

9. $f(x, y) = 2x^2y - 4xy^2, P = (2, 3).$

$$\langle 1, 0, -12 \rangle$$

$$\langle f_x, f_y, -1 \rangle$$

$$\langle 0, 1, -40 \rangle$$

$$\langle -12, -40, -1 \rangle$$

$$\langle 2, 3, -48 \rangle$$

$$\langle 2, 3, -48 \rangle + \langle -12, -40, -1 \rangle t$$

11. $f(x, y) = 3x - 5y, P = (4, 2).$

$$\frac{\partial f}{\partial x}$$

$$f_x = 3$$

$$\langle 3, -5, -1 \rangle$$

$$f_y = -5$$

$$\langle 4, 2, 2 \rangle$$

$$\langle 4, 2, 2 \rangle + \langle 3, -5, -1 \rangle t$$

17. $f(x, y) = 2x^2y - 4xy^2, P = (2, 3).$

$$\langle -12, -40, -1 \rangle$$

$$\langle 2, 3, -48 \rangle$$

$$-12(x-2) + -40(y-3) + -1(z+48) = 0$$

$$-12x + 24 - 40y + 120 - z - 48 = 0$$

$$-12x - 40y - z = -96$$

19.

$$\langle 3, -5, -1 \rangle$$

$$\langle 4, 2, 2 \rangle$$

$$0 = 3(x-4) + 5(y-2) + (z-2)$$

$$3x - 5y = z$$

$$(a) \int_2^5 (6x^2 + 4xy - 3y^2) dy$$

$$(b) \int_{-3}^{-2} \int_2^5 (6x^2 + 4xy - 3y^2) dy dx$$

$$a) 6x^2y + 2xy^2 - y^3 \Big|_2^5$$

$$\int_{-3}^{-2} 18x^2 + 42x - 117 dx$$

$$6x^3 + 21x^2 - 117x + C \Big|_{-3}^{-2}$$

$$-48 + 84 - 351 + C$$

$$378$$

$$270 - 378 = -108$$

$$9. (a) \int_0^y (\cos x \sin y) dx$$

$$(b) \int_0^\pi \int_0^y (\cos x \sin y) dx dy$$

$$a) \sin x \sin y \Big|_0^y = \sin^2 y$$

$$\int_0^\pi \sin^2(y) dy = \int_0^\pi \frac{1 - \cos(2y)}{2} dy$$

$$\frac{1}{4} (y - \sin(2y)) \Big|_0^\pi = \frac{\pi}{4}$$



$$\nabla f(x, y)$$

$$[f_x(x, y), f_y(x, y)]$$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle$$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\nabla \cdot \vec{F}(x, y, z)$$

$$\frac{\partial}{\partial x} \vec{F} + \frac{\partial}{\partial y} \vec{F} + \frac{\partial}{\partial z} \vec{F}$$

$$\vec{F} = \langle x^2 y, \sin(x) y^2 \rangle$$

$$\frac{\partial}{\partial x} x^2 y + \frac{\partial}{\partial y} \sin(x) y^2$$

$$2xy + 2y \sin(x)$$

$$\vec{F} = \langle M, N \rangle$$

$$\text{curl: } N_x - M_y$$

$$\vec{F} = \langle M, N, P \rangle$$

$$\nabla \times \langle M, N, P \rangle$$

Diagram illustrating the curl of a vector field $\vec{F} = \langle M, N, P \rangle$ using the determinant method:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

Red arrows indicate the expansion along the first row:

- $M_y \hat{k}$ (from $\frac{\partial}{\partial y} P$)
- $N_z \hat{i}$ (from $\frac{\partial}{\partial z} M$)
- $P_x \hat{j}$ (from $\frac{\partial}{\partial x} N$)

Green arrows indicate the expansion along the first row:

- $P_y \hat{i}$ (from $\frac{\partial}{\partial y} P$)
- $M_z \hat{j}$ (from $\frac{\partial}{\partial z} M$)
- $N_x \hat{k}$ (from $\frac{\partial}{\partial x} N$)

The resulting curl vector is shown in green:

$$\langle P_y - N_z, M_z - P_x, N_x - M_y \rangle$$