



# **Tutorial 4**

## **Normalisation**

**CZ2007**  
**Introduction to Databases**



# Tutorial 4

- BCNF: In all dependencies (FDs), LHS must contain key (cannot depend on partial key)

## Third Normal Form (3NF)

- Definition: A table satisfies 3NF, if and only if for every non-trivial  $X \rightarrow Y$ 
  - Either X contains a key
  - Or each attribute in Y is contained in a key
- Example:
  - Given FDs:  $C \rightarrow B$ ,  $AB \rightarrow C$ ,  $BC \rightarrow C$
  - Keys:  $\{AB\}$ ,  $\{AC\}$
  - $AB \rightarrow C$  is OK, since **AB** is a key of R
  - $C \rightarrow B$  is OK, since **B** is in a key of R
  - $BC \rightarrow C$  is OK, since **C** is in AC and in BC (it is also trivial)
  - So R is in 3NF

R	A	B	C

## Minimal Basis

- Given a set S of FDs, the **minimal basis** of S is a **simplified** version of S
- Previous example:
  - $S = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
  - A minimal basis:  $\{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$
- Three conditions.
- Condition 1: For any FD in the minimal basis, its right hand side has only one attribute.
- Condition 2: No FD in the minimal basis is redundant.
- Condition 3: For each FD in the minimal basis, none of the attributes on the left hand side is redundant

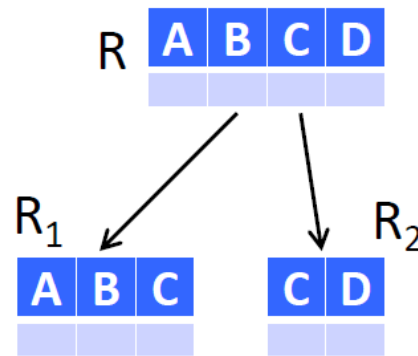
## Algorithm for Minimal Basis

- Step 1: Transform the FDs, so that each right hand side contains only one attribute
- Step 2: Remove redundant FDs
- Step 3: Remove redundant attributes on the left hand side (lhs) of each FD

# Tutorial 4

## 3NF Decomposition Algorithm

- Given: A table R, and a set S of FDs
  - e.g.,  $R(A, B, C, D)$   
 $S = \{A \rightarrow BD, AB \rightarrow C, C \rightarrow D, BC \rightarrow D\}$
- Step 1: Derive a **minimal basis** of S
  - e.g., a minimal basis of S is  $\{A \rightarrow B, A \rightarrow C, C \rightarrow D\}$
- Step 2: In the minimal basis, combine the FDs whose left hand sides are the same
  - e.g., after combining  $A \rightarrow B$  and  $A \rightarrow C$ , we have  $\{A \rightarrow BC, C \rightarrow D\}$
- Step 3: Create a table for each FD remained
  - $R_1(A, B, C), R_2(C, D)$
- Step 4: If none of the tables contain a key of the original table R, create a table that contains a key of R
- Step 5: Remove redundant tables (schema is a subset of another)



## BCNF vs. 3NF

- BCNF Decomposition:
  - Avoids insertion, deletion, and update anomalies
  - Eliminates most redundancies
  - But does not always preserve all FDs
- 3NF Decomposition:
  - Avoids insertion, deletion, and update anomalies
  - May lead to a bit more redundancy than BCNF
  - Always preserve all FDs

# Question 1

- A medical clinic database schema contains the following:
  - ▣ APPOINTMENT (patient-id, patient-name, doctor-id, doctor-name, appointment-date, appointment-time, clinic-room-no)
- Identify the functional dependencies in the schema, stating any assumptions made.
- Using these functional dependencies, normalise the schema to Third Normal Form.

# Question 1

- APPOINTMENT(patient-id, patient-name, doctor-id, doctor-name, appointment-date, appointment-time, clinic-room-no)
- Identified FDs are:
  - ▣ patient-id → patient-name
  - ▣ doctor-id → doctor-name
  - ▣ appointment-date, appointment-time, clinic-room-no → doctor-id, patient-id
- Is APPOINTMENT in BCNF, 3NF?
  - ▣ Key: patient-id
  - ▣ Not in BCNF - doctor-id → doctor-name: doctor-id is not the key
  - ▣ Not in 3NF - doctor-id → doctor-name: doctor-name is not in a key of Appointment
- Do the FDs form a minimal basis? → NO

# Question 1

- APPOINTMENT(A, B, C, D, E, F, G)
- Identified FDs are:
  - $S = \{A \rightarrow B, C \rightarrow D, EFG \rightarrow AC\}$
- A minimal basis
  - Step 1:  $\{A \rightarrow B, C \rightarrow D, EFG \rightarrow A, EFG \rightarrow C\}$ 
    - Step 1: Transform the FDs, so that each right hand side contains only one attribute
  - Step 2:  $A \rightarrow B, C \rightarrow D$  not redundant,
    - Step 2: Remove redundant FDs $EFG \rightarrow A$  not redundant,  $\{EFG\}^+ = \{EFGC\}$  does not contain A  
 $EFG \rightarrow C$  not redundant,  $\{EFG\}^+ = \{EFGAB\}$  does not contain C
  - Step 3: Remove redundant attributes on the left hand side (lhs) of each FD
  - Step 3:  $EFG \rightarrow A$  has more than one attribute on the lhs,
    - remove E,  $FG \rightarrow A$ ; given S,  $\{FG\}^+ = \{FG\}$ , does not contain A, attribute E not redundant
    - remove F,  $EG \rightarrow A$ ; given S,  $\{EG\}^+ = \{EG\}$ , does not contain A, attribute F not redundant
    - remove G,  $EF \rightarrow A$ ; given S,  $\{EF\}^+ = \{EF\}$ , does not contain A, attribute G not redundant
    - $EFG \rightarrow C$ , attributes on lhs are not redundant, similarly
  - Minimal basis:  $\{A \rightarrow B, C \rightarrow D, EFG \rightarrow A, EFG \rightarrow C\}$

# Question 1

- Step 2: In the minimal basis, combine the FDs whose left hand sides are the same
    - e.g., after combining  $A \rightarrow B$  and  $A \rightarrow C$ , we have  $\{A \rightarrow BC, C \rightarrow D\}$
  - Step 3: Create a table for each FD remained
    - $R_1(A, B, C), R_2(C, D)$
  - Step 4: If none of the tables contain a key of the original table  $R$ , create a table that contains a key of  $R$
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- Minimal basis:  $\{A \rightarrow B, C \rightarrow D, EFG \rightarrow A, EFG \rightarrow C\}$
  - 3NF decomposition –  $S = \{A \rightarrow B, C \rightarrow D, EFG \rightarrow AC\}$
  - Using the above identified FDs, we decompose the schema into 3NF:
    - $PATIENT(\underline{patient-id}, patient-name)$
    - $DOCTOR(\underline{doctor-id}, doctor-name)$
    - $APPT(doctor-id, patient-id, \underline{appointment-date}, \underline{appointment-time}, clinic-room-no)$

## Question 2

2. Consider the relation `Courses` (`C`, `T`, `H`, `R`, `S`, `G`) whose attributes may be thought informally as course, teacher, hour, room, student, and grade. Let the set of FD's of `Courses` be:

$C \rightarrow T, HR \rightarrow C, HT \rightarrow R, HS \rightarrow R, \text{ and } CS \rightarrow G.$

- (a) What are all the keys for `Courses`?
- (b) Verify that the given FDs are their own minimal basis.
- (c) Use the 3NF decomposition algorithm to find a lossless-join, dependency-preserving decomposition.



## Question 2(a)

(a) What are all the keys for Courses?

$C \rightarrow T, HR \rightarrow C, HT \rightarrow R, HS \rightarrow R, \text{ and } CS \rightarrow G.$

- The usual procedure to find the keys is to take the closure of all 63 nonempty subsets.
- However, if we notice that **none of the right sides of the FDs contains attributes H and S.**
- Thus, we know that attributes **H and S** must be part of any key.
- We eventually will find out that **HS** is the only key for the Courses relation.

i.e.  $\{HS\}^+ = \{HSRCTG\}$

## Question 2(b)

(b) Verify that the given FDs are their own minimal basis.

$C \rightarrow T$ ,  $HR \rightarrow C$ ,  $HT \rightarrow R$ ,  $HS \rightarrow R$ , and  $CS \rightarrow G$ .

- Check if any of the FDs can be removed.
  - If we remove any one of the five FDs, the remaining four FDs do not imply the removed FD.
  - For example: Try  $C \rightarrow T$ , if removed, then  $HR \rightarrow C$ , and etc.  $\{C\}^+ = \{C\}$ , does not contain  $T$ , so cannot be removed. Similarly for other FDs.
- Check if any of the LHS attribute of an FD can be removed without losing the dependencies.
  - This is not the case for the four FDs that contain two attributes on the left side. Thus, the given set of FDs is a minimal basis.
  - For example,  $HR \rightarrow C$ , (i) remove  $H$ , we have  $R \rightarrow C$ , given the set of FDs,  $\{R\}^+ = \{R\}$ , does not contain  $C$ , so cannot be removed. (ii) remove  $R$ , we have  $H \rightarrow C$ , given the set of FDs,  $\{H\}^+ = \{H\}$ , does not contain  $C$ , so cannot be removed. Similarly for other FDs with more than one attributes on lhs.

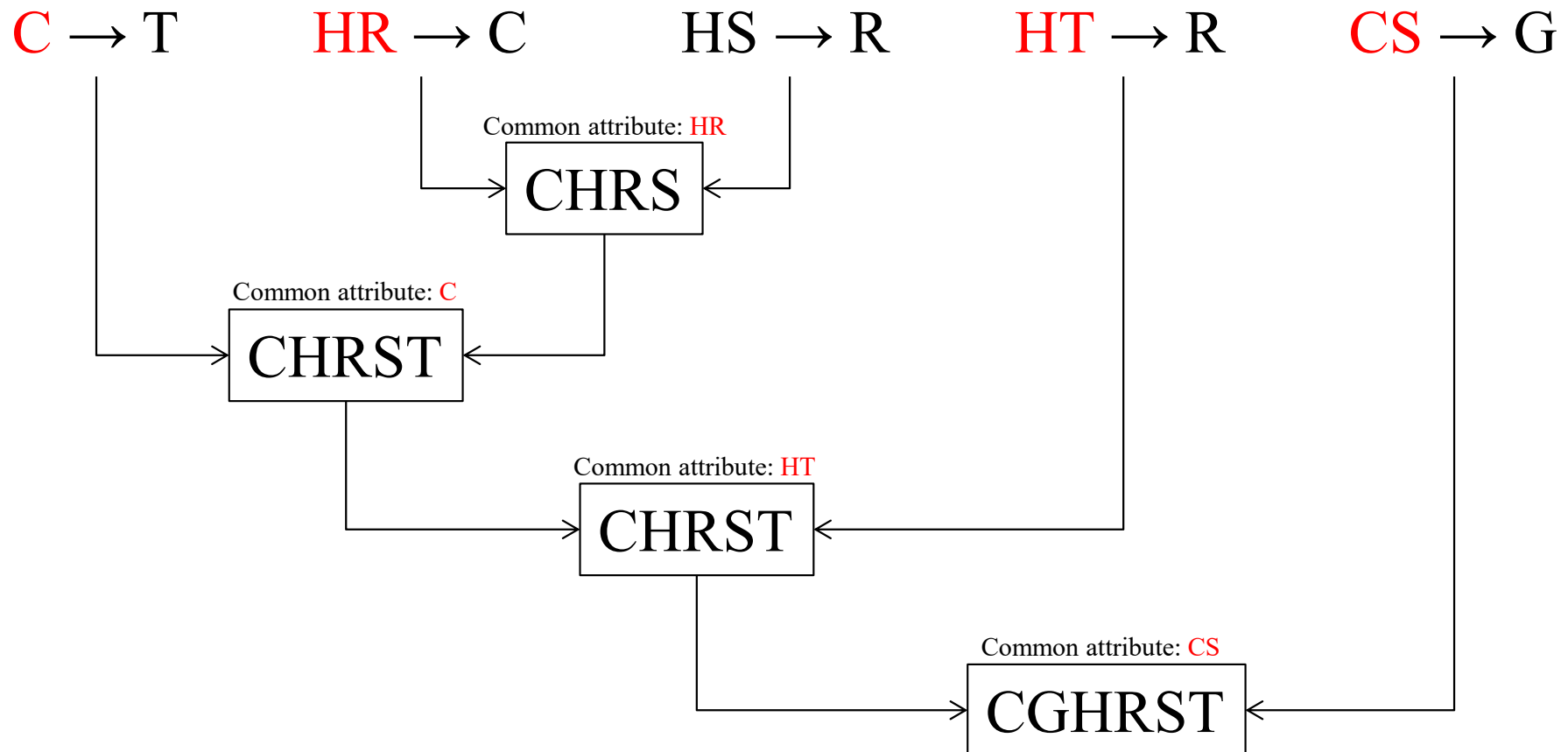
## Question 2(c)

(c) Use the 3NF decomposition algorithm to find a lossless-join, dependency-preserving decomposition.

- Since the only key is **HS**, the given set of FDs has some dependencies that violate 3NF.
- We also know that the given set of FDs is a minimal basis.
- Thus, the **decomposed relations** are CT, HRC, HTR, HSR and CSG.
- Since the relation **HSR** contains a key, we do not need to add an additional relation.
- The final set of decomposed relations is CT, HRC, HTR, HSR and CSG.
- Since each decomposed relation came from a FD, the decomposition is FD preserving.

## Question 2(c)

- The following sequence of joins shows that the decomposition is loss-less:



## Question 3

3. Consider a relation  $R(W, X, Y, Z)$  which satisfies the following set of FDs  $G = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$ , where  $G$  is a minimal basis.
- (i) Decompose  $R$  into a set of relations in 3NF.
  - (ii) Is the decomposition also in BCNF? Explain your answer.

## Question 3

(i) Decompose R into a set of relations in 3NF.  $R(W, X, Y, Z)$

$$G = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$$

- $G = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$ , where G is a minimal basis
- Use **closure** method to find keys
- Possible keys: **Y, WX, XZ**, i.e.  $\{Y\}^+ = \{WXYZ\}$ ,  $\{WX\}^+ = \{WXYZ\}$ ,  $\{XZ\}^+ = \{WXYZ\}$
- Minimal basis:
  - Step 1: Transform the FDs, so that each right hand side contains only one attribute
  - Step 2: Remove redundant FDs
  - Step 3: Remove redundant attributes on the left hand side (lhs) of each FD
  - 1.  $M = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$
  - 2.  $M = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$  – no redundant FD
  - 3.  $M = \{Z \rightarrow W, Y \rightarrow X, Y \rightarrow Z, XW \rightarrow Y\}$  – no redundant attribute lhs
- Combine FDs:  $\{Z \rightarrow W, Y \rightarrow XZ, XW \rightarrow Y\}$
- Decomposed relations:  $R1(Z, W), R2(X, Y, Z), R3(X, Y, W)$

## Question 3

(ii) Is the decomposition also in BCNF? Explain your answer.

□ FD:  $\{Z \rightarrow W, Y \rightarrow XZ, XW \rightarrow Y\}$

Then all LHS of FDs are superkeys; therefore relations are in BCNF

## Question 4

4. Consider a relation  $R(A, B, C, D, E)$  and FD's  $A \rightarrow BC$ ,  $CD \rightarrow E$ ,  $E \rightarrow A$ , and  $B \rightarrow D$ .
- (a) Is the decomposition  $R_1(A, B, C)$  and  $R_2(A, D, E)$  of  $R$  lossless or lossy? Justify your answer. Is this decomposition dependency preserving? If your answer is NO, then what is not preserved?
- (b) Is the decomposition  $R_3(A, B, C, D)$  and  $R_4(C, D, E)$  of  $R$  lossless or lossy? Justify your answer. Is this decomposition dependency preserving? If your answer is NO, then what is not preserved?



## Question 4(a)

Consider a relation  $R(A, B, C, D, E)$  and FD's  $A \rightarrow BC$ ,  $CD \rightarrow E$ ,  $E \rightarrow A$ , and  $B \rightarrow D$ .

(a) Is the decomposition  $R_1(A, B, C)$  and  $R_2(A, D, E)$  of  $R$  lossless or lossy? Justify your answer. Is this decomposition dependency preserving? If your answer is NO, then what is not preserved?

- FDs:  $A \rightarrow BC$ ,  $E \rightarrow A$ ,  $CD \rightarrow E$ ,  $B \rightarrow D$
- Keys?  $A$ ?  $E$ ?  $CD$ ?  $BC$ ?
- (a) Decomposition:  $R_1(A, B, C)$  and  $R_2(A, D, E)$
- (b) Decomposition:  $R_3(A, B, C, D)$  and  $R_4(C, D, E)$
- Is decomposition lossless (can be joined back)?
- Is decomposition dependency preserving (no FDs lost)?

## Question 4(a)

- **Decomposition is lossless** since

- The two tables have a common attribute A, and
- A is a superkey for R1(A,B,C)

- FDs that hold on R1(A,B,C)

- $A \rightarrow BC$  since R1 contains A, B, and C

$A \rightarrow BC, E \rightarrow A, CD \rightarrow E, B \rightarrow D$

- FDs that hold on R2(A,D,E)

- $E \rightarrow A$ , since R2 contains A and E

- Two other FDs need to be checked:  $CD \rightarrow E, B \rightarrow D$

- Given  $A \rightarrow BC$  and  $E \rightarrow A$ , we have:
- $\{B\}^+ = \{B\}$ , so  $B \rightarrow D$  is not preserved
- $\{CD\}^+ = \{CD\}$ , so  $CD \rightarrow E$  is not preserved

- **Decomposition is NOT dependency-preserving**

## Question 4(b)

(b) Is the decomposition  $R_3(A, B, C, D)$  and  $R_4(C, D, E)$  of  $R$  lossless or lossy? Justify your answer. Is this decomposition dependency preserving? If your answer is NO, then what is not preserved?

□ **Decomposition is lossless** since

- The two tables have common attributes  $CD$ , and
- $CD$  is a superkey for  $R_4(C, D, E)$

$A \rightarrow BC, E \rightarrow A, CD \rightarrow E, B \rightarrow D$

□ FDs that hold on  $R_3(A, B, C, D)$

- $A \rightarrow BC, CD \rightarrow A$ , and  $B \rightarrow D$  since  $R_3$  contains  $A, B, C$ , and  $D$

□ FDs that hold on  $R_4(C, D, E)$

- $CD \rightarrow E, E \rightarrow CD$

□ One other FDs needs to be checked:  $E \rightarrow A$

- Given  $A \rightarrow BC, B \rightarrow D, CD \rightarrow E, E \rightarrow A$  and, we have:
- $\{E\}^+ = \{A, B, C, D, E\}$ , so  $E \rightarrow CD$  in  $R_4$ ,  $CD \rightarrow A$  in  $R_3$ ,  $E \rightarrow A$  is preserved

□ **Decomposition is dependency-preserving**