5. $f(x, y) = x^2y - x + 2y + 3$ at $(1, 2)$

$f_x(x,y) = 2xy - 1$ $\quad$ 3

$f_y(x,y) = x^2 + 2$ $\quad$ 3

9. $f(x, y) = x^2y + 3x^2 + 4y - 5$

$f_x(x,y) = 2xy + 6x$

$f_y(x,y) = x^2 + 4$

$f_{xx}(x,y) = 2y + 6$

$f_{yy}(x,y) = 0$

$f_{xy}(x,y) = 2x$

$f_{yx}(x,y) = 2x$

11. $f(x, y) = \dfrac{x}{y}$

$f_x(x,y) = \dfrac{1}{y}$

$f_y(x,y) = \dfrac{-x}{y^2}$

$f_{xx}(x,y) = 0$

$f_{yy}(x,y) = \dfrac{2x}{y^3}$

$f_{xy}(x,y)$ $-\dfrac{1}{y^2}$

7. $z = 3x + 4y, \quad x = t^2, \quad y = 2t; \quad t = 1$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\left\langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right\rangle \cdot \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

$$3(2t) + 4(2) = 6t + 8 = 14$$

Directional Derivatives

$$D_{\vec{u}} f(x_0, y_0) = f_x(x_0, y_0)u_1 + f_y(x_0, y_0)u_2.$$

$$\left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$$

$z = 14 - x^2 - y^2 \qquad P(1, 2)$

direction: $\langle 2, -1 \rangle$

$$\left\langle \underset{u_1}{\frac{2}{\sqrt{5}}}, \underset{u_2}{\frac{-1}{\sqrt{5}}} \right\rangle$$

$f_x = -2x$

$f_y = -2y$

$$-2x \cdot \frac{2}{\sqrt{5}} + -2y\left(\frac{-1}{\sqrt{5}}\right)$$

$$\frac{-4}{\sqrt{5}} + \frac{4}{\sqrt{5}} = 0$$

$f(x,y) = \cos(xy)$ $\vec{v} = \langle \pi, -2 \rangle$

$f_x = -y\sin(xy)$ $p = (1, \frac{\pi}{2})$

$f_y = -x\sin(xy)$ $\hat{v} = \left\langle \frac{\pi}{\sqrt{\pi^2+4}}, \frac{-2}{\sqrt{\pi^2+4}} \right\rangle$

$$\frac{\pi}{\sqrt{\pi^2+4}} \cdot -y\sin(xy) + \frac{-2}{\sqrt{\pi^2+4}} \cdot -x\sin(xy)$$

$$= \frac{-\pi^2}{2\sqrt{\pi^2+4}} \cdot 1 + \frac{2}{\sqrt{\pi^2+4}} \cdot 1$$

$$\frac{4-\pi^2}{2\sqrt{\pi^2+4}}$$

Gradients

$\nabla f = \langle f_x(x,y), f_y(x,y) \rangle$

$\nabla f \cdot \vec{u} = f_x(x,y) \cdot u_1 + f_y(x,y) \cdot u_2$

$$f(x,y) = x^3 + 2x^2y + 3y^2$$

$$\nabla f = \langle 3x^2 + 4xy \,,\, 2x^2 + 6y \rangle$$

$$\nabla f(3,1) = \langle 39, 24 \rangle$$

Tangent lines

$f(x,y) = \sin(x)\cos(y)$ $\quad\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$f_x = \cos(x)\cos(y)$$

$$f_y = -\sin(x)\sin(y)$$

x-slope $= 0$ $\quad$ Through: $\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$

direction: $\langle 1, 0, 0 \rangle$

x-tangent $\longrightarrow \langle \frac{\pi}{2}, \frac{\pi}{2}, 0 \rangle + t\langle 1, 0, 0 \rangle$

y-slope $= -1$ $\quad$ Through: $\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$

direction: $\langle 0, 1, -1 \rangle$

y-tangent: $\langle \frac{\pi}{2}, \frac{\pi}{2}, 0 \rangle + t\langle 0, 1, -1 \rangle$

$\langle -1, 1 \rangle$ $\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ $\quad \nabla f \cdot \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

$-\frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{\sqrt{2}} \cdot -1$ $\quad \langle 0, -1 \rangle \cdot \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

$\langle -1, 1 \rangle$-slope $= \frac{-1}{\sqrt{2}}$ $\quad \left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$

$$\left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle$$

$\langle \frac{\pi}{2}, \frac{\pi}{2}, 0 \rangle + t \langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$