SC2207/CZ2007 Introduction to Database Systems (Week 2)

Topic 2: Functional Dependencies (1)

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This Lecture

- Data anomalies
- Functional dependencies
- Armstrong's axioms

Database Design

- Applications → ER diagrams → tables
- It works in general, but sometimes things may go wrong:
  - ER diagrams may not be well designed
  - Not all requirements can be represented in ER diagram
  - Conversion from ER diagrams to tables may not be properly done
- As a result, resulting tables may have various problems

Data Anomalies

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:
  (NRIC, PhoneNumber), one composite key
- There are several anomalies in the table
- First, redundancy:
  - Alice's address is duplicated, attribute values appear multiple times in different rows (columns) in table

Data Anomalies

Name	<u>NRIC</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:
  (NRIC, PhoneNumber), one composite key
- Second, update anomalies:
  - We may accidentally update one of Alice's addresses, leaving the other unchanged

Data Anomalies

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:
  (NRIC, PhoneNumber), one composite key
- Third, deletion anomalies:
  - Bob no longer uses a phone
  - Can we remove Bob's phone number?
  - No. (Note: Primary key attributes cannot be NULL)

Data Anomalies

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:
  (NRIC, PhoneNumber), one composite key
- Fourth, insertion anomalies:
  - Name = Cathy, NRIC = 9394, HomeAddress = YiShun
  - Can we insert this information into the table?
  - No. (Note: Primary key attributes cannot be NULL)

Normalization

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- How do we get rid of those anomalies?
- Normalize the table (i.e., decompose it)

Name	NRIC	HomeAddress
Alice	1234	Jurong East
Bob	5678	Pasir Ris

NRIC	PhoneNumber
1234	67899876
1234	83848384
5678	98765432

Effects of Normalization

Name	NRIC	HomeAddress
Alice	1234	Jurong East
Bob	5678	Pasir Ris

NRIC	PhoneNumber
1234	67899876
1234	83848384
5678	98765432

- Duplication?
  - No. (Alice’s address is no longer duplicated.)
- Update anomalies?
  - No. (There is only one place where we can update the address of Alice)
- Deletion anomalies?
  - No. (We can freely delete Bob’s phone number)
- Insertion anomalies?
  - No. (We can insert an individual without a phone)

This Lecture

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Road Map

- We will discuss
  - How to decide whether a table is good
  - If a table is not good, how to normalize it
- The plan
  - First, basic concepts (e.g., functional dependencies)
  - Second, reasoning with basic concepts
  - Finally, the real meat (e.g., normalization)

Functional Dependencies: Intuition

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Why was this table bad?
- It has a lot of anomalies
- Why does it have those anomalies?
- Intuitive answer: It contains a bad combination of attributes

Functional Dependencies: Intuition

Name	NRIC	PhoneNumber	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- In general, how do we know whether a combination of attributes is bad?
- We need to check the correlations among those attributes
- What kind of correlations?
- Functional dependencies (FD)

Functional Dependencies (FD)

- Consider two attributes in practice: NRIC, Name
- Given an NRIC, can we always uniquely identify the name of the person?
- Theoretically yes -- We just need help from ICA
- Given a Name, can we always uniquely identify the NRIC of the person
- In general no -- Different people can have the same name
- Therefore
  - NRIC determines Name, but not vice versa
  - Functional dependencies:
    NRIC $\rightarrow$ Name, but not Name $\rightarrow$ NRIC

Functional Dependencies (FD)

- Consider three attributes in practice:
  - StudentID, Name, Address, PostalCode
- Functional Dependencies:
  - StudentID → Name, Address, PostalCode
  - Address → PostalCode

Formal Definition of FD

- Attributes $\boxed{A_1, A_2, \ldots, A_m}$, $\boxed{B_1, B_2, \ldots, B_n}$
- $\boxed{A_1A_2\ldots A_m} \rightarrow \boxed{B_1B_2\ldots B_n}$
- Meaning: There do not exist two objects that
  - Have the same values on $A_1, A_2, \ldots, A_m$
  - but different values on $B_1, B_2, \ldots, B_n$
- Previous example: NRIC → Name
- Meaning: There do not exist two persons that
  - have the same values on NRIC
  - but different values on Name

Name	Category	Color	Department	Price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office Supplies	59

- Functional dependencies that hold on the table:
  - Category → Department ✓
  - Category, Color → Price ✓
  - Price → Color ✓
  - Name → Color ✓
  - Department, Category → Name ✗
  - Color, Department → Name, Price, Category ✓

Where Do FDs Come From?

- From common sense, knowledge in real world
- From application's requirements
- Example
  - Purchase(CustomerID, ProductID, ShopID, Price, Date)
  - Requirement: Each shop can sell at most one product
  - FD implied: ShopID → ProductID

Where Do FDs Come From?

ShopID	ProductID
S1	P1
S1	P2

ShopID	ProductID
S1	P1
S2	P1

- F kn
- F s re
- Example
  - Purchase(CustomerID, ProductID, ShopID, Price, Date)
  - Requirement: Each shop can sell at most one product
  - FD implied: ShopID → ProductID

Where Do FDs Come From?

- Example
  - Purchase(CustomerID, ProductID, ShopID, Price, Date)
  - Requirement: No two customers buy the same product
  - FD implied: ProductID $\rightarrow$ CustomerID

Where Do FDs Come From?

- Example
  - Purchase(CustomerID, ProductID, ShopID, Price, Date)
  - Requirement: No two customers buy the same product
  - FD implied: ProductID → CustomerID

CustomerID	ProductID
C1	P1
C2	P1

CustomerID	ProductID
C1	P1
C1	P2

Where Do FDs Come From?

- Example
  - Purchase(CustomerID, ProductID, ShopID, Price, Date)
  - Requirement: No shop will sell the same product to the same customer on the same date at two different prices
  - FD implied:
    CustomerID, ProductID, ShopID, Date → Price

Where Do FDs Come From?

CustomerID	ProductID	ShopID	Date	Price
C1	P1	S1	D1	99
C1	P1	S1	D1	33

- E…
  - Purchase(CustomerID, ProductID, ShopID, Price, …

CustomerID	ProductID	ShopID	Date	Price
C1	P1	S1	D1	99
C2	P1	S1	D1	33

  - …luct …vo different prices
  - FD implied: CustomerID, ProductID, ShopID, Date → Price

Roadmap

- To decide whether or not a table is good, we need to examine the relationships among attributes, i.e., the FDs
- Now we know what FDs are, and where they are from
- How do we use FDs to check whether a table is good?
- We need to learn how to reason with FDs

This Lecture

- Data anomalies
- Functional dependencies
- Armstrong's axioms

Reasoning with FDs

- How to reason with FDs
- Example:
  - Given:
    NRIC → Address,   Address → PostalCode
  - We have: NRIC → PostalCode
- In general
  - Given A→B, B→C
  - We always have A→C
- We will discuss how we can do this kind of derivations

Reasoning with FDs

- Why do we care about such derivations?
- Answer: it is needed in deciding whether a table is "bad" or not
- Intuitive explanation:
  - Suppose that $A \rightarrow C$ is an FD that makes a table "bad"
  - But we are only given $A \rightarrow B$ and $B \rightarrow C$
  - If we do not know that $A \rightarrow C$ is implied by $A \rightarrow B$ and $B \rightarrow C$, then we may misjudge the table to be a "good" one

Reasoning with FDs

- Armstrong’s Axioms
  - Three axioms for FD reasoning
  - Easy to understand, but not easy to apply

Armstrong's Axioms

- Axiom of Reflexivity
  - A set of attributes → A subset of the attributes
- Example
  - NRIC, Name → NRIC
  - StudentID, Name, Age → Name, Age
  - ABCD → ABC
  - ABCD → BCD
  - ABCD → AD

Armstrong's Axioms

- Axiom of Augmentation
  - Given A → B
  - We always have AC → BC, for any C
- Example
  - If NRIC → Name
  - Then NRIC, Age → Name, Age
  - and NRIC, Salary, Weight → Name, Salary, Weight
  - and NRIC, Addr, Postal → Name, Addr, Postal

Armstrong's Axioms

- Axiom of Transitivity
  - Given A → B and B → C
  - We always have A → C
- Example
  - If NRIC → Addr, and Addr → Postal
  - Then NRIC → Postal

Reasoning with FDs

- Given $A \rightarrow B$, $BC \rightarrow D$
- Can you prove that $AC \rightarrow D$?
- Proof
  - Given $A \rightarrow B$, we have $AC \rightarrow BC$ (Augmentation)
  - Given $AC \rightarrow BC$ and $BC \rightarrow D$, we have $AC \rightarrow D$ (Transitivity)

Reasoning with FDs

- Given A→B, D→C
- Can you prove that AD → BC?
- Proof
  - Given A→B, we have AD → BD (Augmentation)
  - Given AD→BD, we have AD → B (Reflexivity)
  - Given D→C, we have AD → AC (Augmentation)
  - Given AD → AC, we have AD → C (Reflexivity)
  - In other words, AD decides B and C
  - Therefore, AD → BC

Reasoning with FDs

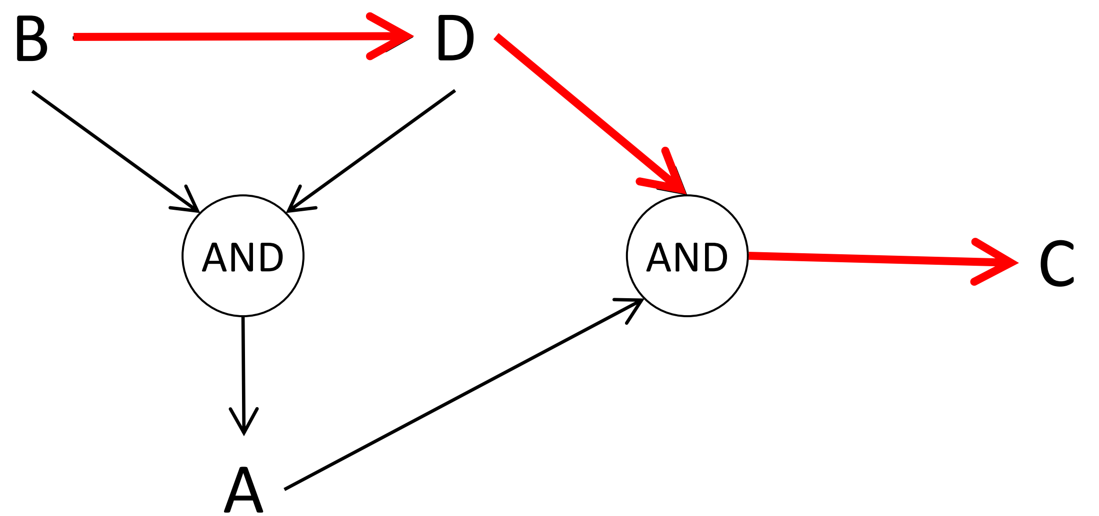
- Given $A \rightarrow C$, $AC \rightarrow D$, $AD \rightarrow B$
- Can you prove that $A \rightarrow B$?
- Proof
  - Given $A \rightarrow C$, we have $A \rightarrow AC$ (Augmentation)
  - Given $A \rightarrow AC$ and $AC \rightarrow D$, we have $A \rightarrow D$ (Transitivity)
  - Given $A \rightarrow D$, we have $A \rightarrow AD$ (Augmentation)
  - Given $A \rightarrow AD$ and $AD \rightarrow B$, we have $A \rightarrow B$ (Transitivity)

Reasoning with FD

- Four attributes: A, B, C, D
- Given: B→D, DB→A, AD→C
- Can you prove B→C?
- Doable with Armstrong's axioms, but troublesome
- We will discuss a more convenient approach

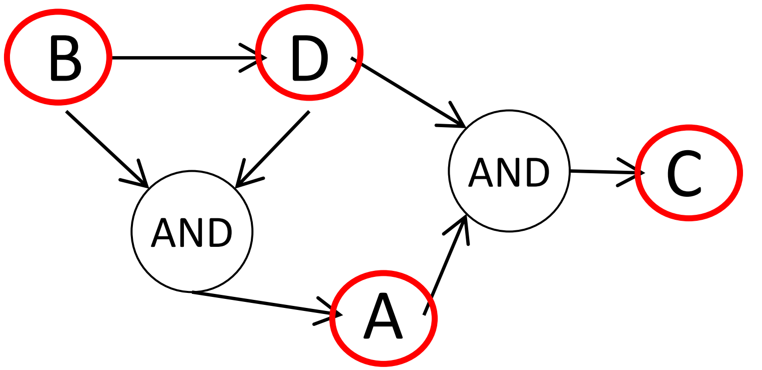
An Intuitive Solution

- Four attributes: A, B, C, D
- Given: B→D, DB→A, AD→C
- Can you prove B→C?



Steps of the Intuitive Solution

- Four attributes: A, B, C, D
- Given: B→D, DB→A, AD→C
- Can you prove B→C?



- First, activate B
  - Activated set = { B }
- Second, activate whatever B can activate
  - Activated set = { B, D }, since B→D
- Third, use all activated elements to activate more
  - Activated set = { B, D, A }, since DB→A
- Repeat the third step, until no more activation is possible
  - Activated set = { B, D, A, C }, since AD→C; done

Exercise

- Given: A→C, C→B, B→D, D→E, E→A
- Can you prove C→ABE?
- We start with {C}
- Since C→B, we have {C, B}
- Since B→D, we have {C, B, D}
- Since D→E, we have {C, B, D, E}
- Since E→A, we have {C, B, D, E, A}
- A, B, E are all in the set, so C→ABE holds

Exercise

- Given: $C \rightarrow D$, $AD \rightarrow E$, $BC \rightarrow E$, $E \rightarrow A$, $D \rightarrow B$
- Can you prove $C \rightarrow A$?
- We start with {C}
- Since $C \rightarrow D$, we have {C, D}
- Since $D \rightarrow B$, we have {C, D, B}
- Since $BC \rightarrow E$, we have {C, D, B, E}
- Since $E \rightarrow A$, we have {C, D, B, E, A}
- A is in the set, so $C \rightarrow A$ holds

Exercise

- Given: C→D, AD→E, BC→E, E→A, D→B, B→F
- Can you prove D→C?
- We start with {D}
- Since D→B, we have {D, B}
- Since B→F, we have {D, B, F}
- What else?
- No more.
- {D, B, F} is all what can be decided by D
- We refer to {D, B, F} as the closure of D

To continue in

Topic 2: Functional Dependencies (2)