



Math 151
Week-In-Review 8
Exam 2 Review
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Problem Statements

1. Find the derivative of the following functions.

(a) $f(x) = 5^x - \log_5(\sin(5x + 5)) + \frac{1}{5\sqrt[5]{x}} + \arcsin(5x^5) - \arctan(5)$

(b) $g(x) = \arctan(\ln(e^{x \sec(3x)}))$

(c) $g(x) = \arctan(\ln(e^{x \sec(3x)}))$



2. Find $\frac{dy}{dx}$ for the following equations.

(a) $e^{x^4}y^3 - \cos^2(y) = \sin^2(x) + \arcsin(y)$

(b) $y = (\sqrt{x})^{\left(\frac{x}{\cot(x)}\right)}$



3. Find all points on the curve $f(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{7}{4}x - 2024$ where the tangent line to the curve is parallel to the line $\mathbf{r}(t) = \langle 17 - 4t, -13 + 5t \rangle$.
4. Find the equation of *both* tangent lines to the circle $x^2 + y^2 = 1$ that pass through the point $(0, 2)$.



5. Find the t -values corresponding to all points where the curve $x = 2t^3 - 6t, y = (t^2 + t - 6)^{13}$ has a horizontal or vertical tangent.

6. Find the points where the curve $x = \ln t, y = 2t^2 + 6t$ has a horizontal tangent line.



7. Find a unit tangent vector to the curve $\mathbf{r}(t) = \langle \sqrt{t^2 + 5}, t \rangle$ when $t = 2$.
8. The height in meters of projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 24.5 m/s is $h = 2 + 24.5t - 4.9t^2$ after t seconds. Potentially useful information for this question: $h(2.5) = 32.625$, $h(4) = 21.6$ and $h(-0.08) = h(5.08) = 0$ (approximately).
- (a) What is the maximum height of the projectile?
- (b) What is the velocity of the projectile when it hits the ground?
- (c) What is the total distance covered by the object after 4 seconds?



9. Consider the piecewise function below.

$$f(x) = \begin{cases} x^2 + x + 2 & \text{if } x \leq -1 \\ -x + 1 & \text{if } -1 < x \leq 0 \\ -x + 5 & \text{if } 0 < x < 2 \\ \sqrt{x+7} & \text{if } 2 \leq x \end{cases}$$

(a) Determine $f'(x)$ for all x -values other than $x = -1$, $x = 0$, and $x = 2$.

(b) Is $f(x)$ differentiable at $x = -1$?

(c) Is $f(x)$ differentiable at $x = 0$?

(d) Is $f(x)$ differentiable at $x = 2$?

(e) Draw a rough sketch of $f(x)$.



10. A population grows at a rate proportional to its size. If the population is 8000 in 1990 and 20000 in 2001. In what year will the population reach 40000?

11. The half life of a substance is 60 years.

(a) How long will it take the substance to decay to 20% of its original amount?

(b) How long will it take the substance to decay to $\frac{1}{16}$ of its original amount?



12. During a low tide, a boat is being towed to the dock by a rope. The rope is pulled from a position that is 7-ft above the water level at a rate of 2 ft/s. How fast is the boat approaching the ladder at the base of the dock when the boat is 24 ft from the ladder?
13. A street light is mounted at the top of a 20-ft pole. A 6-ft man walks away from the pole with a speed of 5 ft/s. How fast is the tip of his shadow moving when he is 30 ft from the pole? Note: This question is trickier than it seems.