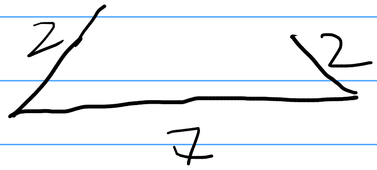
For all natural #'s ....

"Any 3 sides make a triangle"
↑lengths



Any 3 side lengths a, b, c make a triangle iff

$a+b > c$
$a+c > b$
& $b+c > a$

$\mathbb{N}$ $\mathbb{N}$ 0, 1, 2, 3 ...

... −3, −2, −1, 0, 1, 2, 3 ...

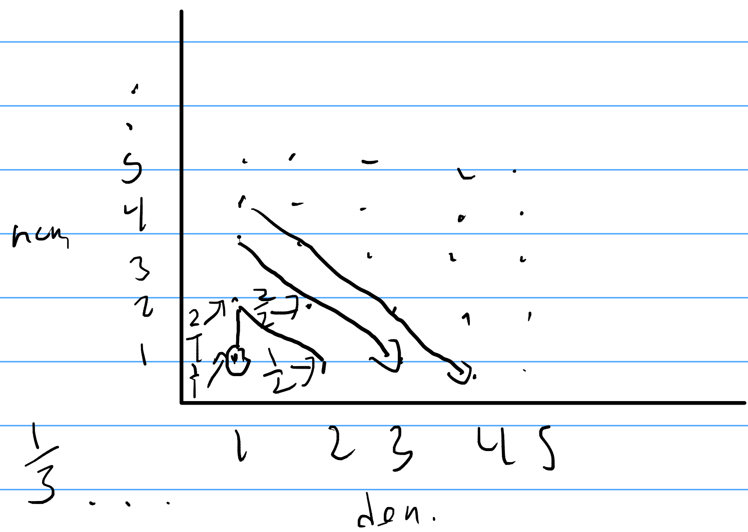
0, 2, 4, 6, 8 ...

0, 1, 2, 3, 4

0, 2, 4, 6, 8

0, 1, -1, 2, -2, 3, -3, 4, -4

$\frac{a}{b}$



$1, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{2}, \frac{1}{3} \ldots$

0, 1, 2, 3 . . .

0, 1, 2

Hilbert's hotel

Real #'s "Countable infinity"

"Listable infinity"

Prove: there is a real not on this list

"Uncountable"
unlistable

0.120935...

0.3045 60...

3.141592...

4.29 06

⋮

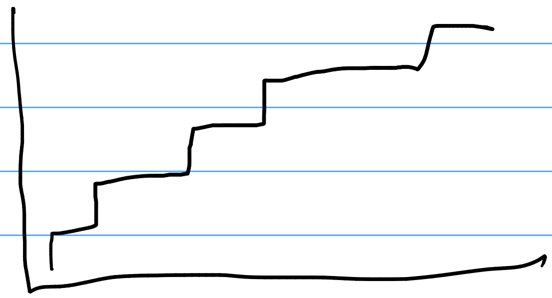
0.2127...

$\aleph_0$

Prime counting function

$\pi(x)$

$\pi(10)=4$ 2, 3, 5, 7



$\int_0^x \frac{1}{\ln(x)}\,dx$ $li(x)$

$li(1{,}000{,}000)$

"Conjecture" Skewe's #

$10^{10^{10^{964}}}$

10^{384}

Disprove a

There exists

Perfect #'s

6   1, 2 3 $1+2+3=6$

28   1, 2, 4, 7, 14 $1+2+4+7+14=28$

are there odd perfect #'s? We don't know!

Collatz Conjecture

even: $\frac{x}{2}$

odd: $3x+1$

$6 \to 3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$

10^{20}

Riemann hypothesis

Fermat's last theorem

pyth: $a^2+b^2=c^2$

$n>2$ $a^n+b^n=c^n$

then a, b, c are not all positive integers

$12^3+1^3=10^3+9^3$

1729   1729

$$a^4 + b^4 + c^4 = d^4$$

$$a^5 + b^5 + c^5 + d^5 = e^5$$

Clock arithmetic

modular arithmetic

11 0 1

10 2

9 3

8 4

7 6 5

0 1 "mod 6"

5 2

4 3

4 0 1

3 2

0

1

0

units: 1, 5

2, 4

3

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

mod 10

$9 \equiv -1$

$1 \equiv 1$

is -1 prime

-1 , 1

Any # that's a factor of 1

"unit"

Fermat's little theorem

for a prime p $\qquad a^p \bmod p = a \bmod p$

$16^5 \bmod 5 = 16 \bmod 5$

Combinatorial proof

A B C $\qquad 3^7 \bmod 7 = 3 \bmod 7$

A B C A C B A $\qquad 3^7$

A A A A A A A

B B B B B B B $\rightarrow 3$

C C C C C C C

A

A B

B C

C A

A

B A

C B

A C