N and returns a Python list consisting of the first N integers in the sequence defined above starting at a. For example, if $a = 20$ and $N = 5$ then the sequence goes $x_1 = 20$, $x_2 = 10$, $x_3 = 5$, $x_4 = 24$, and $x_5 = 12$ and so `a_sequence(20,5)` returns `[20, 10, 5, 24, 12]`. (Hint: Use the quotient operator // to produce an integer when dividing integers.)

5. (a) Define a function called `next_prime` which takes a positive integer N and returns the smallest prime number p strictly greater than N (ie. the next prime after N).
   (b) Find the next prime number after 1327.

6. (a) Write LaTeX code in a markdown cell to display the following definition and theorem:

   The **Riemann zeta function** is defined by the infinite series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

   and satisfies the functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\,\zeta(1-s)$$

   (Hints: `\Gamma`, `\zeta`, `\left(` ... `\right)`)

   (b) Define a function called `partial_zeta` which takes a number $s > 1$ and an integer N (in that order `partial_zeta(s,N)`) and returns the partial sum

$$\sum_{n=1}^{N} \frac{1}{n^s}$$

   For example, `partial_zeta(2,4)` returns the value `1.4236111111111112` which is the float representation of

$$\sum_{n=1}^{4} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$$

   (c) Use the function `partial_zeta` from part (b) to compute the 1000th partial sum of the series defining $\zeta(4)$

$$\sum_{n=1}^{1000} \frac{1}{n^4}$$

   and compare your result to the special value $\zeta(4) = \dfrac{\pi^4}{90}$.

7. Plot the parametric curve given by $x = \cos(t) - \cos^3(3t)$ and $y = \sin(3t) - \sin^3(t)$ for $t \in [0, 2\pi]$.