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Class:- 11th

Subjective Test

Trigonometric Equations

Q-1 $\sin x \tan x - 1 = \tan x - \sin x$

$\sin x \tan x - \tan x + \sin x - 1 = 0$

$\tan x (\sin x - 1) + 1(\sin x - 1) = 0$

$(\tan x + 1)(\sin x - 1) = 0$

$\sin x = 1$

$x = \pi/2$

$$\boxed{x = 2n\pi + (-1)^n \left(\frac{\pi}{2}\right)}$$

$\tan x = -1$

$x = -\frac{\pi}{4}$

$$\boxed{x = n\pi - \frac{\pi}{4}}$$

When $x = \frac{\pi}{2}$ & multiples of $\frac{\pi}{2}$, $\tan x$ is not defined

$\therefore$ $$\boxed{x = n\pi - \frac{\pi}{4}}$$

Q-2. $\tan x + \tan 2x + \sqrt{3}\tan x \tan 2x = \sqrt{3}$

$\Rightarrow \tan x + \tan 2x = \sqrt{3} - \sqrt{3}\tan x \tan 2x$

$\Rightarrow \tan x + \tan 2x = \sqrt{3}(1 - \tan x \tan 2x)$

$\Rightarrow \dfrac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \sqrt{3}$

$\Rightarrow \tan(x + 2x) = \sqrt{3}$

$\tan 3x = \sqrt{3}$

$\alpha = \dfrac{\pi}{3}$

$\Rightarrow 3x = n\pi + \dfrac{\pi}{3}$

$x = \dfrac{n\pi}{3} + \dfrac{\pi}{9}$

$\Rightarrow \boxed{x = (3n+1)\dfrac{\pi}{9}}$

Q-3. $3(\sec\theta - 1) = \tan^2\theta$

$\Rightarrow 3\sec\theta - 3 = \sec^2\theta - 1$

$\Rightarrow \sec^2\theta - 3\sec\theta + 2 = 0$

Put $\sec\theta = t$

$\Rightarrow t^2 - 3t + 2 = 0$

$\Rightarrow t^2 - 2t - t + 2 = 0$

$\Rightarrow t(t-2) - 1(t-2) = 0$

$\Rightarrow (t-2)(t-1) = 0$

$\Rightarrow t = 1$

$\sec\theta = 1$

$\cos\theta = 1$

$\theta = 2n\pi \pm 0$

$\boxed{\theta = 2n\pi}$

$t = 2$

$\sec\theta = 2$

$\cos\theta = \frac{1}{2}$

$\boxed{\theta = 2m\pi \pm \frac{\pi}{3}}$

$\theta = \{2n\pi\} \cup \{2m\pi \pm \pi/3\}$

Q-4. $\sin 7\theta \cdot \sin 5\theta = \sin 3\theta \cdot \sin\theta$

Mult. both sides by 2

$2\sin 7\theta \cdot \sin 5\theta = 2\sin 3\theta \cdot \sin\theta$

$\cos 12\theta - \cos 2\theta = \cos 4\theta - \cos 2\theta$

$12\theta = 2n\pi \pm 4\theta$

$12\theta = 2n\pi + 4\theta$

$8\theta = 2n\pi$

$\theta = \frac{n\pi}{4}$

$12\theta = 2n\pi - 4\theta$

$16\theta = 2n\pi$

$\theta = \frac{n\pi}{8}$

$\therefore \quad \theta = \frac{n\pi}{8}$

Q-6. $\sin 2\theta + \sin 4\theta + \sin 6\theta = 0$

$2 \sin 4\theta \cos 2\theta + \sin 4\theta = 0$

$\sin 4\theta (2\cos 2\theta + 1) = 0$

$\sin 4\theta = 0$

$4\theta = 2n\pi + (-1)^n (0)$

$\boxed{\theta = \frac{n\pi}{2}}$

$\cos 2\theta = -\frac{1}{2}$

$2\theta = 2m\pi \pm \frac{\pi}{3}$

$\boxed{\theta = m\pi \pm \frac{\pi}{6}}$

$\therefore \quad \theta = \left\{\frac{n\pi}{2}\right\} \cup \left\{m\pi \pm \frac{\pi}{6}\right\}$

Q-7. $\cot^2\theta + 3\,\mathrm{cosec}\,\theta + 3 = 0$

$\mathrm{cosec}^2\theta - 1 + 3\,\mathrm{cosec}\,\theta + 3 = 0$

$\mathrm{cosec}^2\theta + 3\,\mathrm{cosec}\,\theta + 2 = 0$

Put $\mathrm{cosec}\,\theta = t$

$t^2 + 3t + 2 = 0$

$\Rightarrow t(t+2) + 1(t+2) \Rightarrow (t+1)(t+2) = 0$

$\operatorname{cosec}\theta = -1$
$\sin\theta = -1$
$\theta = 2n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

$\operatorname{cosec}\theta = -2$
$\sin\theta = \frac{-1}{2}$
$\theta = 2m\pi + (-1)^n\left(\frac{5\pi}{3}\right)$

$\theta = \left\{2n\pi + (-1)^n\left(-\frac{\pi}{2}\right)\right\} \cup \left\{2m\pi + (-1)^n\left(\frac{5\pi}{3}\right)\right\}$

Q-8. $\sin x + \cos x = 1 + \sin x \cos x$

$(\sin x - \sin x \cos x) + \cos x - 1 = 0$

$\Rightarrow \sin x(1 - \cos x) - 1(1 - \cos x) = 0$

$\Rightarrow (\sin x - 1)(1 - \cos x) = 0$

$\sin x = 1$
$x = 2n\pi + (-1)^n\frac{\pi}{2}$

$\cos x = 1$
$x = 2m\pi \pm 0$
$x = 2m\pi$

$x = \left\{2n\pi + (-1)^n\frac{\pi}{2}\right\} \cup \{2m\pi\}$