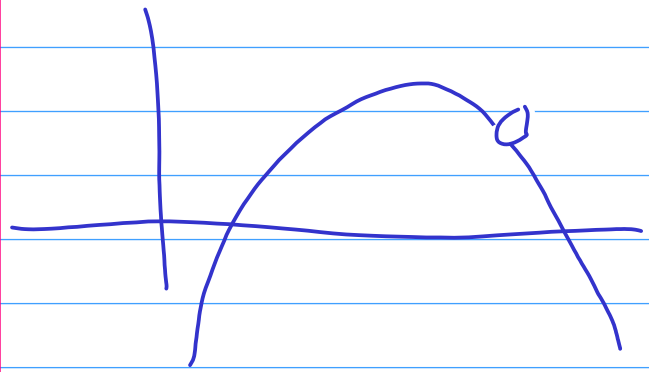
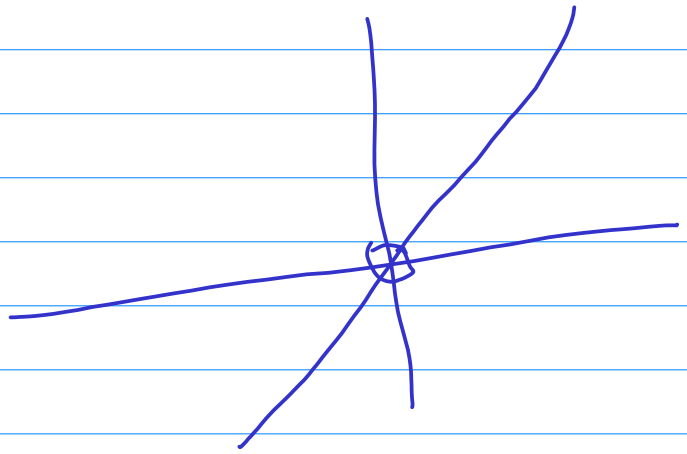
Limits



$$\frac{x^2}{x} = x \qquad x \neq 0$$



$\delta \qquad \varepsilon$

$\lim_{x \to 0} \frac{\sin x}{x}$ well, try

0.001

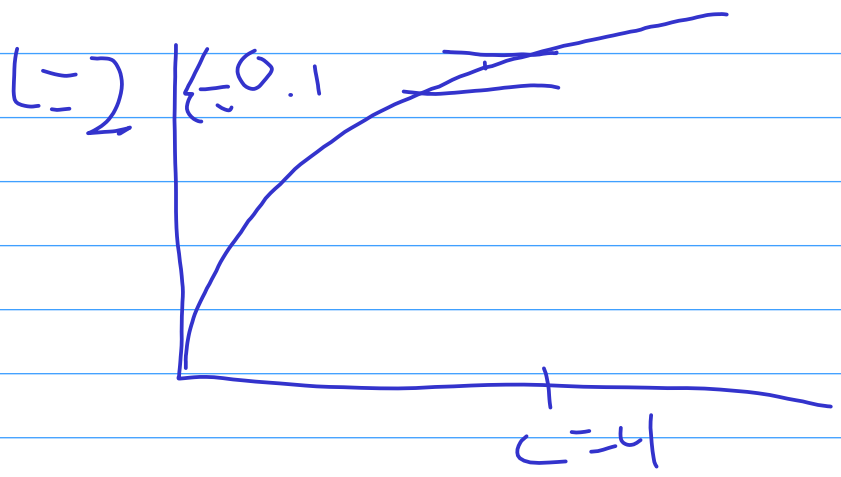
0 -0.001

$\delta = \varepsilon \cdot 0.1$

Epsilon = y-tolerance
Delta = x-tolerance

If I pick any epsilon, no matter how small,
I can find a delta such that:
~~if x is within Delta units of our input,~~
y is within epsilon units of our output

$\lim_{x \to 4} \sqrt{x} = 2$



$(4-\delta, 4+\delta)$

$\sqrt{4-\delta} = 1.9$

or $\sqrt{4+\delta} = 2.1$

$4-\delta = 3.61$ $\quad 4 = 3.61 + \delta$

$0.39 = \delta$

$4 + \delta = 4.41$

$0.41 = \delta$

if $\varepsilon = 0.1$ $\quad \delta \leq 0.39$

$\sqrt{4-\delta} = 2-\varepsilon$

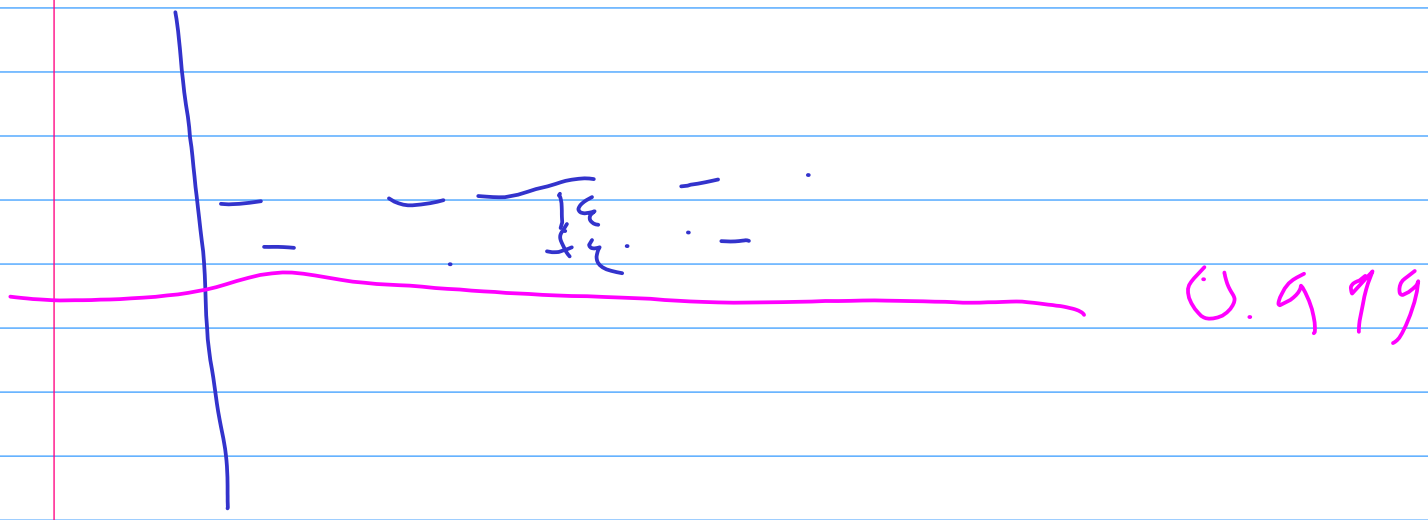
$\sqrt{4+\delta} = 2+\varepsilon$

$y = 0.999$

Does

$\lim_{x \to 3} y = 1?$

$\varepsilon = 0.0005$



$$f(x) = \frac{x^2 - 4}{(x+2)^2} \qquad \lim_{x \to -2} f(x) = ?$$

$$\frac{1}{0} \qquad \frac{0}{0} \text{ Indeterminate form}$$

$$\frac{(x+2)(x-2)}{(x+2)(x+2)} \qquad \frac{(x-2)}{(x+2)} \quad \frac{-4}{0}$$

$$g(x) = \frac{(x-2)(x+2)}{(x+2)(x-1)} \qquad \frac{-4}{-3} = \frac{4}{3}$$

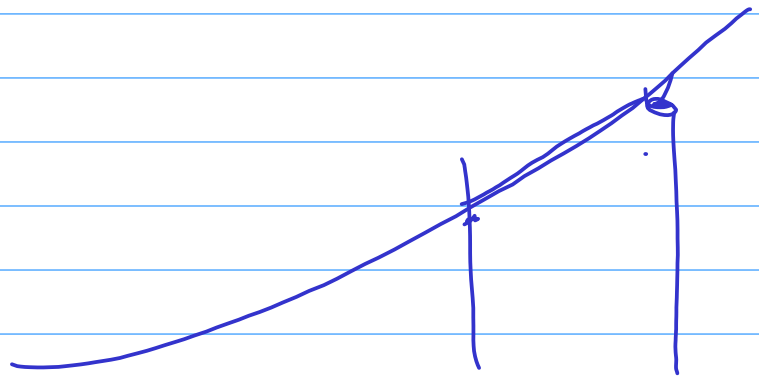
$$\lim_{x \to -2} g(x) = \frac{4}{3}$$

$s = 2t + 3$

$t = 2$ seconds

$$\frac{s(3) - s(1)}{2} = 2$$

$$\frac{s(2+\epsilon) - s(2-\epsilon)}{2\epsilon}$$



$t = 2 \quad t = 2+h$

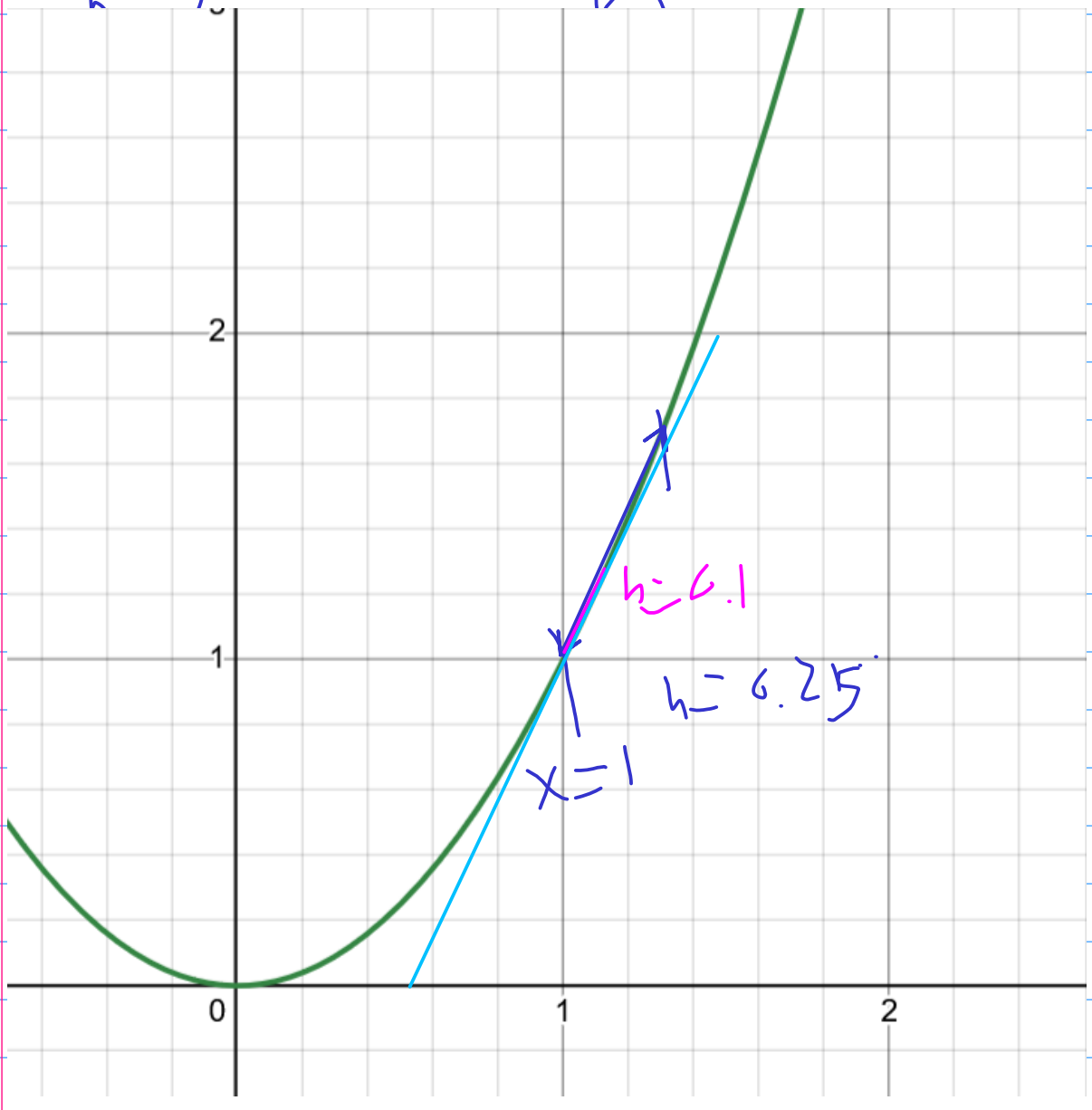
$$\frac{s(2+h) - s(2)}{h}$$

h

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$f(x)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Derivative:
Instantaneous rate of change
Slope of the tangent line

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x)$ $f'(x)$

y $\frac{dy}{dx}$

$f(x) = 2x + 3$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{2(x+h)+3 - (2x+3)}{h}$$

$$\frac{2x + 2h + 3 - (2x+3)}{h}$$

2

$$f(x) = 2x^2$$

$$\lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$$

$$\frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$$

$$\frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$$

$$\frac{4xh + 2h^2}{h} \qquad 4x + 2h$$

$$\lim_{h \to 0} 4x + 2h = 4x$$

$$f'(x) = 4x$$

$$\lim_{x \to 2} x^2 = 4$$

$\varepsilon = 0.01$

$$(2+\delta)^2 = 4.01$$
$$\therefore 2-\delta^2 = 3.99$$

$$2+\delta = \sqrt{4.01} = 2.002498$$
$$\text{or } 2-\delta = \sqrt{3.99} = 1.997498$$

$$\delta = 0.002$$

$$(2+\delta)^2 = 4+\varepsilon \qquad 2+\delta = \sqrt{4+\varepsilon}$$
$$\text{or } (2-\delta)^2 = 4-\varepsilon \qquad 2-\delta = \sqrt{4-\varepsilon}$$

$$\delta = \sqrt{4+\varepsilon} - 2$$
$$\delta = 2 - \sqrt{4-\varepsilon}$$