Example ϵ: 0.001

δ

$2(x+\delta)+5 = 13.001$

$-5 \quad -5$

$2(x-\delta)+5 = 12.999$

$-5 \quad -5$

$2(x-\delta) = 7.999$

$2(x+\delta) = 8.001$

$2(4-\delta) = 11$

$2(4+\delta) = 11$

$4-\delta = 3.9995$

$4+\delta = 4.0005$

$\delta = 0.0005$

$\delta = 0.0005$

$\delta = \frac{\epsilon}{2} \qquad \delta \leq \frac{\epsilon}{2}$

$2(x+\delta)+5 = 13+\epsilon$

$2(x-\delta)+5 = 13-\epsilon$

$2(x+\delta) = \delta+\epsilon$

$x+\delta = 4+\frac{\epsilon}{2}$

$2(x-\delta) = \delta-\epsilon$

$x-\delta = 4-\frac{\epsilon}{2}$

$4+\delta = 4+\frac{\varepsilon}{2}$ $\qquad \delta \leq \frac{\varepsilon}{2}$

$4-\delta = 4-\frac{\varepsilon}{2}$

$\lim_{x \to 3} x^2 - 3 = 6$

$(x+\delta)^2 - 3 = 6+\varepsilon$

$(x-\delta)^2 - 3 = 6-\varepsilon$

$(x+\delta)^2 = 9+\varepsilon$

$(x-\delta)^2 = 9-\varepsilon$

$x+\delta = \sqrt{9+\varepsilon}$

$x-\delta = \sqrt{9-\varepsilon}$

$3+\delta = \sqrt{9+\varepsilon}$ $\qquad \delta = \sqrt{9+\varepsilon} - 3$

$3-\delta = \sqrt{9-\varepsilon}$ $\qquad \delta = 3 - \sqrt{9-\varepsilon}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \frac{6-6}{h}$$

$$\lim_{h \to 0} \frac{0}{h} = 0$$

$$\lim_{h \to 0} \frac{4-3(t+h) - (4-3t)}{h}$$

$$\frac{4-3t-3h-4+3t}{h} = -3$$

$$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$\lim_{h \to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$3x^2$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

x^3 $\quad 3x^2$

x^5 $\quad 5x^4$

$x^{0.5}$ $\quad \frac{1}{2}x^{-\frac{1}{2}}$

x^{-2} $\quad -2x^{-3}$

$\frac{1}{x} = x^{-1}$ $\quad -x^{-2} = \frac{-1}{x^2}$

$\frac{1}{\sqrt[3]{x}}$ $\quad x^{-\frac{1}{3}}$ $\quad -\frac{1}{3}x^{-\frac{4}{3}}$

Power rule

2^x $\quad \frac{d}{dx} e^x = e^x$

$e^{(\ln(2))x}$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} \sin x = \cos x$$

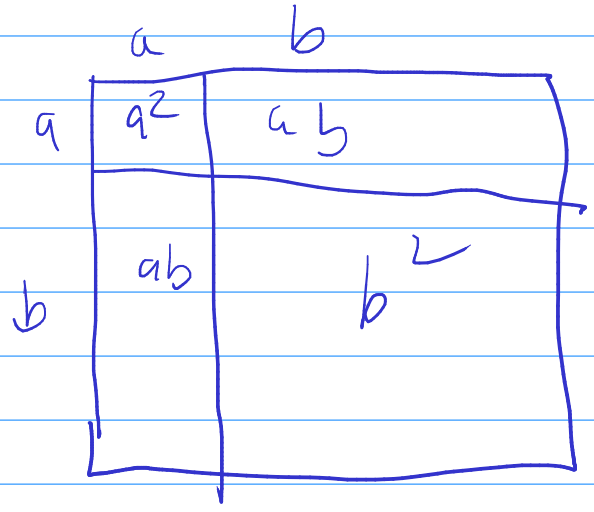
$$\frac{d}{dx} \cos x = -\sin$$

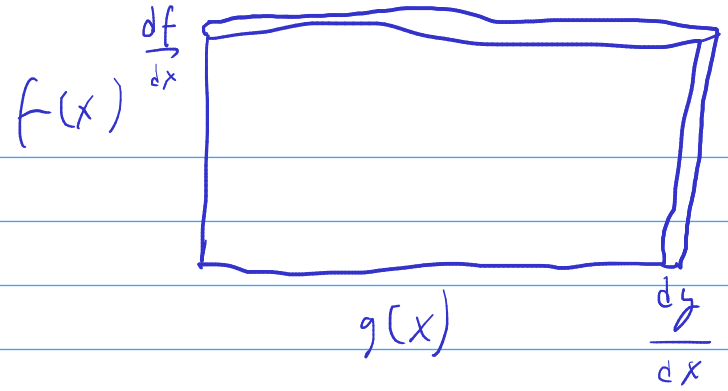
$$\frac{d}{dx} 3\sin x = 3\cos x$$

$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$

$$\frac{d}{dx} -e^x + 3\sin x - 4$$

$$-e^x + 3\cos x + 0$$





$f'(x)g(x) + g'(x)f(x)$

$$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + g'(x)f(x)$$

$\frac{\sin(x)}{x}$ $\quad x^{-1}\sin x$

$-x^{-2}\sin x + \cos(x)\,x^{-1}$

$f(x) \quad g(x)$

$e^x \cdot \cos x$

$e^x(-\sin x) + \cos(x)e^x$

$f(x)\,g'(x) \quad + \quad g(x)\,f'(x)$

$f(x) \quad g(x)$

$3x^3 \ln(x)$

$3\left(3x^2\ln(x) + \frac{1}{x}x^3\right)$

$f'(x)\,g(x)$

$3(3x^2\ln x + x^2)$

$9x^2\ln x + 3x^2$

Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Quotient Rule

$\frac{\sin(x)}{x}$ $\quad \frac{x(\cos(x) - \sin(x)\cdot 1}{x^2}$ $\quad \frac{x\cos x}{x^2} - \frac{\sin x}{x^2}$

$-x^{-2}\sin x + \cos(x)\,x^{-1}$

$\frac{-\sin x}{x^2} + \frac{\cos x}{x}$

$8\cdot 4\cdot x^3 \qquad 5\cdot 2\cdot x$

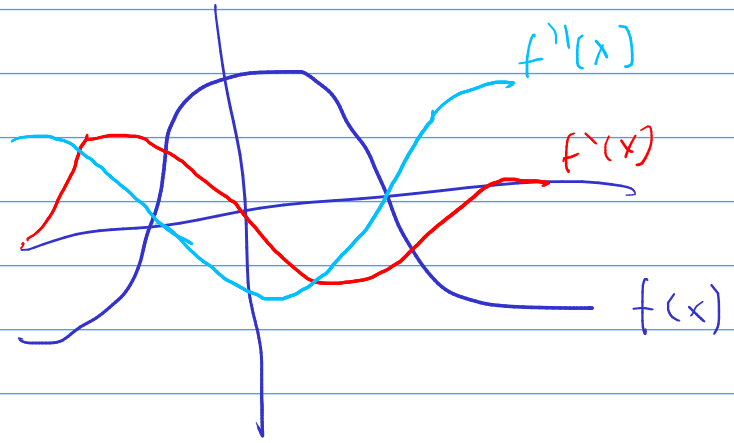
$\frac{-\cos x}{8x^4+5x^2}$

$\frac{(8x^4+5x^2)(\sin x) - (-\cos x)(32x^3+10x)}{(8x^4+5x^2)^2}$

$f(x)$ $x^3 + \sin x + 2x + 1$

$f'(x)$ $3x^2 + \cos x + 2$

$f''(x)$ $6x - \sin x$



$f''(x)$ tells you about CONCAVITY

$f(x)$ Position

$f'(x)$ Velocity

$f''(x)$ Acceleration

$f'''(x)$ Jerk

$f^{(4)}(x)$ Snap

Crackle

Pop

$f^{(5)}x$

$\frac{d}{dx}\tan(x)$ $\frac{d}{dx}\frac{\sin(x)}{\cos x}$ $\frac{\cos x(\cos x) - \sin(x)(-\sin x)}{(\cos x)^2}$

$\frac{d}{dx}\tan(x) = \sec^2(x)$ $\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{1}{\cos^2 x}$

cot $\frac{\cos x}{\sin x}$ $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ $\frac{-1}{\sin^2 x}$

$-\csc^2 x$

$\csc x = \frac{1}{\sin(x)}$ $\frac{\sin x(0) - \cos x}{\sin^2(x)}$

$\frac{-\cos x}{\sin^2 x}$ $-\cot x \csc x$

$\sec x = \frac{1}{\cos x}$ $\frac{\cos x(0) + 1(\sin x)}{\cos^2 x}$ $\frac{\sin x}{\cos^2 x}$ $\tan x \sec x$