↔	E	Y	YY	Z	YZ	YYZ
E	E	Y	YY	Z	YZ	YYZ
Y	Y	YY	E	YZ	YYZ	Z
YY	YY	E	Y	YYZ	Z	YZ
Z	Z			E	Y	YY
YZ	YZ				YY	E
YYZ	YYZ					Y

zy ~ yz~~yyy~~ yz

yyz → yyyzy → zy

1 2 3 4 5 6 7 8 9

1 2 3 4 5 6 7 8 9

8 1 6

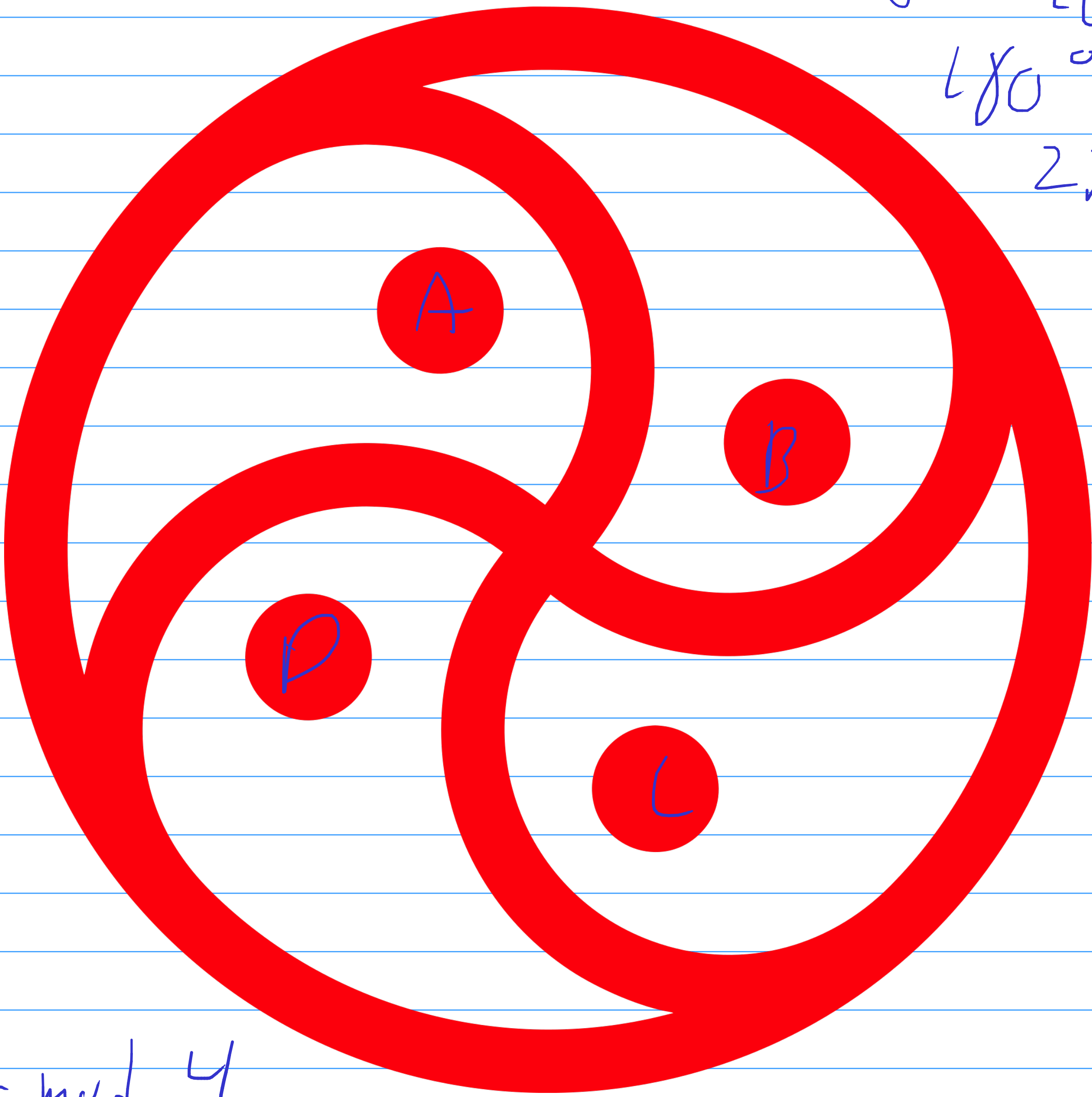
3 5 7

4 9 2

mod 7

calendar math is
ISOMORPHIC
To $\oplus$ mod 7
$\otimes$

0° 90°
180°
270°



$+$ mod 4

$0^\circ \longleftrightarrow 0$ $\quad$ $180^\circ \longleftrightarrow 2$

$90^\circ \longleftrightarrow 1$ $\quad$ $270^\circ \longleftrightarrow 3$

$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

	0°	90°	180°	270°
0°	0	90	180	270
90°	90	180	270	0
180°	180	270	0	90
270°	270	0	90	180

$\otimes_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$\oplus_2$	0	1
0	0	1
1	1	0

$\otimes_2$	0	1
0	0	0
1	0	1

Definition: a *group* is a set of elements, together with an operation that satisfies the following rules.

- *closure:* using the operation on two elements of the group yields an element of the group.
- *associative law:* (a b) c = a (b c).
- *identity element:* one of the elements, e, is such that a e = e a = a, for any element a in the group.
- *inverse element:* every element a has an inverse a' such that a a' = a' a = e

Some groups are *commutative* (a b = b a) and some are not.

$f_1 c f_3$

$3 + 4 + 9$

$(f_1 c)\ f_3$

$f_1\ (c f_3)$

$f_1\ f_2\ f_3$

$a \in Z\ c$

S

0	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

mod 15

$3 \otimes 5 = 0$

1 ÷

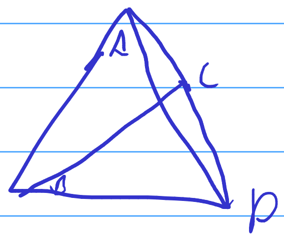
1 Trivial group

$1 \div 1 = 1$

$1 \div 1 \div 1 = 1$

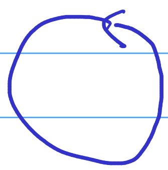
$(1, -1)$

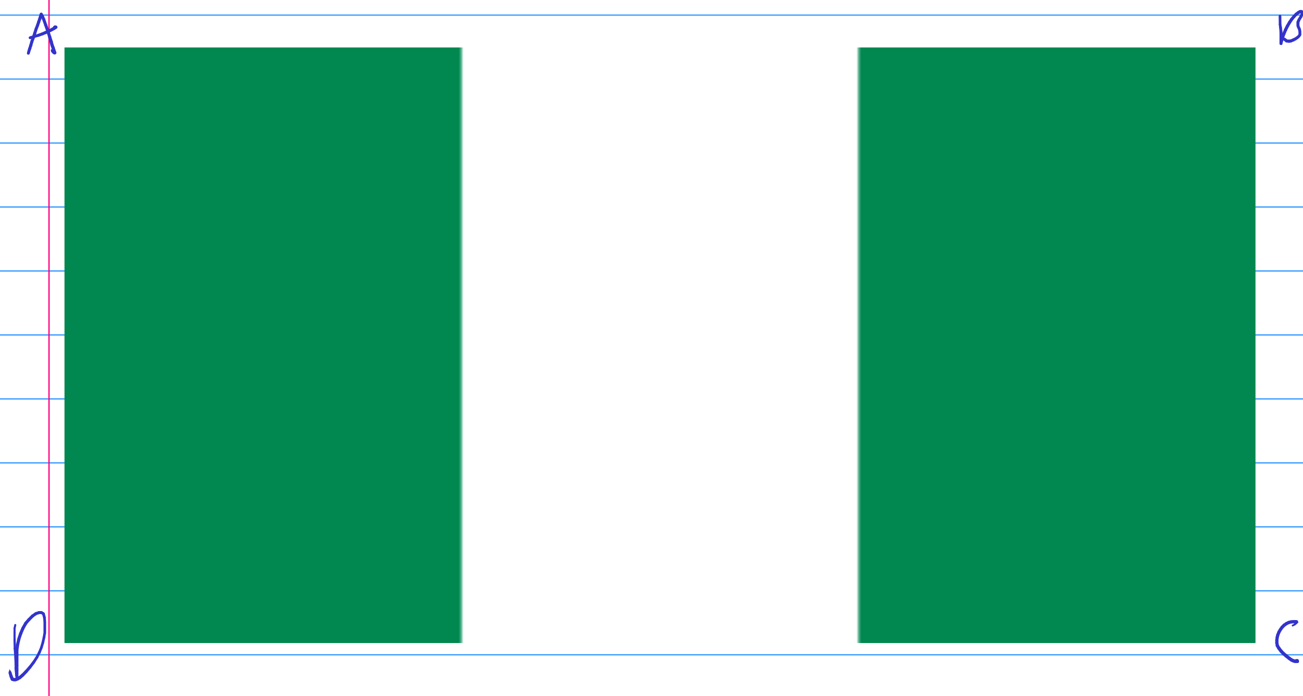
$1, i, -1, -i$ ×

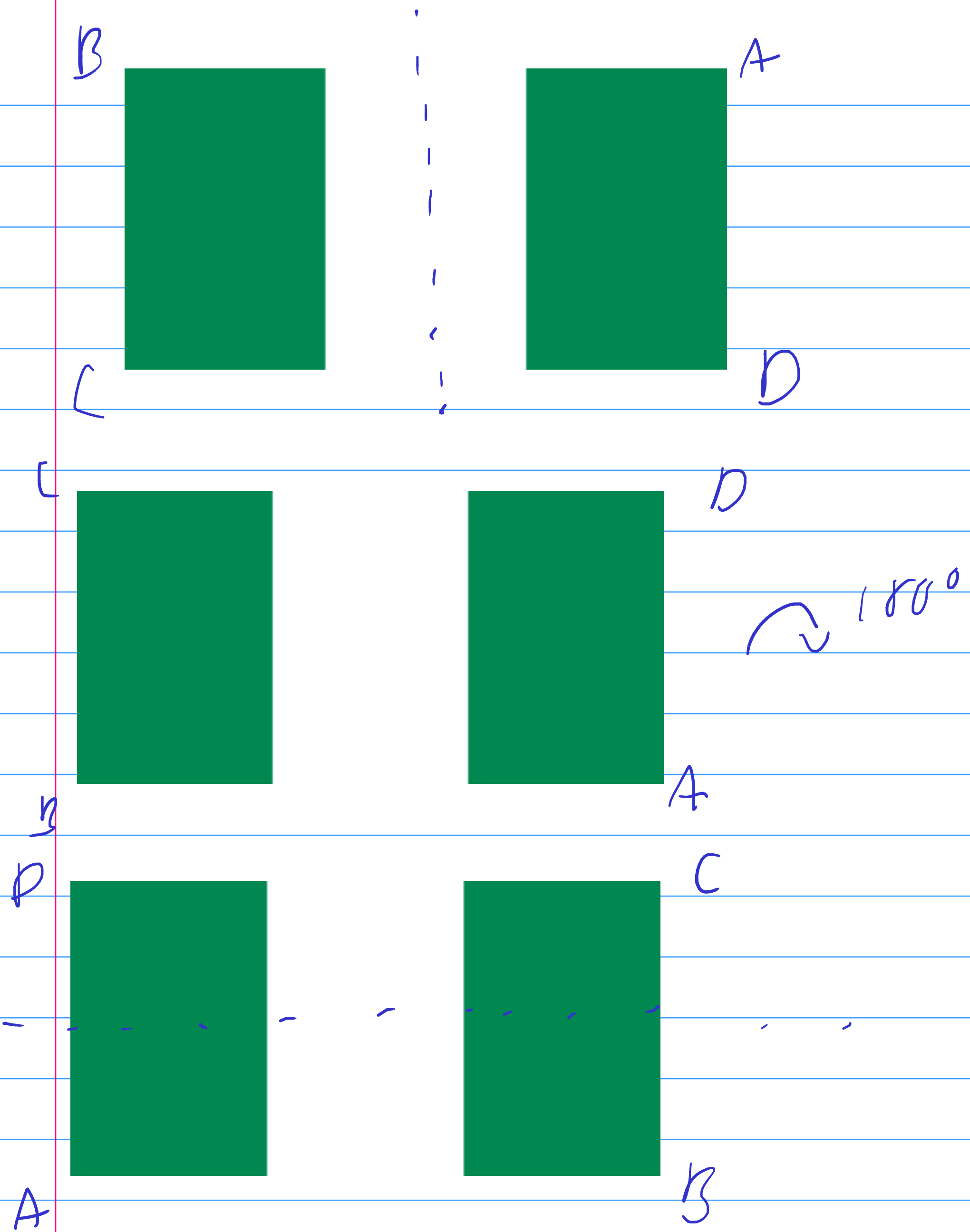


A B C D

B D C A







2nd

1st	S	X	Y	180
S	S	X	Y	180
X	X	S	180	Y
Y	Y	180	S	X
180	180	Y	X	S

Klein 4-group

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

+ mod 4

$\mathbb{Z}_4$ +

$\mathbb{N}$ $\mathbb{Q}$ $\mathbb{R}$ $\mathbb{C}$

$\frac{3}{4}$