2025 Weston Calculus & Groups Pretest

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Each subquestion is worth 5 points. No points off for incorrect answers, so ***FEEL FREE TO GUESS***. I will also give partial credit so ***SHOW AS MUCH WORK AS YOU CAN!*** A calculator should not be necessary, but you may use one if you have one.

The point of this pretest is to see what you already know so I can pace this course. I expect you may not know the answers to many questions, but you will learn them if you take this course.

1. (a) Find the following limit: $\lim_{x\to 1} \frac{2x^2-2}{2x^2+x-3}$

$ab=-6$

$a+b=1$

$(2x^2-2x)+(3x-3)$

$(x-1)(2x+3)$

$$\frac{2(x+1)\cancel{(x-1)}}{\cancel{(x-1)}(2x+3)}$$

l'Hospital's rule

$$\lim_{x\to 1} \frac{2(x+1)}{2x+3} \to \frac{4}{5}$$

$$\frac{4x}{4x+1} \to \frac{4}{5}$$

(b) For the same equation $y = \frac{2x^2-2}{2x^2+x-3}$, find a tolerance δ such that for every x between $1-\delta$ and $1+\delta$, y is no more than 0.01 away from the answer you found in part (a). For example, if your answer for (a) was 4.2, then your y values would have to stay between 4.19 and 4.21.

$y(1-\delta) \geq .79$

$y(1+\delta) \leq .81$

$|f(x) - l| < \varepsilon$

$-0.8 <$

$$0.79 = \frac{2x+2}{2x+3}$$

$0.79(2x+3) < 2x+2$

$1.58x + 2.37 < 2x+2$

$0.37 < 0.42x$

$\frac{0.37}{0.42} < x$

$$0.81 = \frac{2x+2}{2x+3}$$

$1.62x+2.43 = 2x+2$ 0.881

$0.43 = 0.38x$

$\frac{0.43}{0.38}$ 1.132

$1-\delta > 0.881$

$1+\delta < 1.132$

$\delta < 0.132$ $\delta < 0.119$

2. Find the derivative of the function $f(x) = 6(x-3)^2 - x + 4$

$12(x-3)\cdot 1 - 1$

$12(x-3)-1$

$12x-36-1$

$12x-37$

3. Find the derivative of the function $f(x) = \dfrac{e^x \sin(x)}{x^2}$

$$\frac{x^2(e^x \cos x + e^x \sin x) - e^x \sin x \cdot 2x}{x^4}$$

4. (a) Find the indefinite integral (antiderivative) $\int -\sin(x) + x^3 \; dx$

$$\cos x + \frac{x^4}{4} + C$$

(b) Find the definite integral $\int_0^\pi -\sin(x) + x^3 \; dx$

$$\cos x + \frac{x^4}{4} + C \Big|_0^\pi$$

$$-1 + \frac{\pi^4}{4} - (1 + 0)$$

$$-2 + \frac{\pi^4}{4}$$

Name:______________________

5. Consider this Taiwanese recycling symbol


(symbol)

(a) List all the "moves" you can do (reflections or rotations) that leave the symbol appearing identical. (Don't forget to include leaving it still!). Moves that leave the symbol in the same position as each other should be treated as the same move (e.g. a 180° clockwise turn, a 540° clockwise turn, and a 180° counterclockwise turn are all the same move)

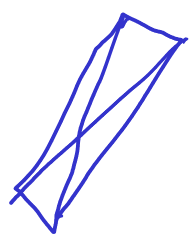
still, 90°, 180°, 270°
0°

(b) To "combine" two moves, you perform one and then the other, and see what single move would leave the symbol in that same new position. For instance, a 180° turn plus another 180° turn equals leaving the symbol still. Make a full table of all possible combinations of two moves of the symbol.

	0°	90°	180°	270°
0°	0°	90°	180°	270°
90°	90°	180°	270°	0°
180°	180°	270°	0°	90°
270°	270°	0°	90°	180°

6. Consider the flag of Scotland


(flag)



(a) List all the "moves" you can do (reflections or rotations) that leave the flag appearing identical. Note that the flag is rectangular, and not square. (Again, don't forget to include leaving it still!)

still 180° flipx flipy

(b) Make a full table of all possible combinations of two moves of the flag.

	S	180	fx	fy
S	S	180	fx	fy
180	180	S	fy	fx
fx	fx	fy	S	180
fy	fy	fx	180	S

7. The moves on the shapes in the two previous problems, plus the rules for combining them, are those shapes' "symmetry groups", a common kind of "group". In general, groups consist of some basic objects ("elements") and rules for combining them (an "operation"). For symmetry groups, the elements are moves and the operation is combining them as described in question 5.

   Two groups A and B are said to be "isomorphic" to each other if you can pair up the elements of A with those of B so that the results of the operation always match up correctly (i.e. if in A, $uv = w$ and in B, $xy = z$, and u is paired with x and v is paired with y, then w must be paired with z).

   (a) Are the symmetry groups of the symbol and the flag isomorphic to each other? If so, give one appropriate pairing between their elements (moves). If not, explain why they are not.

   No: for the flag, all the moves undo themselves (are their own inverse), but not for the symbol.

   (b) Consider the group (named $\mathbb{Z}_4$) whose table is below. Its elements are the numbers $0, 1, 2, 3$ and its operation is adding two numbers and taking the remainder when you divide the result by 4.

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

   Is $\mathbb{Z}_4$ isomorphic to either the flag's or the recycling symbol's symmetry group? If so, give one appropriate pairing between their elements. If not, explain why not.

   yes: symbol

   0 → still
   1 → 90°
   2 → 180°
   3 → 270°

   or

   0 → still
   1 → 270°
   2 → 180°
   3 → 90°