MAT 186 Module E4-4: Modelling with Riemann Sums Part 4
Section LEC 0108

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1. Warm-Up Problems
2. Interpretive problem (hopefully a fun one)

Warm-Up Problem 1

Problem

Recall Question 1 of the PCE: "We wish to empty a vertically-oriented, cylindrical tank. The tank has a height of 3m and a radius of 2m, while the pipe to empty it brings liquid up to 3m above the top of the tank. ... [Determine the]work done to empty the entire tank." The density of liquid in the tank is ρ, and g is the acceleration due to gravity.

Your friend Germund wrote the following solution to this quiz question: "There is a total of $m = \rho \times \pi \times 2^2 \times 3 = 12\pi\rho$ kilograms of mass in the tank. Since work is simply force times distance and all the material has to pumped 3m, we have that the total work done is $W = 36\pi\rho g$ Joules."

What do you think of this argument? Is it correct?

Problem

True or false: when using a Riemann sum strategy to approximate the work required to move a continuously distributed substance against gravity (acting in the y-direction), you *must* slice parallel to the y-axis (that is, the planar "knife" doing the slicing cuts parallel to the y-axis).
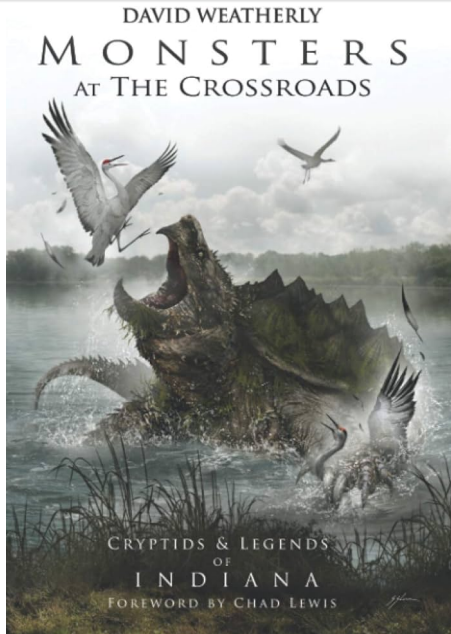
- In spring 1949 two men claimed to have seen a car-sized turtle in Fulk Lake near Churubusco, Indiana
- Eyewitness testimony: "We saw the big waves a-rolling and up came that turtle" (Ora Blue, 1949)
- The lake's owner Gale Harris also claimed to have seen the "Beast of 'Busco" (aka "Oscar" (?)) and organized expeditions (including divers) to try and catch it
- Eventually Harris just paid to have the lake drained! You will be unsurprised to hear Oscar was not found.
- For more details, see B. J. Hollars' article here (click), or this episode of the MonsterTalk podcast (click).

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Interpretive Problem, Part 1

Problem

The goal of this problem is to roughly estimate how much work it would take to drain Lake Fulk of all its water and try to find Oscar.

First, let's assume Lake Fulk is linearly vertically mass-stratified: the mass density $\rho(y)$ (in kg/m^3) of water is a degree-one polynomial in the vertical distance y from the bottom of the lake (measured in m). Let's suppose Lake Fulk is bowl-shaped and has a maximum depth of H metres.

If the density of lake water near the bottom of the lake is ρ_b and the density of water near the surface is ρ_s (with $\rho_b > \rho_s$), write out $\rho(y)$.

Problem

Assume the interior of Lake Fulk is the solid of revolution associated to rotating the graph of $f(x) = x^2$ defined on $\left[0, \sqrt{H}\right]$.

Find the work required to drain all the water out of Lake Fulk (that is, to take all the water to the lake's surface $y = H$).