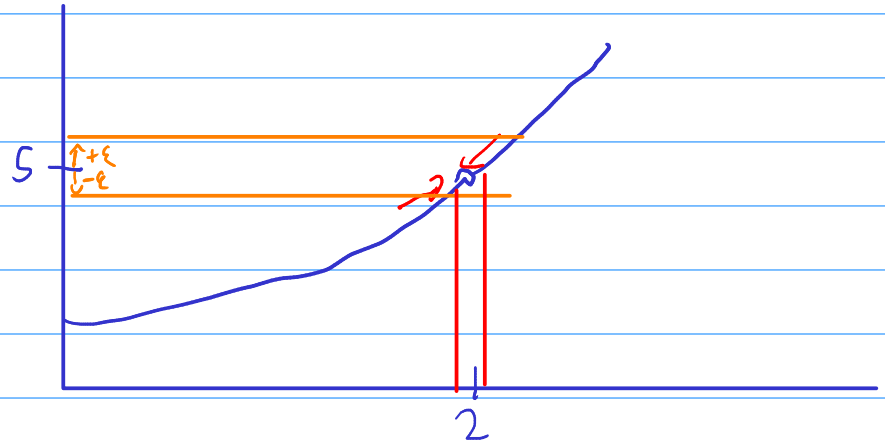
$y = x^2 + 1$

$y = \dfrac{(x^2+1)(x-2)}{(x-2)}$

$\lim_{x \to 2} y = 5$

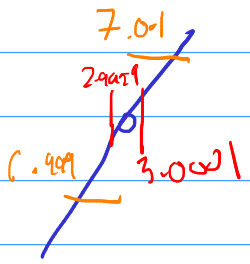
Δ δ Delta

E ε Epsilon



$y = \frac{(2x+1)(x-3)}{(x-3)}$ $\lim_{x\to 3} y = 7$

.999
7.001 $\varepsilon = 0.001$



$3-\delta$ 0.0001
$3+\delta$ 2.9999
3.0001

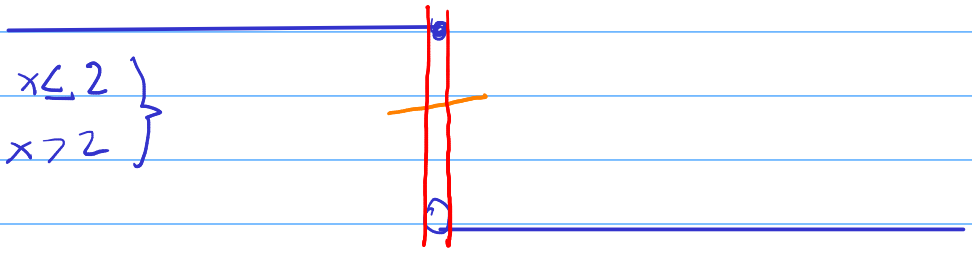
$f(x) = 0.99999$

$\varepsilon = 0.0000001$

$\lim_{x\to 3} f(x) \stackrel{?}{=} 1$

$\varepsilon = 1$

$f(x) = \begin{cases} 1, & x \le 2 \\ -1, & x > 2 \end{cases}$



$\lim_{x\to 2} f(x)$ DNE $\lim_{x\to 2^-} = 1$

$\lim_{x\to 2^+} = -1$

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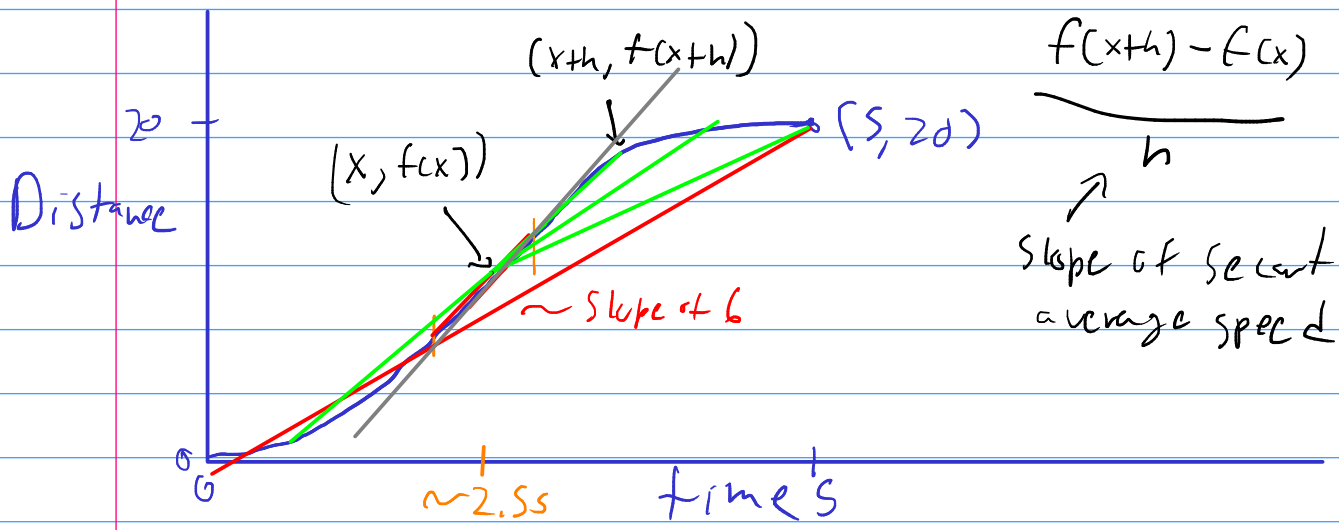
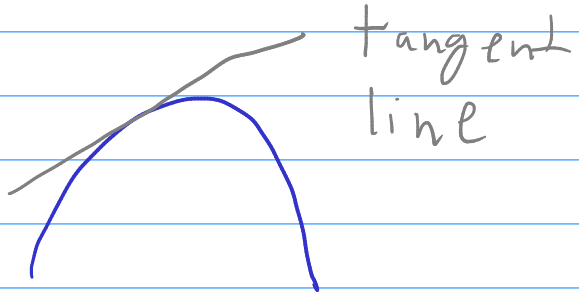


$\frac{f(x+h)-f(x)}{h}$

↑ Slope of secant line
average speed

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ← limit definition of the derivative



$f(x) = 2x^2+1$

$x = -1$

$\lim_{h\to 0} \frac{2(x+h)^2+1-(2x^2+1)}{h}$

@ $x=-1$

$f'(x) = 4x$

$\frac{2(x^2+2xh+h^2)+1-(2x^2+1)}{h}$

$\frac{2x^2+4xh+2h^2+1-2x^2-1}{h}$ $\quad 4x+2h$

$4x$

-4