7. $\vec{F} = \langle x+y, x-y\rangle$; C is the curve with initial and terminal points $(3,-2)$ and $(3,2)$, respectively, parametrized by $\vec{r}(t) = \langle 3t^2, 2t\rangle$ on $-1 \le t \le 1$.

$$\int_{-1}^{1} M g'(t) - N f'(t)\, dt$$

$$\int_{-1}^{1} (3t^2+2t)(2) - (3t^2-2t)(6t)\, dt$$

$$\int_{-1}^{1} 6t^2 + 4t - 18t^3 + 12t^2\, dt$$

$$\int_{-1}^{1} -18t^3 + 18t^2 + 4t\, dt$$

$$-\frac{9}{2}t^4 + 6t^3 + 2t^2 + C \Big|_{-1}^{1}$$

$$\left(\frac{7}{2} + C\right) - \left(\frac{-17}{2} + C\right)$$

$$= \frac{24}{2} = 12$$

$\langle t, t^2\rangle$ from $0 \le t \le 2$

$\langle t, 2t\rangle$ from $2 \le t \le 0$

$$\int_0^2 \vec{F}(r(t)) \cdot r'(t)\, dt$$

$$\langle t-t^2, t+t^2\rangle \cdot \langle 1, 2t\rangle$$

$$t - t^2 + 2t^2 + 2t^3$$

$$\int_0^2 t + t^2 + 2t^3\, dt$$

$$\frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{1}{2}t^4 + C \Big|_0^2$$

$2 + \frac{8}{3} + 8$

$\frac{38}{3} - 0 \quad = \frac{38}{3}$

$\int_2^0 \vec{F}(\vec{r}(t)) \cdot r'(t)\, dt$

$\langle -t, 3t \rangle \cdot \langle 1, 2 \rangle$

$\int_2^0 5t \, dt$

$\frac{5}{2} t^2 \Big|_2^0 \qquad -10$

$\frac{8}{3}$

$N_x - M_y$

$1 - -1$

2

$\int_0^2 \int_{x^2}^{2x} 2 \, dy \, dx$

$2y \Big|_{x^2}^{2x}$

$\int_0^2 4x - 2x^2 \, dx$

$2x^2 - \frac{2}{3}x^3 + C \Big|_0^2$

$8 - \frac{16}{3}$

$\frac{8}{3} - 0 \qquad \frac{8}{3}$

$$\int_0^2 (t-t^2)(2t) - (t+t^2)\,dt$$

$$\int_0^2 2t^2 - 2t^3 - t - t^2\,dt$$

$$\int_0^2 t^2 - 2t^3 - t\,dt$$

$$\frac{t^3}{3} - \frac{t^4}{2} - \frac{t^2}{2} \Big|_0^2$$

$$\frac{8}{3} - 8 - 2 = -2\frac{2}{3}$$

$$\int_2^0 -t \cdot (2) - 3t\,dt$$

$$\frac{8}{3}$$

$$\int_2^0 -5t\,dt$$

$$10 \qquad 8$$

$$\langle x-y, x+y \rangle$$

$$\nabla \cdot \vec{F}$$

$$1 + 1 = 2$$

$$2 \cdot \frac{4}{3} \qquad \frac{8}{3}$$

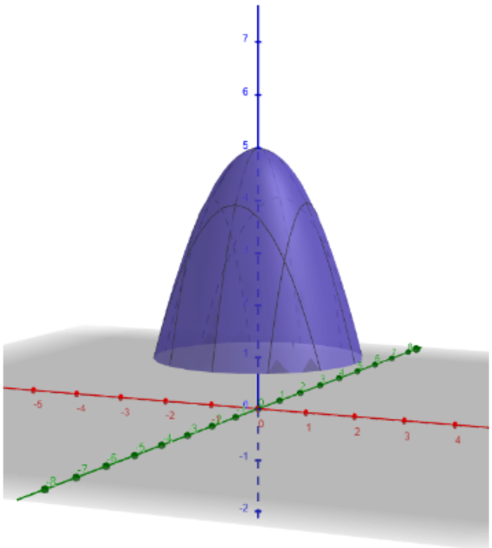
$$\vec{F}(x,y,z) = \begin{bmatrix} 3x+y \\ y^2 \\ x\sqrt{z} \end{bmatrix}$$

Let the surface S be the shape (circular paraboloid) defined by

$$z = -(x^2+y^2)+5, z \geq 1$$

Using Stokes' theorem and the curl function from question 7,[1] find the sum of the curl of $\vec{F}$ across S.[2]

You may find the following 3D graph of S helpful:



$\nabla \times \vec{F}$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$ $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$

$3x+y$ y^2 $x\sqrt{z}$ $3x+y$ y^2

$\langle 0, -\sqrt{z}, -1 \rangle$

$\oint_C \langle 0, -\sqrt{z}, -1 \rangle$

$-(x^2+y^2)+5 = 1$

$x^2+y^2 = 4$

$x^2+y^2 = r^2$

$z = 1$

$\langle 0, -1, -1 \rangle$

-1

4π

$\boxed{-4\pi}$