MATH 210 Assignment 5

Numerical integration and differentiation

INSTRUCTIONS

- Create a new Python 3 Jupyter notebook
- Answer each question in the Jupyter notebook and clearly label the solutions with headings
- Functions should include documentation strings and comments
- There are 25 total points and each question is worth 5 points
- Submit the `.ipynb` file to Connect by **11pm Monday, March, 13, 2017**
- You may work on these problems with others but you must write your solutions on your own

QUESTIONS

1. (a) Write LaTeX code to display the formula

$$\int_0^\infty \frac{\arctan(px)\arctan(qx)}{x^2} dx = \frac{\pi}{2} \ln\left(\frac{(p+q)^{p+q}}{p^p q^q} \right) \quad , \ p > 0, \ q > 0$$

   (b) Write a function called `pq_integral` which takes input parameters p and q and returns an approximation of the infinite integral (using the function `scipy.integration.quad`)

$$\int_0^\infty \frac{\arctan(px)\arctan(qx)}{x^2} dx$$

   If $p \leq 0$ or $q \leq 0$, then the function should display an error message and return `None`.

2. (a) Write LaTeX code to display the formula

$$\int_0^\infty \frac{\ln x}{a^2 + b^2 x^2} = \frac{\pi}{2ab} \ln\left(\frac{a}{b} \right) \quad , \ ab > 0$$

   (b) Write a function called `ab_integral` which takes input parameters a and b and returns an approximation of the infinite integral (using the function `scipy.integration.quad`)

$$\int_0^\infty \frac{\ln x}{a^2 + b^2 x^2}$$

   If $ab \leq 0$, then the function should display an error message and return `None`.

3. Write a function called `derivatives` which takes input parameters f, a, n and h (with default value `h = 0.001`) and returns approximations of the derivatives $f'(a)$, $f''(a)$, $\dots$, $f^{(n)}(a)$ (as a NumPy array) using the formula

$$f^{(n)}(a) \approx \frac{1}{2^n h^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f\left(a + (n-2k)h\right)$$

   Use either `scipy.misc.factorial` or `scipy.misc.comb` to compute n choose k: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

4. Write a function called `taylor` which takes input parameters f, a, n and L and plots both $f(x)$ and the Taylor polynomial $T_n(x)$ of $f(x)$ at $x = a$ of degree n

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

   on the interval $[a - L, a + L]$ (in the same figure).

5. Write a function called `newton` which takes input parameters f, x_0, `tolerance` and `max_iter` and performs Newton's method to approximate a root r where $f(r) = 0$. In other words, compute the values of the recursive sequence starting at x_0 and defined by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

   The desired result is that the method converges to an approximate root of $f(x)$ however there are several possibilities:

   (a) The sequence reaches the desired tolerance $f(x_n) <$ `tolerance` and `newton` returns the value x_n

   (b) The number of iterations exceeds the maximum number of iterations `max_iter` and `newton` returns `None`

   (c) A zero derivative is computed $f'(x_n) = 0$ and `newton` returns `None`