

Tutorial 6: Introduction to NP

This tutorial helps you develop skills in the learning outcome of the course: “Able to classify some decision problems into P or NP problems and apply greedy heuristic approach to solve NP-complete problems”.

1. Problem: Given a network of cities G and a positive integer k . Are the shortest paths between all pairs of cities not longer than k ? Is this problem in the class of P or NP? Justify your answers.
2. Problem: Given a graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, and a positive integer k . Is there a way to colour the vertices of the graph using k colours or less such that adjacent vertices have different colours? Show that this K -colouring problem is in NP.
3. Why do we say NP-Complete problems are the hardest problems in NP?
4. Implement the `shortestLinkTSP()` algorithm below (slide 29 of lecture notes) to find a TSP tour in graph G . You may consider using a minimizing heap, a union-find data structure and other data structures in your implementation of the algorithm.

```

shortestLinkTSP(V, E, W)
{
    R = E;
    C = empty; // C is a forest

    while (no. of edges in C < |V| - 1) {
        remove the lightest edge vw from R;
        if (vw does not form a cycle in C and
            vw would not be the third edge in C incident on v or w)
            add edge vw to C;
    }

    add edge connecting the end points to C;
    return C;
}

```

5. Greedy heuristic algorithms are often used to solve problems because of its simplicity. Design a greedy heuristic method to solve the chain matrix multiplication problem where array d is used to store the dimensions of n matrices.