Challenge problem:

Prove $\log_{10} 2$ is irrational

Suppose a b are smallest integers such that $\frac{a}{b} = \log_{10} 2$

$a = b \log_{10} 2$

$a = \log_{10} 2^b$

$10^a = 10^{\log_{10}(2^b)}$

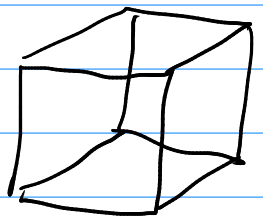
$10^a = 2^b$

$5^a = 2^{b-a}$

5^a: odd, 2^{b-a}: even

contradiction so no $\frac{a}{b} = \log_{10} 2$

$V - E + F =$ euler characteristic

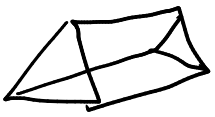


$F = 6$

$V = 8$

$E = 12$

$8 - 12 + 6 = 2$

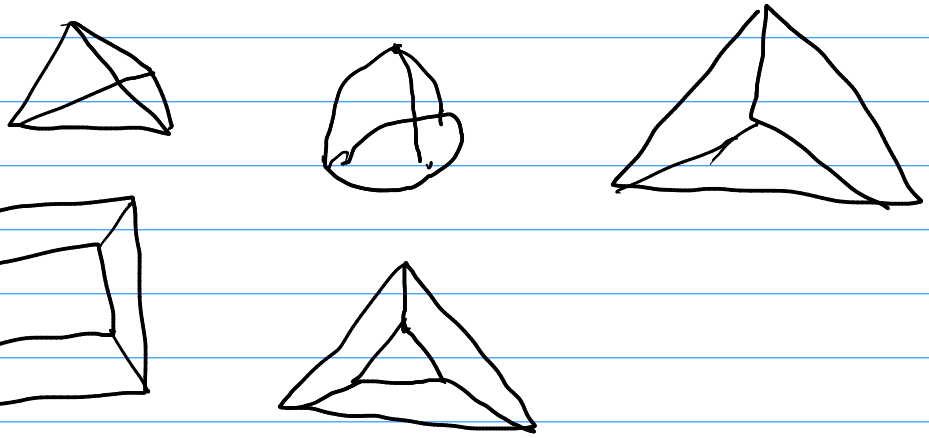


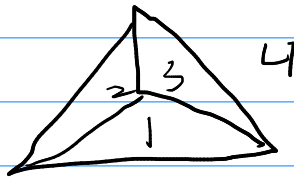
$V = 6$
$E = 9$
$F = 5$

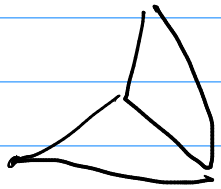
$V - E + F = 2$

$V = 5$
$E = 8$
$F = 5$

$V - E + F = 2$





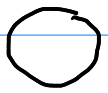


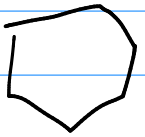
$F \to \downarrow 1$
$E \downarrow 1$
$V - =$

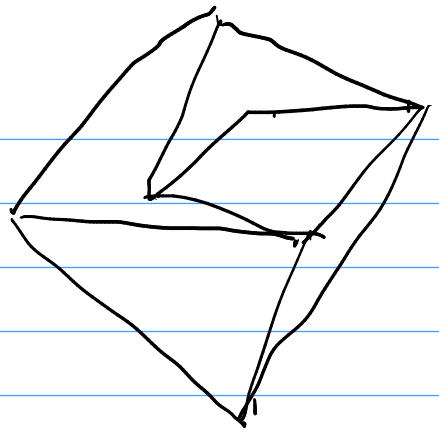
$F =$
$E \downarrow 1$
$V \downarrow 1$

$V - E + F$

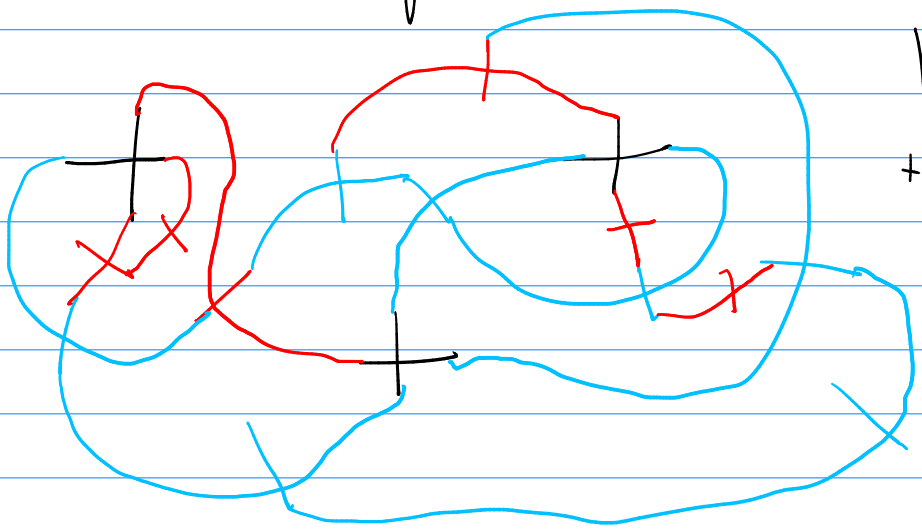
$0 + 2$





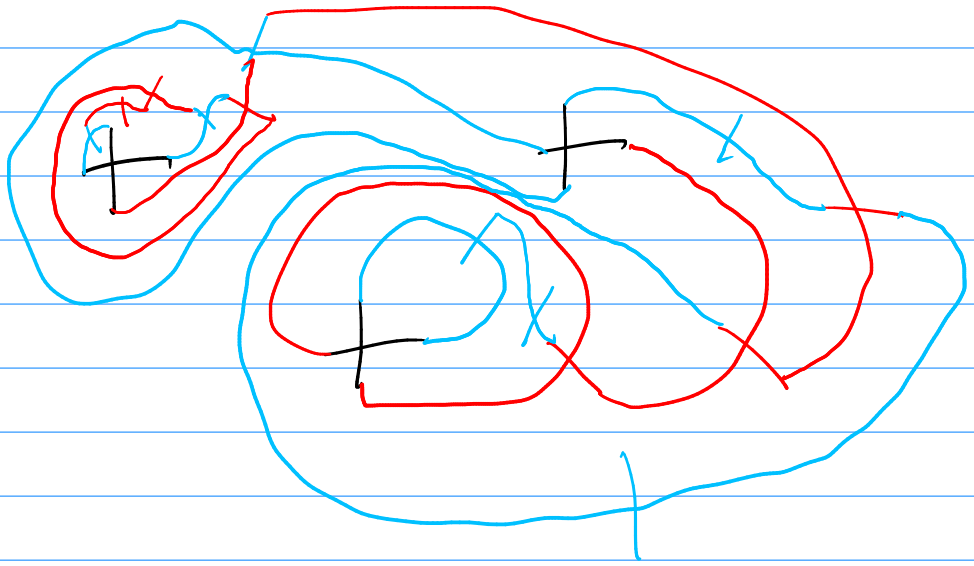


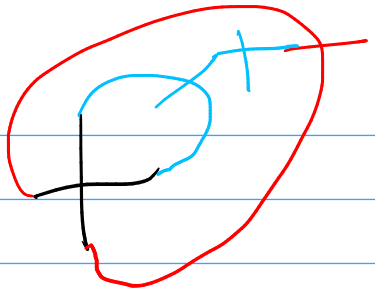
Brussels sprouts

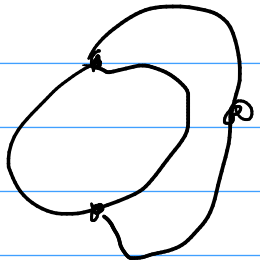


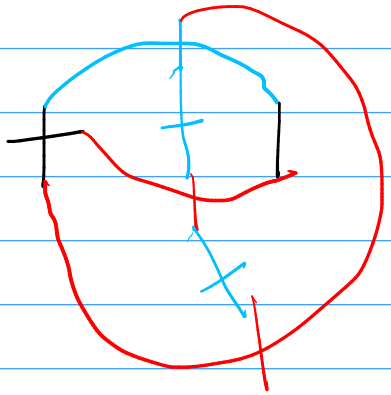
$V - E + F$

$+1 \quad +2 \quad +1$









Peano Arithmetic

1. 0 is a natural number.
2. There is a relation "=" between natural numbers with the properties:
   - x=x
   - x=y => y=x
   - x=y & y=z => x=z
   - natural numbers only equal other natural numbers

$x \in \mathbb{N}$

3. There is a function called the "Successor function" [S(n)] with the properties:
   - if n is a natural number, S(n) is a natural number
   - if S(m)=S(n), then m=n
   - There is no natural number n such that S(n) is 0
   - 0, S(0), S(S(0)), etc. are all the natural numbers

Define "+":

$$a + 0 = a$$
$$S(a) + b = S(a+b)$$

1: $S(0)$ 2: $S(S(0))$

$S(0) + S(0) = S(S(0))$

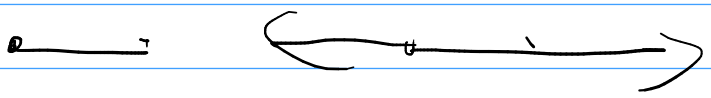
$S(0 + S(0))$

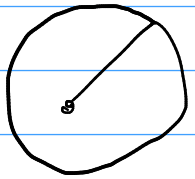
$S(S(0)) = S(S(0))$

Define ".": $0 \cdot a = 0$

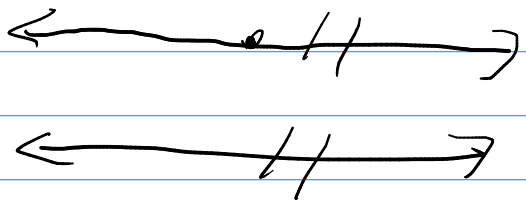
$a \cdot S(b) = a \cdot b$ then $+a$

Euclid's Postulates

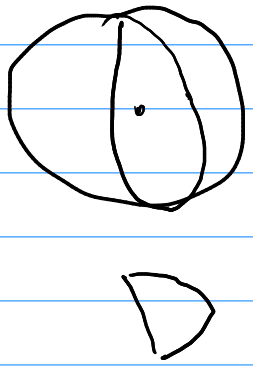




"fifth postulate"



Elliptic



Hyperbolic