CZ2007 Introduction to Database Systems (Week 4)

Topic 4: Third Normal Form (1)

This Lecture

- 3NF
- 3NF Decomposition

Third Normal Form (3NF)

- A relaxation of BCNF that
  - Is less strict
  - Allows decompositions that always preserve functional dependencies

1NF, 2NF

- **Key-attribute**: An attribute in a multi-attribute key
- Key-attribute(s) = Partial key or part of a key
- 1NF: All attributes have atomic values
- 2NF: Every non-key attribute is dependent on the whole of **EVERY** candidate key
  - Even so, may still have additional dependencies, such as (non-key-attribute X) → (non-key-attribute Y) in relation

Third Normal Form (3NF)

- Definition: A table satisfies 3NF, if and only if for every non-trivial X→Y
  - Either X contains a key
  - Or each attribute in Y is contained in a key
- Example:
  - Given FDs: C→B, AB→C, BC→C
  - Keys: {AB}, {AC}
  - AB→C is OK, since AB is a key of R
  - C→B is OK, since B is in a key of R
  - BC→C is OK, since C is in AC and in BC (it is also trivial)
  - So R is in 3NF

R

A	B	C

3NF, BCNF

- 3NF: 2NF + all key-attributes determined **ONLY** by candidate keys (in whole or in part)
  - (non-key-attribute X) → (non-key-attribute Y) cannot exist anymore, RHS must be key attribute in 3NF
  - But candidate keys may have **overlapping** attributes
  - May result in key-attribute(s) of one key depends on key-attribute(s) of another key (this dependency is eliminated in BCNF)
- BCNF: In all dependencies (FDs), LHS must contain key (cannot depend on partial key)

Third Normal Form (3NF)

- Definition: A table satisfies 3NF, if and only if for every non-trivial $X \rightarrow Y$
  - Either X contains a key
  - Or each attribute in Y is contained in a key
- Another Example:
  - Given FDs: $A \rightarrow B$, $B \rightarrow C$
  - Keys: {A}
  - $A \rightarrow B$ is OK, since A is a key of R
  - $B \rightarrow C$ is not OK, since C is NOT in a key of R, and it is NOT in the left hand side
  - So R is NOT in 3NF

R

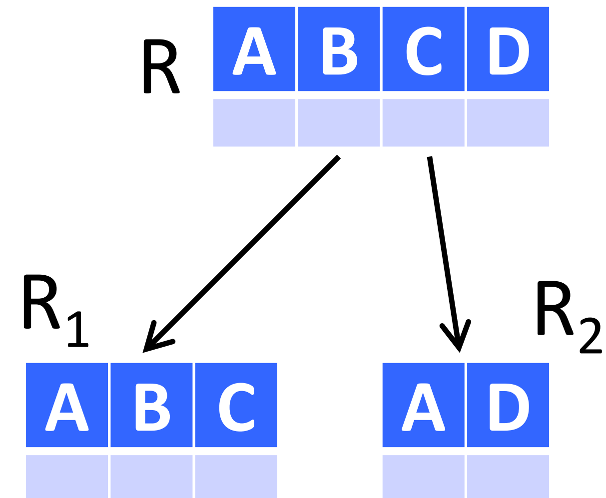
A	B	C

This Lecture

- 3NF
- 3NF Decomposition

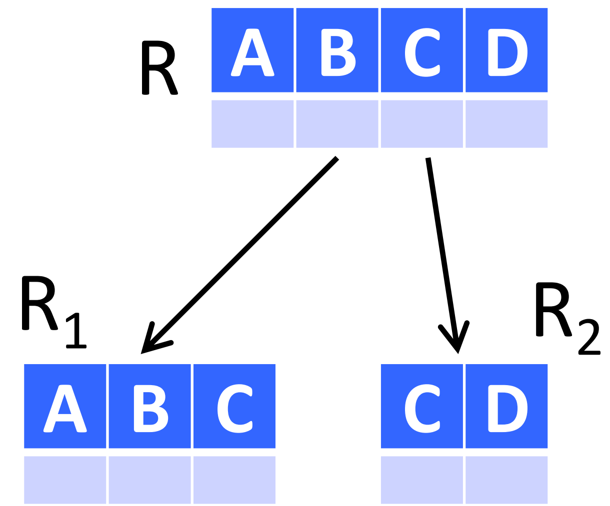
3NF Decomposition

- Given: A table NOT in 3NF
- Objective: Decompose it into smaller tables that are in 3NF
- Example
  - Given: R(A, B, C, D)
  - FDs: AB→C, C→B, A→D
  - Keys: {AB}, {AC}
  - R is not in 3NF, due to A→D
  - 3NF decomposition of R: R_1(A, B, C), R_2(A, D)



3NF Decomposition Algorithm

- Given: A table R, and a set S of FDs
  - e.g., R(A, B, C, D)
    S = {A→BD, AB→C, C→D, BC→D}
- Step 1: Derive a minimal basis of S
  - e.g., a minimal basis of S is
    {A→B, A→C, C→D}
- Step 2: In the minimal basis, combine the FDs whose left hand sides are the same
  - e.g., after combining A→B and A→C, we have {A→BC, C→D}
- Step 3: Create a table for each FD remained
  - R_1(A, B, C), R_2(C, D)
- Step 4: If none of the tables contain a key of the original table R, create a table that contains a key of R
- Step 5: Remove redundant tables (schema is a subset of another)



Minimal Basis

- Given a set S of FDs, the minimal basis of S is a simplified version of S
- Previous example:
  - S = {A→BD, AB→C, C→D, BC→D}
  - A minimal basis: {A→B, A→C, C→D}
- How simplified?
- Three conditions.
- Condition 1: For any FD in the minimal basis, its right hand side has only one attribute.
- Example in S: A→BD does not satisfy this condition
- That is why A→BD is not in the minimal basis

Minimal Basis

- Previous example:
  - S = {A→BD, AB→C, C→D, BC→D}
  - A minimal basis: {A→B, A→C, C→D}
- Condition 2: No FD in the minimal basis is redundant.
- That is, no FD in the minimal basis can be derived from the other FDs.
- Example in S: BC→D can be derived from C→D
- That is why BC→D is not in the minimal basis

Minimal Basis

- Previous example:
  - S = {A→BD, AB→C, C→D, BC→D}
  - A minimal basis: {A→B, A→C, C→D}
- Condition 3: For each FD in the minimal basis, none of the attributes on the left hand side is redundant
- That is, for any FD in the minimal basis, if we remove an attribute from the left hand side, then the resulting FD is a new FD that cannot be derived from the original set of FDs
- Example:
  - S contains AB→C
  - If we remove B from the left hand side, we have A→C
  - A→C can be derived from S, as $\{A\}^+ = \{ABDC\}$
  - This indicates that A→C is "hidden" in S
  - Hence, we can replace AB→C with A→C, as A→C is "simpler"
  - This is why AB→C is not in the minimal basis

Algorithm for Minimal Basis

- Given: a set S of FDs
- Example: S = {A→BD, AB→C, C→D, BC→D}
- Step 1: Transform the FDs, so that each right hand side contains only one attribute
- Result: S = {A→B, A→D, AB→C, C→D, BC→D}
- Reason:
  - Condition 1 for minimal basis:
    The right hand side of each FD contains only one attribute

Algorithm for Minimal Basis

- Result of Step 1:
  - S = {A→B, A→D, AB→C, C→D, BC→D}
- Step 2: Remove redundant FDs
- Is A→B redundant?
- i.e., is A→B implied by other FDs in S?
- Let's check
- Without A→B, we have {A→D, AB→C, C→D, BC→D}
- Given those FDs, we have $\{A\}^+ = \{AD\}$, which does not contain B
- Therefore, A→B is not implied by the other FDs

Algorithm for Minimal Basis

- Result of Step 1:
  - S = {A→B, A→D, AB→C, C→D, BC→D}
- Continue Step 2: Remove redundant FDs
- Is A→D redundant?
- i.e., is A→D implied by other FDs in S?
- Let's check
- Without A→D, we have {A→B, AB→C, C→D, BC→D}
- Given those FDs, we have $\{A\}^+ = \{ABCD\}$, which contains D
- Therefore, A→D is implied by the other FDs
- Hence, A→D is redundant and should be removed
- Result: S = {A→B, AB→C, C→D, BC→D}

Algorithm for Minimal Basis

- Result of the last step:
  - S = {A→B, AB→C, C→D, BC→D}
- Continue Step 2: Remove redundant FDs
- Is AB→C redundant?
- i.e., is AB→C implied by other FDs in S?
- Let's check
- Without AB→C, we have {A→B, C→D, BC→D}
- Given those FDs, we have $\{AB\}^+ = \{AB\}$, which does not contain C
- Therefore, AB→C is NOT implied by the other FDs
- Hence, AB→C is not redundant and should not be removed

Algorithm for Minimal Basis

- Result of the last step:
  - S = {A→B, AB→C, C→D, BC→D}
- Continue Step 2: Remove redundant FDs
- Is C→D redundant?
- i.e., is C→D implied by other FDs in S?
- Let's check
- Without C→D, we have {A→B, AB→C, BC→D}
- Given those FDs, we have $\{C\}^+ = \{C\}$, which does not contain D
- Therefore, C→D is NOT implied by the other FDs and should not be removed

Algorithm for Minimal Basis

- Result of the last step:
  - S = {A→B, AB→C, C→D, BC→D}
- Continue Step 2: Remove redundant FDs
- Is BC→D redundant?
- i.e., is BC→D implied by other FDs in S?
- Let's check
- Without BC→D, we have {A→B, AB→C, C→D}
- Given those FDs, we have $\{BC\}^+ = \{BCD\}$, which contains D
- Therefore, BC→D is implied by the other FDs
- Hence, BC→D is redundant and should be removed
- Result: S = {A→B, AB→C, C→D}

Algorithm for Minimal Basis

- Result of Step 2:
  - S = {A→B, AB→C, C→D}
- Step 3: Remove redundant attributes on the left hand side (lhs) of each FD
- Only AB→C has more than one attribute on the lhs
- Let's check
- Is A redundant?
- If we remove A, then AB→C becomes B→C
- Whether this removal is OK depends on whether B→C is "hidden" in S already
- If B→C is "hidden" in S, then the removal of A is OK, (since the removal does not add extraneous information into S)
- Is B→C "hidden" in S?
- Check: Given S, we have $\{B\}^+ = \{B\}$, which does NOT contain C
- Therefore, B→C is not "hidden" in S, and hence, A is NOT redundant

Algorithm for Minimal Basis

- Result of Step 2:
  - S = {A→B, AB→C, C→D}
- Step 3: Remove redundant attributes on the left hand side (lhs) of each FD
- Only AB→C has more than one attribute on the lhs
- Let’s check
- Is B redundant?
- If we remove B, then AB→C becomes A→C
- Whether this is OK depends on whether A→C is “hidden” in S
- Is A→C “hidden” in S?
- Check: Given S, we have $\{A\}^+$ = {ABCD}, which contains C
- Therefore, A→C is “hidden” in S
- Hence, we can simplify AB→C to A→C
- Final minimal basis: S = {A→B, A→C, C→D}

To continue in

Topic 4: Third Normal Form (2)