7. $\vec{u} = \langle 3, 2, -2 \rangle, \quad \vec{v} = \langle 0, 1, 5 \rangle$

$0\hat{k} \quad -2\hat{i} \quad 15\hat{j}$

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & 2 & -2 & 3 & 2 \\ 0 & 1 & 5 & 0 & 1 \end{array}$$

$10\hat{i} \quad 0\hat{j} \quad 3\hat{k}$

$\langle 10, 0, 3 \rangle - \langle -2, 15, 0 \rangle = \langle 12, -15, 3 \rangle$

9. $\vec{u} = \langle 4, -5, -5 \rangle, \quad \vec{v} = \langle 3, 3, 4 \rangle$

$-15\hat{k} \quad -15\hat{i} \quad 16\hat{j}$

$$\begin{array}{ccccc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 4 & -5 & -5 & 4 & -5 \\ 3 & 3 & 4 & 3 & 3 \end{array}$$

$-20\hat{i} \quad -15\hat{j} \quad 12\hat{k}$

$\langle -20, -15, 12 \rangle - \langle -15, 16, -15 \rangle$

$\langle -5, -31, 27 \rangle$

Prove: if x is odd then x+1 is even

$x = 2y+1$

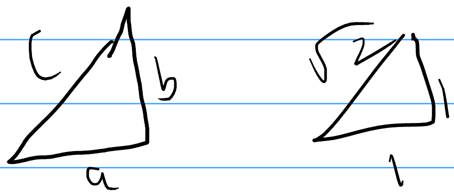
$x+1 = 2y+1+1 = 2y+2$

$x+1 = 2(y+1)$ $\square$ QED

Prove $\sqrt{2}$ is irrational

$\sqrt{2}$: x where $x^2 = 2$

irrational: a quantity thet can't be written as a fuction $\frac{a}{b}$, a+b are integers



Proof by contradiction

Suppose (for contradiction) $\exists\ \frac{a}{b} = \sqrt{2}$, where $a, b \in \mathbb{Z}$

we can assume b is +, if b were negative, use

$\frac{-a}{-b}$ assume a, b as small as possible

$$\left(\frac{a}{b}\right)^2 = 2$$

$$\frac{a^2}{b^2} = 2 \qquad a^2 = 2b^2$$

is b odd or even?

if b is odd: $2b^2$ is a multiple of 2, but not 4

a^2 must be a multiple of 2 and not 4

is a even or odd

If a is odd then a^2 is not a multiple of 2

if a is even then a^2 is a multiple of 4

thus a is not an integer, and can't exist by our assumptions

thus b is not odd

if b is even:

$a^2 = 2b^2$ $\qquad$ b^2 is a multiple of 4

a^2 is also a multiple of 4

$\Rightarrow \frac{a}{2}$ & $\frac{b}{2}$ are integers

that means $\left(\frac{a}{2}\right)^2 = 2\left(\frac{b}{2}\right)^2$ which contradicts

the assumption that a & b are as small as possible group: set of elements with an operation $a \circ b$

1. $(a \circ b) \circ c = a \circ (b \circ c)$
2. there is an "identity" e
   where $e \circ a = a$ and $a \circ e = a$
3. "closed": if a & $b \in G$ then $a \circ b \in G$
4. "inverse": for a, $\exists\, a^{-1}$: $a \circ a^{-1} = a^{-1} \circ a = e$

abstract

$\mathbb{Z}, +$ is a group

$a \in \mathbb{Z} + b \in \mathbb{Z} = c \in \mathbb{Z}$

identity: 0

$(a+b)+c = a+(b+c)$ $\qquad a \neq b$

$a^{-1} = -a$ $\qquad a \notin \mathbb{Z}$

integers, times is not a group

because the inverse of, 2 is not an integer

Triangle #'s $1+2+3+4+\dots$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

induction

base case ✓ Prove the formula works for $n=1$

induction step Prove that if the formula works for $n=a$, it also works for $n=a+1$

$n=1$ $$1 = \frac{1(1+1)}{2} = \frac{1\cdot 2}{2} = 1$$

Suppose $1+2+\dots a = \frac{a(a+1)}{2}$

Show: $1+2+\dots a+a+1 = \frac{(a+1)(a+1+1)}{2}$

$$\frac{a(a+1)}{2} + a+1 \qquad \frac{a^2+a}{2} + a+1$$

$$\frac{a^2+a}{2} + \frac{2(a+1)}{2} \qquad \frac{a^2+a+2(a+1)}{2} = \frac{a^2+3a+2}{2}$$

$$\frac{(a+1)(a+2)}{2} = \frac{a^2+3a+2}{2}$$

QED

"Prove": all horses are the same color

base case: 1 horse is the same color as itself

induction step: if we have all groups of n horses the same color then all groups of $n+1$ horses are the same color