SC2001/CX2101 Algorithm Design and Analysis

Tutorial 3: Analysis Techniques

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This tutorial helps you develop skills in the learning outcome of the course: "Able to conduct complexity analysis of recursive algorithms: solve recurrences using the substitution method, the iteration method, the master theorem, the characteristic equation."

1. Solve the following recurrences by the iteration method
    1) $T(1) = 1$, and for $n \geq 2$, $T(n) = 3T(n-1) + 2$
    2) $T(1) = 1$, and for $n \geq 2$, a power of 2, $T(n) = 2T(n/2) + 6n$

2. Solve the recurrences in Question 1 by the substitution method.

3. Solve the following recurrences by the master method.
    1) $W(n) = W(n/3) + 5$
    2) $T(n) = 2T(n/2) + n/4$
    3) $W(n) = 2W(n/4) + \sqrt{n^3}$

4. Determine which of the following are linear homogeneous recurrence relations with constant coefficients. Also find the degree of those that are.
    1) $a_n = 4a_{n-2} + 5a_{n-3}$
    2) $a_n = 2na_{n-1} + a_{n-2}$
    3) $a_n = a_{n-1} + a_{n-4}$
    4) $a_n = a^2_{n-1} + a_{n-2}$
    5) $a_n = a_{n-2} + n$

5. Solve the following recurrence relations together with the initial conditions given. (Due to time constraints, we may not cover every part in the tutorial class.)
    1) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$
    2) $a_n = 4a_{n-2}$ for $n \geq 2$, $a_0 = 6$, $a_1 = 8$
    3) $a_n = 2a_{n-1} - a_{n-2}$ for all $n \geq 2$, $a_0 = 1$, $a_1 = 3$