7. $\vec{F} = \langle y, y^2 \rangle$; C is the line segment from $(0,0)$ to $(3,1)$.

$\langle 3t, t \rangle \quad t = [0,1]$

$\langle t, \frac{t}{3} \rangle \quad t = [0,3]$

$\int_C \vec{F} \cdot d\vec{r}$

$\int_0^1 \vec{F}(\vec{r}(t)) \cdot r'(t)\, dt$

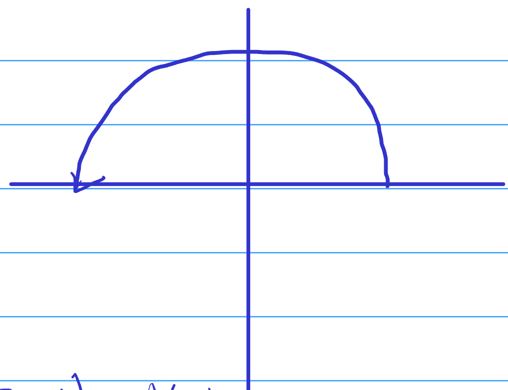
$\langle 3t, t^2 \rangle \cdot \langle 3, 1 \rangle$

$\int_0^1 9t + t^2\, dt$

$\frac{9}{2}t^2 + \frac{t^3}{3} + C \Big|_0^1$

$\frac{9}{2} + \frac{1}{3} = \frac{29}{6}$

9. $\vec{F} = \langle y, x \rangle$; C is the top half of the unit circle, beginning at $(1,0)$ and ending at $(-1,0)$.



$\langle \cos(t), \sin(t) \rangle$

$t = [0, \pi]$

$\int_0^\pi \vec{F}(\vec{r}(t)) \cdot r'(t)\, dt$

$\langle \sin(t), \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle\, dt$

$\int_0^\pi \cos^2(t) - \sin^2(t)\, dt \qquad \int_0^\pi \cos(2t)\, dt$

$$\frac{\sin(2t)}{2} + C \Big|_0^{2\pi}$$

$= 0$

In Exercises 17 – 20, a conservative vector field $\vec{F}$ and a curve C are given.

1. **Find a potential function f for $\vec{F}$.**
2. **Compute** curl **$\vec{F}$.**
3. **Evaluate $\int_C \vec{F} \cdot d\vec{r}$ directly, i.e., using Key Idea 14.3.1.**
4. **Evaluate $\int_C \vec{F} \cdot d\vec{r}$ using the Fundamental Theorem of Line Integrals.**

17. $\vec{F} = \langle y + 1, x \rangle$, C is the line segment from $(0, 1)$ to $(1, 0)$.

1. $xy + x$

4. $1 - 0 = 1$

2. $N_x - M_y$

$1 - 1$

3 $\langle t, 1-t \rangle$

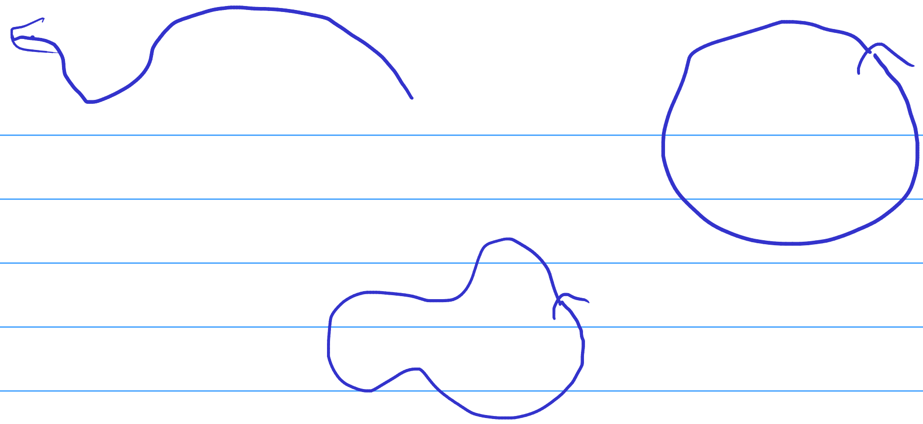
$$\int_0^1 \langle 2-t, t \rangle \cdot \langle 1, -1 \rangle \, dt$$

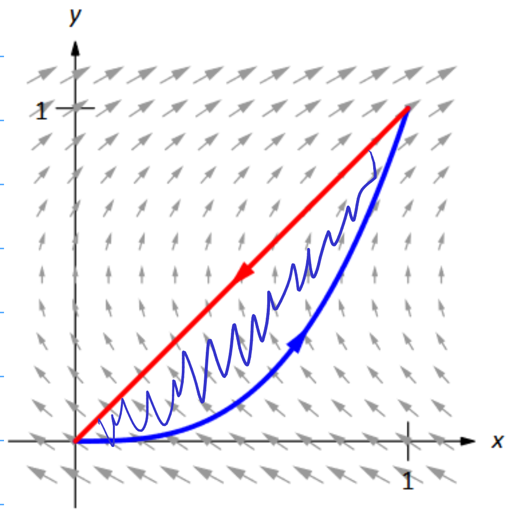
$$\int_0^1 2-t-t$$

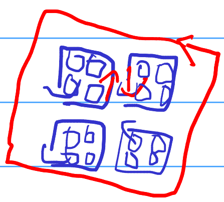
$$\int_0^1 2-2t \, dt$$

$$2t - t^2 + C \Big|_0^1$$

1







$\vec{F} = \langle -y, x^2+1 \rangle$

$\vec{r}_1(t) = \langle 2t-1, 0\rangle,$
$\vec{r}_2(t) = \langle 1-t, 2t\rangle,$
$\vec{r}_3(t) = \langle -t, 2-2t\rangle,$

$(-1,0)$, $(1,0)$, $(0,2)$

$$\int_0^1 \vec{F}(r_1(t)) \cdot r_1'(t)\,dt$$

$$\langle 0, 4t^2-4t+2\rangle \cdot \langle 2, 0\rangle$$

0

r_2: $\langle -2t, 2-2t+t^2 \rangle \cdot \langle -1, 2 \rangle$

$4-4t+2t^2+2t$

$\int_0^1 4-2t+2t^2 \, dt$

$4t-t^2+\frac{2}{3}t^3+C \Big|_0^1$

$4-1+\frac{2}{3}-0$

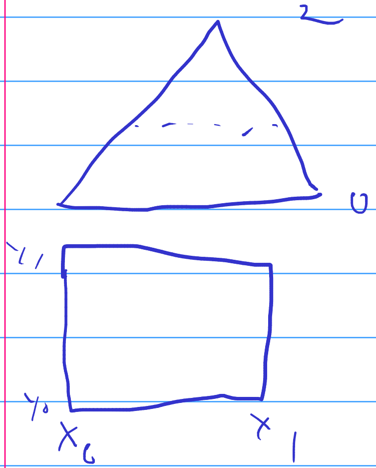
$\frac{11}{3}$

$F=\langle -y, x^2+1 \rangle$

$N_x - M_y$

$2x+1$

$$\int_0^2 \int_{y/2-1}^{1-y/2} (2x+1)\,dx\,dy = \int_0^2 (2-y)\,dy = 2,$$



$x^2+x \Big|_{y/2-1}^{1-y/2}$

$\int_0^2 2-y \, dy = 2$

$y=2x+2$

$y-2=2x$

$\frac{y}{2}-1=x$

Circle: $\langle 2\cos(t), 2\sin(t) \rangle$

Field: $\langle x-y, x+y \rangle$

Mg': $(2\cos(t) - 2\sin(t))(2\cos(t))$

Nf': $(2\cos(t) + 2\sin(t))(-2\sin(t))$

$\langle M, N \rangle$: Field

$\langle f(t), g(t) \rangle$: Curve

$[2\cos(t) - 2\sin(t))[2\cos(t)] - (-[2\cos(t) + 2\sin(t)][2\sin(t)])$

$4\cos^2(t) - 4\sin(t)\cos(t) + 4\sin(t)\cos(t) + 4\sin^2(t)$

$= 4$

$$\int_0^{2\pi} 4 \, dt$$

$$4t + C \Big|_0^{2\pi} = 8\pi$$

$\nabla \cdot \vec{F}$

$$\frac{\partial}{\partial x} M + \frac{\partial}{\partial y} N = 1 + 1 = 2$$

$$\iint_R \nabla \cdot \vec{F} \, dA = 8\pi$$