

Solution 5: BFS & DFS

Q1 Manually execute breadth-first search on the undirected graph in Figure 5.1, starting from vertex s . Then, use it as an example to illustrate the following properties:

- (a) The results of breadth-first search may depend on the order in which the neighbours of a given vertex are visited.
- (b) With different orders of visiting the neighbours, although the BFS tree may be different, the distance from starting vertex s to each vertex will be the same.

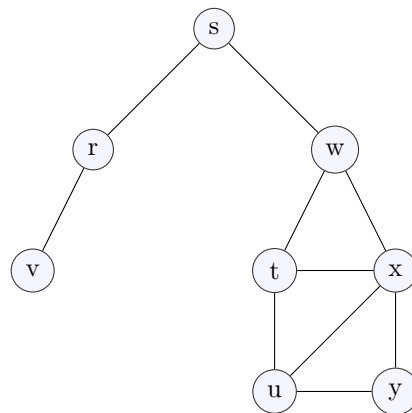


Figure 5.1: The graph for Q1

Figure 5.2: Graph for S1

- S1**
- When the queue is empty, the BFS is finished.
 - The edges of BFS tree are shown in red.
 - Likewise, a BFS tree can be constructed if the neighbors are visited in the reverse alphabetical order (an exercise for the students).
 - The two trees differ in that vertex *u* is adjacent with *t* in the left tree, but adjacent with *x* in the right tree.
 - The distance from starting vertex *s* to each other vertex is equal in the two trees.

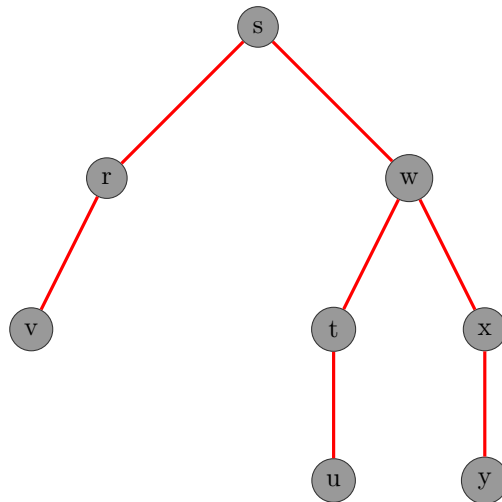


Figure 5.3: Visiting neighbors in alphabetical order

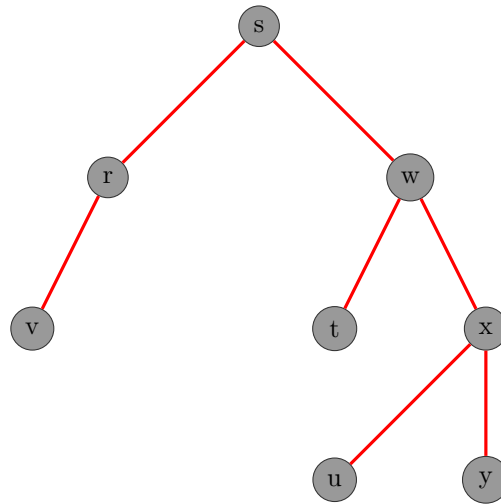


Figure 5.4: Visiting neighbors in reverse alphabetical order

Q2 Give a pseudocode of checking whether an undirected graph is connected or not by using breadth-first search.

S2 Idea: Apply BFS in any vertex to find the spanning tree, and check whether all the vertices of the graph are in this tree.

Algorithm 1 Check connected-graph (BFS)

```

function CONNECTGRAPH(Graph  $G$ )
    create a Queue,  $Q$ 
    select an arbitrary vertex  $v$ 
    enqueue  $v$  into  $Q$ 
    mark  $v$  as visited
    while  $Q$  is not empty do
        dequeue a vertex from  $Q$  and denoted as  $w$ 
        for each unvisited vertex  $u$  adjacent to  $w$  do
            mark  $u$  as visited
            enqueue  $u$  into  $Q$ 
        end for
    end while
    for each vertex  $z$  in  $G$  do
        if  $z$  is not visited then
            return False
        end if
    end for
    return True
end function

```

Q3 Give a pseudocode of finding a simple path connecting two given vertices in an undirected graph by using depth-first search.

S3 Idea: Use DFS to traverse the paths starting from v and check whether any of them contains w .

Algorithm 2 Simple path (DFS)

```
function SIMPLEPATH(Graph  $G$ , Vertex  $v$ , Vertex  $w$ )
    create a Stack,  $S$ 
    push  $v$  into  $S$ 
    mark  $v$  as visited
    while  $S$  is not empty do
        peek the stack and denote the vertex as  $x$ 
        if  $x == w$  then
            while  $S$  is not empty do
                pop a vertex from  $S$ 
                peek the stack
                print the link
            end while
            return Found
        end if
        if no unvisited vertices are adjacent to  $x$  then
            pop a vertex from  $S$ 
        else
            push an unvisited vertex  $u$  adjacent to  $x$ 
            mark  $u$  as visited
        end if
    end while
    return Not Found
end function
```
