2023 Weston Calc 3 Pretest

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1. Let $\vec{u} = \langle 2, 0, -1 \rangle$ and $\vec{v} = \langle 0, 1, 3 \rangle$. Find the result of each of the following, and indicate whether each is a vector or a scalar (1 pt for each result, 1 pt for vector/scalar) :

   (a) $3\vec{u}$

   (b) $\vec{u} + \vec{v}$

   (c) $\vec{u} \cdot \vec{v}$

   (d) $\vec{u} \times \vec{v}$

   (e) $\vec{v} \times \vec{u}$

   (f) $\|\vec{u}\|$

   (g) A unit vector parallel to $\vec{u}$

   (h) A unit vector perpendicular to $\vec{v}$

2. Let $f(x)$ be the vector valued function $\begin{bmatrix} \cos(\ln(x)) \\ e^{2x} \end{bmatrix}$

   Find $\frac{df}{dx}$

3. Let $f(x, y, z) = 2x^2 + x\sin(y) + \cos(y) + \ln(z)$
   Find each of the following partial derivatives (4 pts each):

   (a) $\frac{\partial f}{\partial x}$

   (b) $\frac{\partial f}{\partial y}$

   (c) $\frac{\partial f}{\partial z}$

4. Let $f(x, y, z) = 2x^2 + x\sin(y) + \cos(y) + \ln(z)$ and point $p = (1, \frac{\pi}{2}, e)$
   Find the gradient of f at point p

5. Consider the surface S in $\mathbb{R}^3$ described by the function $z = \sin(x) + y^{\frac{1}{3}}$

   (a) Find an equation of the normal (perpendicular) line to S at the point $(\pi, -8)$.

   (b) Find an equation of the tangent plane to S at the same point $(\pi, -8)$.

6. Again, consider the surface S in $\mathbb{R}^3$ described by $z = \sin(x) + y^{\frac{1}{3}}$
   Find the volume between S and the xy plane within the box with corners at the points $(0, 0), (\pi, 0), (0, 1), (\pi, 1)$.

7. Consider the vector field

$$\vec{F}(x, y, z) = \begin{bmatrix} 3x + y \\ y^2 \\ x\sqrt{z} \end{bmatrix}$$

   (a) Find the function for the divergence of $\vec{F}$

   (b) Find the function for the curl of $\vec{F}$

8. Again, consider the vector field

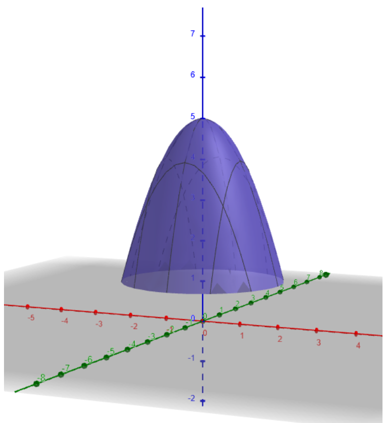
$$\vec{F}(x,y,z) = \begin{bmatrix} 3x+y \\ y^2 \\ x\sqrt{z} \end{bmatrix}$$

Let the surface S be the shape (circular paraboloid) defined by

$$z = -(x^2+y^2)+5, z \geq 1$$

Using Stokes' theorem and the curl function from question 7,[1] find the sum of the curl of $\vec{F}$ across S.[2]

You may find the following 3D graph of S helpful:



[1]This is a hint. How can you get Stokes' theorem to help even more than usual since you already know the curl function? If you find yourself taking the integral of $\sin^2 x$, you are not taking full advantage of the hint, but if you want to continue down that path it's dangerous to go alone, take this: $\sin^2(x) = \frac{1-\cos(2x)}{2}$, $\sin(2x) = 2\sin(x)\cos(x)$

[2]Technically it's the sum of the *flux* of the curl to make it a scalar, but don't worry about the distinction. Stokes' theorem is the last topic we plan to cover in the course.