Υπολογιστική Νοημοσύνη

Εργαστηριακές Ασκήσεις ακ. έτους 2024-25

Ομάδα:

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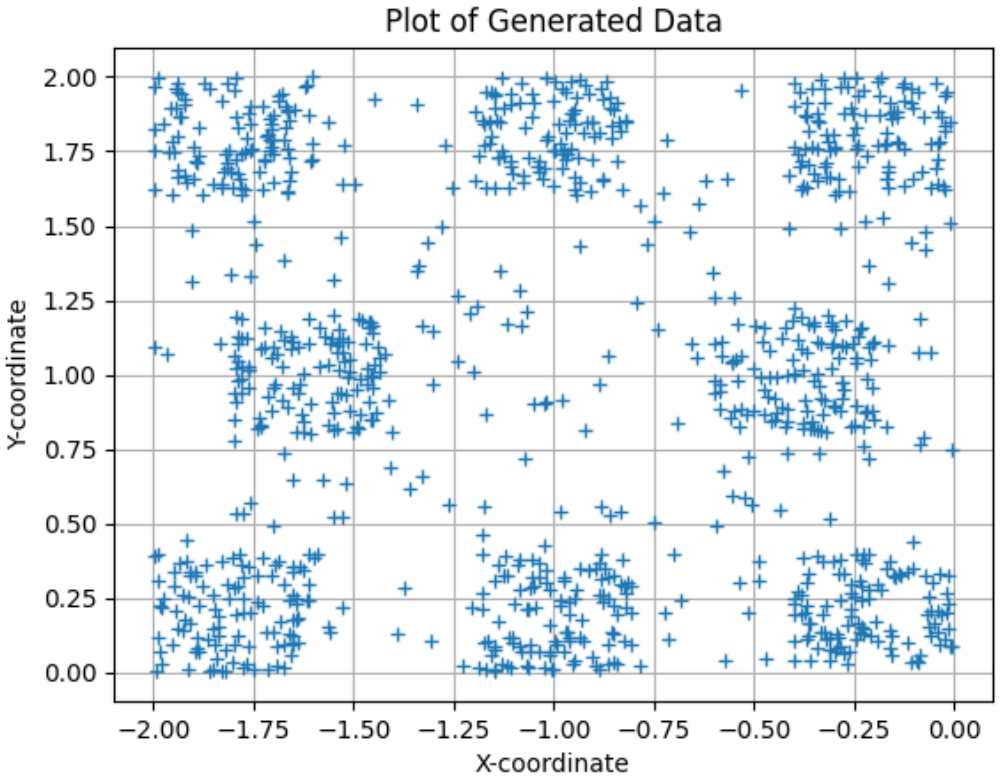
Άσκηση 2:

Για τα γραφήματα χρησιμοποιήσαμε python και πιο συγκεκριμένα τις βιβλιοθήκες pandas και matplotlib. Σε περίπτωση που χρειαστεί να τρέξετε τα προγράμματα python θα πρεπει να εγκατασταθούν οι 2 βιβλιοθήκες. Εναλλακτικά θα έπρεπε να γίνει χρήση gnuplot το οποίο είναι δύσχρηστο και δεν προσφέρει εύκολη μεταχείριση csv αρχείων.
Εντολή μεταγλώττισης: gcc -o Ask2 Ask2.c -lm, gcc -o ODSCollector ODSCollector.c -lm.

Εκτελέσιμο: ./Ask2, ./ODSCollector

Ξεκινήσαμε χρησιμοποιώντας το **ODSCollector** για να φτιάξουμε **random** σημεία μέσα σε ένα επίπεδο. Όπως φαίνεται παρακάτω από το γράφημα τα σημεία στο επίπεδο θα είναι:



Βλέπουμε ότι υπάρχουν **clusters** με **points** καθώς έχει γίνει ομαδοποίηση της τάξεως του **8** κατά την δημιουργία των παραδειγμάτων.

Α) Χρησιμοποιώντας τα παραπάνω παραδείγματα τρέχουμε τον κώδικα τύπου **k-means** για όλες τις ζητούμενες τιμές του M κάνοντας **20** επαναλήψεις πριν διαλέξουμε την καλύτερη επανάληψη ως τελική. Παρακάτω είναι τα αποτελέσματά όλων των επαναλήψεων καθώς και η επιλεγμένη καλύτερη επανάληψη:

```
M = 4:
  Iteration 1: Error = 448.491109, Centers: (-1.388196, 1.786480) (-0.294944, 1.002035) (-1.562832, 1.008011) (-1.367392, 0.210790)
  Iteration 2: Error = 416.873358, Centers: (-1.408126, 1.733673) (-0.604999, 0.227971) (-0.320503, 1.380421) (-1.665478, 0.597217)
  Iteration 3: Error = 416.516129, Centers: (-1.573367, 0.387311) (-0.375747, 1.547744) (-0.469722, 0.370499) (-1.506950, 1.580554)
  Iteration 4: Error = 416.886266, Centers: (-0.319653, 1.382598) (-1.408126, 1.733673) (-1.665478, 0.597217) (-0.604727, 0.230390)
  Iteration 5: Error = 416.165874, Centers: (-1.492843, 0.299367) (-0.472899, 1.633141) (-1.595630, 1.483667) (-0.398994, 0.446363)
  Iteration 6: Error = 416.886266, Centers: (-1.665478, 0.597217) (-0.604727, 0.230390) (-0.319653, 1.382598) (-1.408126, 1.733673)
  Iteration 7: Error = 443.574825, Centers: (-0.624717, 1.794075) (-1.695663, 0.992712) (-0.391301, 1.003960) (-0.627751, 0.217436)
  Iteration 8: Error = 416.165874, Centers: (-0.398994, 0.446363) (-0.472899, 1.633141) (-1.595630, 1.483667) (-1.492843, 0.299367)
  Iteration 9: Error = 416.432116, Centers: (-0.476196, 0.374908) (-1.581768, 0.380080) (-1.498160, 1.580281) (-0.366074, 1.544824)
  Iteration 10: Error = 416.886266, Centers: (-0.604727, 0.230390) (-0.319653, 1.382598) (-1.408126, 1.733673) (-1.665478, 0.597217)
  Iteration 11: Error = 417.505562, Centers: (-0.351168, 0.601837) (-1.418372, 0.242652) (-1.655608, 1.395507) (-0.585556, 1.771689)
  Iteration 12: Error = 417.440189, Centers: (-1.411592, 0.233379) (-1.653715, 1.386476) (-0.585556, 1.771689) (-0.347588, 0.604354)
  Iteration 13: Error = 416.886266, Centers: (-0.319653, 1.382598) (-1.408126, 1.733673) (-1.665478, 0.597217) (-0.604727, 0.230390)
  Iteration 14: Error = 416.873358, Centers: (-0.604999, 0.227971) (-0.320503, 1.380421) (-1.408126, 1.733673) (-1.665478, 0.597217)
  Iteration 15: Error = 417.570785, Centers: (-1.426890, 0.253647) (-0.585556, 1.771689) (-1.656486, 1.407260) (-0.354854, 0.599680)
  Iteration 16: Error = 416.886266, Centers: (-0.604727, 0.230390) (-1.408126, 1.733673) (-0.319653, 1.382598) (-1.665478, 0.597217)
  Iteration 17: Error = 416.886266, Centers: (-0.319653, 1.382598) (-1.408126, 1.733673) (-1.665478, 0.597217) (-0.604727, 0.230390)
  Iteration 18: Error = 416.650779, Centers: (-0.503126, 1.722654) (-1.484334, 0.290027) (-0.392443, 0.535471) (-1.607138, 1.474395)
  Iteration 19: Error = 416.886266, Centers: (-0.319653, 1.382598) (-0.604727, 0.230390) (-1.665478, 0.597217) (-1.408126, 1.733673)
  Iteration 20: Error = 416.886266, Centers: (-1.408126, 1.733673) (-0.319653, 1.382598) (-1.665478, 0.597217) (-0.604727, 0.230390)

  Best Iteration for M = 4:
  Error: 416.165874
  Centers:
  (-1.492843, 0.299367)
  (-0.472899, 1.633141)
  (-1.595630, 1.483667)
  (-0.398994, 0.446363)
```

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M = 6:
Iteration 1: Error = 300.946558, Centers: (-0.408997, 0.979209) (-0.992863, 0.211169) (-1.441737, 1.722824) (-0.316986, 1.777290) (-0.237157, 0.214835) (-1.691725, 0.597698)
Iteration 2: Error = 299.990160, Centers: (-0.222943, 1.780892) (-1.775534, 1.740613) (-0.609859, 0.212662) (-1.671933, 0.602850) (-1.018900, 1.734162) (-0.406503, 0.987659)
Iteration 3: Error = 301.484307, Centers: (-0.293622, 0.482322) (-1.547608, 1.015834) (-0.988027, 0.217950) (-1.781928, 0.221280) (-0.307852, 1.572426) (-1.416467, 1.788430)
Iteration 4: Error = 301.049605, Centers: (-0.397741, 0.997693) (-1.736954, 0.347995) (-1.658553, 1.442759) (-0.597742, 1.795359) (-0.993391, 0.218531) (-0.239417, 0.217798)
Iteration 5: Error = 301.593592, Centers: (-0.292032, 0.473933) (-0.306483, 1.562439) (-1.413798, 1.788576) (-1.547608, 1.015834) (-0.988027, 0.217950) (-1.781928, 0.221280)
Iteration 6: Error = 303.407938, Centers: (-1.009448, 1.777128) (-0.387227, 0.423676) (-1.543292, 1.010546) (-1.780389, 1.777976) (-0.282063, 1.544219) (-1.482399, 0.208403)
Iteration 7: Error = 302.396517, Centers: (-0.237157, 0.214835) (-0.993391, 0.218531) (-1.603911, 1.524278) (-1.725434, 0.416484) (-0.405348, 0.984862) (-0.518783, 1.789563)
Iteration 8: Error = 299.780371, Centers: (-0.486219, 1.746698) (-0.984022, 0.216611) (-1.599992, 1.782891) (-0.303631, 0.603699) (-1.543544, 1.015235) (-1.781928, 0.221280)
Iteration 9: Error = 296.045849, Centers: (-1.549337, 1.013258) (-1.603508, 1.782055) (-1.651068, 0.214737) (-0.491995, 1.784298) (-0.385104, 0.995784) (-0.544436, 0.221186)
Iteration 10: Error = 303.404198, Centers: (-0.289227, 1.508878) (-0.399284, 0.390838) (-1.007244, 1.783162) (-1.538748, 1.003381) (-1.780547, 1.770476) (-1.497116, 0.207051)
Iteration 11: Error = 299.445957, Centers: (-0.406503, 0.987659) (-0.222943, 1.780892) (-0.584715, 0.217744) (-1.682066, 1.416839) (-1.687953, 0.312725) (-1.005159, 1.765751)
Iteration 12: Error = 300.796242, Centers: (-1.005159, 1.765751) (-0.504750, 0.222329) (-0.222943, 1.780892) (-1.588765, 0.289938) (-0.406503, 0.987659) (-1.685061, 1.414290)
Iteration 13: Error = 299.780371, Centers: (-1.781928, 0.221280) (-1.599992, 1.782891) (-0.303631, 0.603699) (-0.486219, 1.746698) (-1.543544, 1.015235) (-0.984022, 0.216611)
Iteration 14: Error = 301.953589, Centers: (-0.237157, 0.214835) (-0.405348, 0.984862) (-0.518783, 1.789563) (-1.604898, 1.512812) (-0.993391, 0.218531) (-1.727630, 0.400069)
Iteration 15: Error = 301.593592, Centers: (-0.988027, 0.217950) (-0.306483, 1.562439) (-1.781928, 0.221280) (-1.413798, 1.788576) (-0.292032, 0.473933) (-1.547608, 1.015834)
Iteration 16: Error = 303.107392, Centers: (-0.251429, 1.683880) (-0.344061, 0.551310) (-1.548851, 1.008930) (-1.780389, 1.777976) (-1.420930, 0.206370) (-1.004274, 1.769413)
Iteration 17: Error = 300.946558, Centers: (-1.691725, 0.597698) (-1.441737, 1.722824) (-0.992863, 0.211169) (-0.408997, 0.979209) (-0.316986, 1.777290) (-0.237157, 0.214835)
Iteration 18: Error = 303.360730, Centers: (-1.780389, 1.777976) (-1.007244, 1.783162) (-0.289227, 1.508878) (-1.543292, 1.010546) (-1.496423, 0.209092) (-0.399284, 0.390838)
Iteration 19: Error = 294.689765, Centers: (-1.421817, 1.787574) (-0.390100, 0.995935) (-0.544436, 0.221186) (-1.553429, 1.013861) (-1.651068, 0.214737) (-0.280985, 1.775357)
Iteration 20: Error = 300.422698, Centers: (-0.239417, 0.217798) (-0.597742, 1.795359) (-1.656063, 1.428589) (-0.990237, 0.219243) (-0.397741, 0.997693) (-1.741846, 0.324858)

Best Iteration for M = 6:
Error: 294.689765
Centers:
(-1.421817, 1.787574)
(-0.390100, 0.995935)
(-0.544436, 0.221186)
(-1.553429, 1.013861)
(-1.651068, 0.214737)
(-0.280985, 1.775357)
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M = 8:
Iteration 1: Error = 183.788188, Centers: (-1.780389, 1.777976) (-0.237157, 0.214835) (-0.222943, 1.780892) (-0.395753, 0.980803) (-0.994990, 0.213954) (-1.802012, 1.776336) (-1.562551, 1.011770) (-1.785243, 0.222062)
Iteration 2: Error = 183.778281, Centers: (-0.395753, 0.980803) (-1.562551, 1.011770) (-0.222943, 1.780892) (-1.780389, 1.777976) (-1.781928, 0.221280) (-0.991849, 0.214629) (-1.802012, 1.776336) (-0.237157, 0.214835)
Iteration 3: Error = 183.778281, Centers: (-0.222943, 1.780892) (-0.237157, 0.214835) (-1.780389, 1.777976) (-1.802012, 1.776336) (-0.395753, 0.980803) (-0.991849, 0.214629) (-1.562551, 1.011770) (-1.781928, 0.221280)
Iteration 4: Error = 235.461669, Centers: (-0.387361, 0.992718) (-1.468290, 1.785146) (-1.552538, 1.015230) (-0.991849, 0.214629) (-1.781928, 0.221280) (-0.338396, 0.237624) (-0.103366, 0.203603) (-0.348956, 1.778302)
Iteration 5: Error = 183.778281, Centers: (-1.781928, 0.221280) (-0.237157, 0.214835) (-1.780389, 1.777976) (-1.802012, 1.776336) (-0.395753, 0.980803) (-0.991849, 0.214629) (-1.562551, 1.011770) (-0.222943, 1.780892)
Iteration 6: Error = 233.089993, Centers: (-1.780389, 1.777976) (-1.611707, 0.998695) (-0.997498, 1.787011) (-1.654259, 0.212443) (-0.370327, 0.990665) (-1.113625, 1.085663) (-0.223040, 1.784447) (-0.544436, 0.221186)
Iteration 7: Error = 183.764727, Centers: (-1.802012, 1.776336) (-1.785243, 0.222062) (-0.223040, 1.784447) (-0.994990, 0.213954) (-0.389504, 0.986977) (-1.780389, 1.777976) (-1.558576, 1.011125) (-0.239417, 0.217798)
Iteration 8: Error = 234.099560, Centers: (-1.561521, 1.027055) (-0.237157, 0.214835) (-1.720790, 0.379007) (-0.398375, 0.984035) (-0.991988, 0.213378) (-1.421817, 1.787574) (-1.812050, 0.131386) (-0.280459, 1.772248)
Iteration 9: Error = 234.106801, Centers: (-0.398375, 0.984035) (-1.720790, 0.379007) (-1.563893, 1.030223) (-0.237157, 0.214835) (-1.419568, 1.788974) (-0.991988, 0.213378) (-1.812050, 0.131386) (-0.280459, 1.772248)
Iteration 10: Error = 183.778281, Centers: (-0.395753, 0.980803) (-1.562551, 1.011770) (-1.780389, 1.777976) (-0.237157, 0.214835) (-0.991849, 0.214629) (-1.781928, 0.221280) (-1.802012, 1.776336) (-0.222943, 1.780892)
Iteration 11: Error = 234.976171, Centers: (-1.556419, 1.015895) (-0.991849, 0.214629) (-0.274885, 1.229049) (-0.232344, 0.209532) (-0.523191, 1.799856) (-0.433092, 0.899377) (-1.614147, 1.780830) (-1.781928, 0.221280)
Iteration 12: Error = 183.764727, Centers: (-1.802012, 1.776336) (-0.223040, 1.784447) (-1.780389, 1.777976) (-0.389504, 0.986977) (-0.994990, 0.213954) (-0.239417, 0.217798) (-1.785243, 0.222062) (-1.558576, 1.011125)
Iteration 13: Error = 183.778281, Centers: (-1.802012, 1.776336) (-0.395753, 0.980803) (-0.237157, 0.214835) (-1.780389, 1.777976) (-0.222943, 1.780892) (-1.781928, 0.221280) (-1.562551, 1.011770) (-0.991849, 0.214629)
Iteration 14: Error = 236.065065, Centers: (-0.863923, 0.695700) (-0.998717, 0.188657) (-0.368131, 1.003741) (-1.785243, 0.222062) (-0.234070, 0.212449) (-1.570038, 1.019248) (-0.499901, 1.787317) (-1.610595, 1.780487)
Iteration 15: Error = 241.645861, Centers: (-0.113640, 0.198831) (-0.344365, 0.243273) (-1.775534, 1.740613) (-0.406503, 0.987659) (-1.018900, 1.734162) (-0.222943, 1.780892) (-0.992863, 0.211169) (-1.684414, 0.689258)
Iteration 16: Error = 234.908526, Centers: (-1.651068, 0.214737) (-0.544436, 0.221186) (-1.559216, 1.009781) (-0.271188, 1.631587) (-1.011296, 1.781694) (-1.780389, 1.777976) (-0.203198, 1.879402) (-0.394568, 0.987165)
Iteration 17: Error = 234.893302, Centers: (-0.202619, 1.881173) (-1.780389, 1.777976) (-1.555290, 1.009153) (-0.270658, 1.633946) (-0.390277, 0.987682) (-1.651068, 0.214737) (-1.011296, 1.781694) (-0.544436, 0.221186)
Iteration 18: Error = 183.778281, Centers: (-1.562551, 1.011770) (-0.991849, 0.214629) (-0.237157, 0.214835) (-0.222943, 1.780892) (-1.781928, 0.221280) (-0.395753, 0.980803) (-1.802012, 1.776336) (-1.780389, 1.777976)
Iteration 19: Error = 183.788188, Centers: (-0.395753, 0.980803) (-0.222943, 1.780892) (-0.994990, 0.213954) (-1.562551, 1.011770) (-1.802012, 1.776336) (-0.237157, 0.214835) (-1.780389, 1.777976) (-1.785243, 0.222062)
Iteration 20: Error = 183.778281, Centers: (-0.991849, 0.214629) (-1.781928, 0.221280) (-0.237157, 0.214835) (-0.395753, 0.980803) (-1.802012, 1.776336) (-0.222943, 1.780892) (-1.780389, 1.777976) (-1.562551, 1.011770)

Best Iteration for M = 8:
Error: 183.764727
Centers:
(-1.802012, 1.776336)
(-1.785243, 0.222062)
(-0.223040, 1.784447)
(-0.994990, 0.213954)
(-0.389504, 0.986977)
(-1.780389, 1.777976)
(-1.558576, 1.011125)
(-0.239417, 0.217798)
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M = 10:
Iteration 1: Error = 174.072321, Centers: (-1.781928, 0.221280) (-0.324854, 0.978363) (-1.010668, 1.785979) (-0.718163, 1.008330) (-1.780389, 1.777976) (-0.991206, 0.210591) (-0.333326, 1.699067) (-0.239417, 0.217798) (-0.144180, 1.843759) (-1.584110, 1.010391)
Iteration 2: Error = 173.286736, Centers: (-1.111210, 1.130008) (-1.781928, 0.221280) (-0.237157, 0.214835) (-0.991206, 0.210591) (-0.995577, 1.792607) (-1.606133, 0.987780) (-0.374968, 0.978812) (-1.788253, 1.652795) (-0.222943, 1.780892) (-1.771925, 1.887223)
Iteration 3: Error = 173.386791, Centers: (-0.237157, 0.214835) (-0.986586, 0.211226) (-1.002012, 1.776336) (-1.835215, 0.167491) (-0.223040, 1.784447) (-1.628086, 0.378477) (-0.285314, 0.999177) (-0.585936, 0.956640) (-1.780389, 1.777976) (-1.576839, 1.027550)
Iteration 4: Error = 173.523045, Centers: (-1.611203, 0.995702) (-1.781928, 0.221280) (-1.111448, 1.105175) (-0.997498, 1.787011) (-1.917660, 1.817520) (-1.704782, 1.751249) (-0.239417, 0.217798) (-0.223040, 1.784447) (-0.991206, 0.210591) (-0.372803, 0.984742)
Iteration 5: Error = 174.857874, Centers: (-0.332126, 0.235725) (-0.203198, 1.879402) (-0.092830, 0.203789) (-0.991849, 0.214629) (-1.780389, 1.777976) (-1.011296, 1.781694) (-1.558576, 1.011125) (-0.390277, 0.987682) (-0.271188, 1.631587) (-1.781928, 0.221280)
Iteration 6: Error = 175.811070, Centers: (-0.237157, 0.214835) (-1.704782, 1.751249) (-1.863793, 0.295622) (-0.988798, 0.214054) (-1.706370, 0.159377) (-0.395753, 0.980803) (-1.561766, 1.009218) (-0.222943, 1.780892) (-1.917660, 1.817520) (-1.002012, 1.776336)
Iteration 7: Error = 172.798302, Centers: (-1.780389, 1.777976) (-0.373423, 0.833186) (-0.994990, 0.213954) (-1.785243, 0.222062) (-0.232344, 0.209532) (-1.562551, 1.011770) (-0.415315, 1.098813) (-1.011296, 1.781694) (-0.335580, 1.705033) (-0.144180, 1.843759)
Iteration 8: Error = 173.313984, Centers: (-1.674896, 1.715796) (-1.781928, 0.221280) (-1.111210, 1.130008) (-1.607338, 0.990491) (-0.237157, 0.214835) (-0.991206, 0.210591) (-0.222943, 1.780892) (-0.992547, 1.791626) (-0.374968, 0.978812) (-1.680789, 1.819143)
Iteration 9: Error = 171.430033, Centers: (-0.372803, 0.984742) (-0.236376, 0.215457) (-1.780389, 1.777976) (-1.611707, 0.998695) (-0.223040, 1.784447) (-1.785243, 0.222062) (-0.997498, 1.787011) (-1.040076, 0.894400) (-0.940029, 0.335621) (-1.111448, 1.105175)
Iteration 10: Error = 173.305523, Centers: (-1.780389, 1.777976) (-0.223040, 1.784447) (-1.808489, 0.114555) (-1.024882, 0.099407) (-0.236376, 0.215457) (-1.573513, 1.021957) (-1.002012, 1.776336) (-0.385669, 0.988242) (-0.950227, 0.396178) (-1.743105, 0.359399)
Iteration 11: Error = 173.313984, Centers: (-0.991206, 0.210591) (-0.374968, 0.978812) (-1.680789, 1.819143) (-0.237157, 0.214835) (-0.992547, 1.791626) (-0.222943, 1.780892) (-1.781928, 0.221280) (-1.674896, 1.715796) (-1.607338, 0.990491) (-1.111210, 1.130008)
Iteration 12: Error = 225.804944, Centers: (-1.606647, 1.817672) (-0.547034, 0.221219) (-1.029426, 1.810943) (-1.607338, 0.990491) (-1.660645, 0.209956) (-0.220000, 1.785520) (-1.074896, 1.715796) (-0.371352, 0.980243) (-0.786375, 1.571469) (-1.144883, 1.058113)
Iteration 13: Error = 173.326291, Centers: (-0.234070, 0.212449) (-1.780389, 1.777976) (-1.808489, 0.114555) (-0.387771, 0.982483) (-1.743105, 0.359399) (-0.222943, 1.780892) (-1.024882, 0.099407) (-1.573513, 1.021957) (-0.950227, 0.396178) (-1.002012, 1.776336)
Iteration 14: Error = 173.503878, Centers: (-0.374968, 0.978812) (-1.593471, 0.430178) (-1.105206, 1.116812) (-0.985889, 0.207095) (-1.780389, 1.777976) (-1.618420, 1.015761) (-0.237157, 0.214835) (-0.222943, 1.780892) (-0.998023, 1.790080) (-1.816203, 0.182220)
Iteration 15: Error = 227.272623, Centers: (-0.134719, 1.578323) (-0.151402, 1.888111) (-0.366812, 1.753694) (-1.011296, 1.781694) (-1.772099, 1.881041) (-1.544347, 0.993966) (-1.788912, 1.646952) (-0.547034, 0.221219) (-0.390277, 0.987682) (-1.660645, 0.209956)
Iteration 16: Error = 173.832081, Centers: (-1.002012, 1.776336) (-1.785243, 0.222062) (-0.336786, 0.205856) (-1.570070, 1.013252) (-0.223040, 1.784447) (-0.115127, 0.227052) (-1.780389, 1.777976) (-0.385669, 0.988242) (-0.947674, 0.380449) (-1.034380, 0.996610)
Iteration 17: Error = 171.150672, Centers: (-1.785243, 0.222062) (-0.232344, 0.209532) (-1.111210, 1.130008) (-0.994377, 0.209943) (-1.611707, 0.998695) (-1.780389, 1.777976) (-0.366282, 1.105544) (-0.223040, 1.784447) (-0.995577, 1.792607) (-0.385880, 0.836575)
Iteration 18: Error = 174.319494, Centers: (-1.144451, 1.113266) (-0.230417, 0.217798) (-1.785243, 0.222062) (-0.994377, 0.209943) (-0.220000, 1.785520) (-1.027702, 1.823783) (-0.794097, 1.575879) (-1.611707, 0.998695) (-0.373812, 0.983341) (-1.780389, 1.777976)
Iteration 19: Error = 171.887723, Centers: (-0.222943, 1.780892) (-0.989013, 0.207735) (-1.111210, 1.130008) (-1.552187, 0.825120) (-0.374968, 0.978812) (-1.786339, 0.213096) (-0.995577, 1.792607) (-0.237157, 0.214835) (-1.782613, 1.780777) (-1.645343, 1.095587)
Iteration 20: Error = 173.420454, Centers: (-1.780389, 1.777976) (-1.657276, 1.014516) (-0.997498, 1.787011) (-0.380681, 0.982000) (-1.641423, 0.369288) (-0.222943, 1.780892) (-0.985889, 0.207095) (-1.291085, 1.045572) (-1.836580, 0.163906) (-0.237157, 0.214835)

Best Iteration for M = 10:
Error: 171.150672
Centers:
(-1.785243, 0.222062)
(-0.232344, 0.209532)
(-1.111210, 1.130008)
(-0.994377, 0.209943)
(-1.611707, 0.998695)
(-1.780389, 1.777976)
(-0.366282, 1.105544)
(-0.223040, 1.784447)
(-0.995577, 1.792607)
(-0.385880, 0.836575)
```

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M = 12:
Iteration 1: Error = 163.969382, Centers: (-1.552187, 0.825120) (-0.223040, 1.784447) (-0.368772, 0.993058) (-1.646769, 1.090978) (-1.111448, 1.105175) (-1.788415, 1.674380) (-0.758975, 0.351202) (-0.997498, 1.787011) (-0.219254, 0.207894) (-1.044908, 0.166404) (-1.789778, 0.213828) (-1.771925, 1.887223)
Iteration 2: Error = 164.745374, Centers: (-0.376523, 0.976374) (-1.007530, 1.795992) (-0.202619, 1.881173) (-1.786019, 0.180916) (-1.878356, 0.313736) (-1.105206, 1.116812) (-0.270658, 1.633946) (-1.618420, 1.015761) (-1.780389, 1.777976) (-0.985889, 0.207095) (-1.588651, 0.406384) (-0.237157, 0.214835)
Iteration 3: Error = 163.592294, Centers: (-1.065660, 0.886100) (-0.998023, 1.790080) (-0.223040, 1.784447) (-1.616743, 1.013128) (-1.842836, 0.153978) (-1.106425, 1.131534) (-0.837552, 0.247743) (-0.373766, 0.981652) (-0.220444, 0.213132) (-1.090992, 0.408613) (-1.780389, 1.777976) (-1.684952, 0.351972)
Iteration 4: Error = 165.789171, Centers: (-0.996111, 0.208778) (-1.611707, 0.998695) (-0.329706, 1.805470) (-1.780389, 1.777976) (-0.170793, 0.178809) (-1.781928, 0.221280) (-0.390091, 0.307098) (-1.006915, 1.882134) (-0.371530, 0.987829) (-0.998791, 1.655411) (-0.113798, 1.751912) (-1.103559, 1.092063)
Iteration 5: Error = 162.879213, Centers: (-1.080028, 0.166410) (-1.125755, 1.103397) (-0.378481, 1.117874) (-0.379262, 0.847619) (-1.780389, 1.777976) (-1.611707, 0.998695) (-0.223040, 1.784447) (-0.115127, 0.227052) (-1.080421, 1.789683) (-0.332546, 0.194555) (-0.858914, 0.208340) (-1.785243, 0.222062)
Iteration 6: Error = 162.654338, Centers: (-1.636628, 0.369691) (-1.112600, 1.144675) (-0.995577, 1.792607) (-0.332126, 0.235725) (-0.370327, 0.990665) (-1.837595, 0.167359) (-1.024284, 0.091136) (-1.780389, 1.777976) (-0.223040, 1.784447) (-0.951566, 0.343018) (-1.616743, 1.013128) (-0.092830, 0.203789)
Iteration 7: Error = 162.555227, Centers: (-0.997498, 1.787011) (-0.522548, 0.920157) (-1.785243, 0.222062) (-1.125755, 1.103397) (-0.278857, 0.851898) (-1.094729, 0.175210) (-0.229367, 0.210991) (-0.319375, 1.141692) (-0.224328, 1.787599) (-0.859094, 0.253844) (-1.780389, 1.777976) (-1.611707, 0.998695)
Iteration 8: Error = 165.031862, Centers: (-1.785243, 0.222062) (-1.709160, 1.025356) (-0.231134, 0.213921) (-0.859094, 0.253844) (-1.917660, 1.817520) (-0.222943, 1.780892) (-1.015645, 1.857265) (-1.094729, 0.175210) (-0.971583, 1.589715) (-1.403593, 0.988018) (-1.705171, 1.756309) (-0.384833, 0.986936)
Iteration 9: Error = 162.017906, Centers: (-0.943745, 0.339285) (-1.788415, 1.674380) (-0.374968, 0.978812) (-1.789778, 0.213828) (-1.111448, 1.105175) (-0.997498, 1.787011) (-1.646769, 1.090978) (-1.557747, 0.830126) (-1.035125, 0.094667) (-0.234070, 0.212449) (-1.771925, 1.887223) (-0.222943, 1.780892)
Iteration 10: Error = 162.379773, Centers: (-1.611707, 0.998695) (-0.198197, 0.123266) (-1.780389, 1.777976) (-0.876333, 0.254653) (-0.377115, 1.124744) (-1.000421, 1.789683) (-1.785243, 0.222062) (-1.100471, 0.164730) (-0.274285, 0.319515) (-1.125755, 1.103397) (-0.223040, 1.784447) (-0.379730, 0.859722)
Iteration 11: Error = 166.565555, Centers: (-1.808314, 0.098812) (-1.621834, 0.327841) (-1.874786, 0.329934) (-1.060107, 0.695305) (-0.375499, 0.985967) (-1.615876, 1.015354) (-0.997572, 1.795702) (-0.223040, 1.784447) (-0.239417, 0.217798) (-1.141411, 1.255007) (-0.987987, 0.187770) (-1.780389, 1.777976)
Iteration 12: Error = 166.850852, Centers: (-0.223040, 1.784447) (-1.016774, 0.091263) (-0.987388, 1.790896) (-1.785243, 0.222062) (-1.395597, 1.048857) (-0.801264, 0.355434) (-1.780628, 0.994510) (-0.231429, 0.211654) (-1.122983, 0.368851) (-0.383453, 0.994043) (-1.490086, 1.753642) (-1.808219, 1.775914)
Iteration 13: Error = 164.662633, Centers: (-1.780389, 1.777976) (-0.339483, 0.873076) (-0.427580, 1.111638) (-0.383414, 0.299163) (-0.223040, 1.784447) (-1.642898, 0.371333) (-0.975943, 0.419315) (-1.836580, 0.163906) (-1.004899, 1.778851) (-1.002848, 0.110795) (-0.167803, 0.179797) (-1.576839, 1.027550)
Iteration 14: Error = 163.589322, Centers: (-1.780389, 1.777976) (-1.007530, 1.795992) (-0.371530, 0.987829) (-0.333326, 1.699067) (-1.105206, 1.116812) (-1.720790, 0.379007) (-0.991346, 0.209331) (-1.614444, 1.010278) (-1.812050, 0.131386) (-0.092830, 0.203789) (-0.332126, 0.235725) (-0.144180, 1.843759)
Iteration 15: Error = 165.331047, Centers: (-0.219254, 0.207894) (-0.220000, 1.785520) (-1.111448, 1.105175) (-1.811087, 0.129108) (-1.614444, 1.010278) (-0.368772, 0.993058) (-1.780389, 1.777976) (-0.860431, 1.733171) (-1.730524, 0.379092) (-1.047228, 0.170642) (-1.082162, 1.828863) (-0.758975, 0.351202)
Iteration 16: Error = 163.920511, Centers: (-1.596624, 1.130708) (-1.782613, 1.780777) (-1.764261, 0.342140) (-0.343134, 0.962773) (-1.804849, 0.109859) (-0.992228, 0.207104) (-0.237157, 0.214835) (-0.271188, 1.631587) (-1.558037, 0.864655) (-1.010113, 1.791997) (-0.283198, 1.879402) (-0.760987, 1.061017)
Iteration 17: Error = 163.618582, Centers: (-0.992228, 0.207104) (-1.764261, 0.342140) (-0.222943, 1.780892) (-1.804849, 0.109859) (-1.637000, 1.131306) (-1.083824, 1.069806) (-0.237157, 0.214835) (-1.569929, 0.868191) (-1.011315, 1.883588) (-0.374968, 0.978812) (-0.986098, 1.657877) (-1.782931, 1.783850)
Iteration 18: Error = 167.213401, Centers: (-1.606587, 0.985132) (-0.237157, 0.214835) (-1.861743, 0.341978) (-0.995869, 1.789400) (-0.222943, 1.780892) (-1.083824, 1.069806) (-0.988124, 0.209970) (-0.374968, 0.978812) (-1.789457, 1.795276) (-1.851743, 0.088316) (-1.683594, 1.455682) (-1.633495, 0.248030)
Iteration 19: Error = 164.392699, Centers: (-1.112600, 1.144675) (-0.995577, 1.792607) (-0.228444, 0.213132) (-1.087402, 0.405653) (-0.222943, 1.780892) (-0.374968, 0.978812) (-1.794668, 1.879943) (-1.065660, 0.886100) (-1.785243, 0.222062) (-1.610395, 0.993499) (-0.834618, 0.246369) (-1.766734, 1.648032)
Iteration 20: Error = 164.333733, Centers: (-0.200930, 1.882644) (-0.274285, 0.319515) (-0.756386, 1.078507) (-0.300402, 1.111787) (-1.012778, 1.788989) (-0.991206, 0.210591) (-0.267206, 1.651313) (-1.584110, 1.010391) (-0.372958, 0.853948) (-1.781928, 0.221280) (-0.204221, 0.121973) (-1.780389, 1.777976)

Best Iteration for M = 12:
Error: 162.017906
Centers:
(-0.943745, 0.339285)
(-1.788415, 1.674380)
(-0.374968, 0.978812)
(-1.789778, 0.213828)
(-1.111448, 1.105175)
(-0.997498, 1.787011)
(-1.646769, 1.090978)
(-1.557747, 0.830126)
(-1.035125, 0.094667)
(-0.234070, 0.212449)
(-1.771925, 1.887223)
(-0.222943, 1.780892)
```
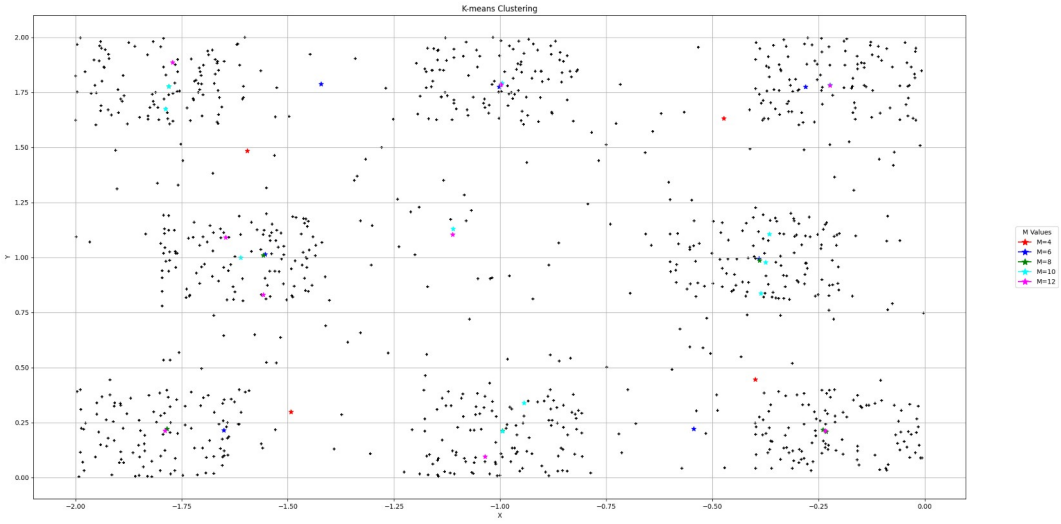
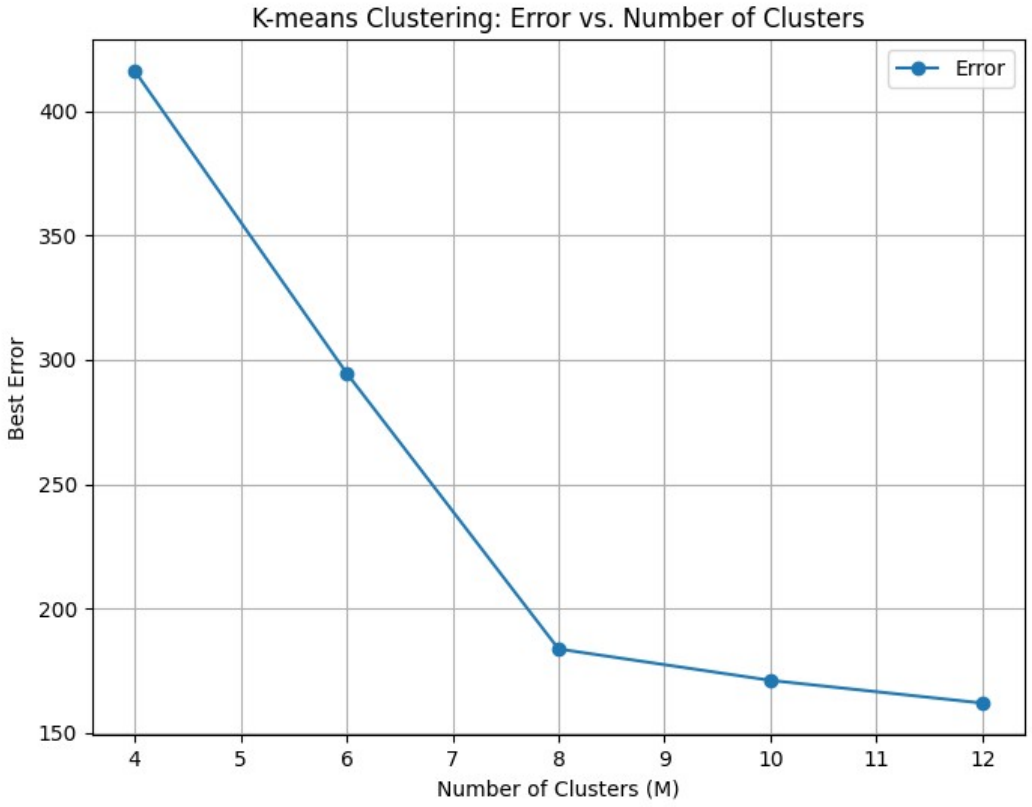

Συμπερασματικά βλέπουμε ότι το **error** γίνεται πιο μικρό όσο το M αυξάνεται που είναι λογικό καθώς η ομαδοποίηση γίνεται πιο ειδική και υπάρχουν περισσότερα και πιο κοντινά κέντρα για τα παραδείγματα.

Β) Παρακάτω είναι το γράφημα που περιλαμβάνει όλα τα παραδείγματα αλλά και όλα τα επιλεγμένα κέντρα για κάθε τιμή του M. Τα κέντρα έχουν διαφορετικό χρώμα με βάση την τιμή του M.



Τέλος το γράφημα για την μεταβολή του σφάλματος όσο αυξάνεται το M:



Με βάση το παραπάνω γράφημα μπορούμε να συμπεράνουμε τον αριθμό που χρησιμοποιήθηκε για την αρχική ομαδοποίηση των παραδειγμάτων καθώς βλέπουμε ότι μέχρι το **M = 8** η μείωση του σφάλματος είναι μεγάλη ενώ μετά από το **8** το σφάλμα μειώνεται αλλά με αρκετά μικρότερο ρυθμό.