Clementi and Palazzo (2016) Replication

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1 Model

1.1 Households:

- Demand for output and supply of capital is perfectly elastic
- Labor supply is given by

$$L_t^s = w_t^\gamma \tag{1}$$

where w_t is the real wage rate with γ being the Frisch elasticity

1.2 Firms:

- There are two kinds of firms: incumbents and potential entrants

1.2.1 Incumbents

- There are perfectly competitive heterogeneous firms - all producing the *same* good with the production technology

$$y_t = X_t s_t \left(k_{t-1}^{\alpha_k} l_t^{\alpha_l}\right)^\theta \tag{2}$$

where X_t: aggregate productivity, s_t: idiosyncratic productivity, and k_{t-1}: capital stock. The idiosyncratic productivity follows an AR(1) process:

$$\log s_{t+1} = \rho_s \log s_t + \sigma_s \epsilon_{s,t+1}$$

- Price of capital is normalized to 1
- **Timeline** of an incumbent is as follows: Let Γ_t denote the distribution of firms over (k_{t-1}, s_t). Given the aggregate state $\lambda_t = (\Gamma_t, X_t)$ and the idiosyncratic state (k_{t-1}, s_t), incumbents
  1. choose labor l_t and produce y_t
  2. decide whether to continue or exit

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3. Continuing firms pay a fixed operating cost c_f and choose capital for the next period k_t discounting the future at rate $\beta_t = 1/R_t$ where R_t is the gross interest rate.[1]
4. If the firm exits, it cannot enter at a later stage and simply recoup the undepreciated capital net of adjustment costs (i.e. $k_t = 0$)

- Incumbent's (perfect-foresight w.r.t aggregates) problem can therefore be written as

$$V_t(k_-, s) = \max\left\{V_t^{exit}(k_-, s), V_t^{cont}(k_-, s) - c_f\right\} \tag{3a}$$

$$V_t^{cont}(k_-, s) = \max_k \left\{\pi(k_-, s) - k + (1-\delta)k_- - g(k - (1-\delta)k_-, k_-) + \frac{1}{R_t}\mathbb{E}_{s'|s}\left[V_{t+1}(k, s') \mid s\right]\right\} \tag{3b}$$

$$V_t^{exit}(k_-, s) = \pi(k_-, s) + (1-\delta)k_- - g(-(1-\delta)k_-, k_-) \tag{3c}$$

where

$$\pi(k_-, s; X) = \max_l \left\{Xs\left(k_-^{\alpha_k} l^{\alpha_l}\right)^\theta - wl\right\}$$

are the maximized real profits and $g(x, k_-) = c_1\left(\frac{x}{k_-}\right)^2 k_-$ are the capital adjustment costs (in terms of final good). This gives

$$\pi(k_-, s; X) = \frac{1-\alpha_l\theta}{\alpha_l\theta} w^{-\frac{\alpha_l\theta}{1-\alpha_l\theta}} \left(\alpha_l\theta X s k_-^{\alpha_k\theta}\right)^{\frac{1}{1-\alpha_l\theta}} \tag{4}$$

$$l(k_-, s; X) = \left(\frac{\alpha_l\theta X s k_-^{\alpha_k\theta}}{w}\right)^{\frac{1}{1-\alpha_l\theta}} \tag{5}$$

$$y(k_-, s; X) = Xs\left(k_-^{\alpha_k} l^{\alpha_l}(k_-, s; X)\right)^\theta$$

Inputs. (w, X, r) and parameters $\left(\alpha_k, \alpha_l, \theta, \rho_s, \sigma_s, \delta, c_f, c_1\right)$ to solve for incumbent policies

Note. Relative to Clementi and Palazzo (2016), there are two simplifying assumptions:

1. I assume that the operating costs c_f are fixed rather than being random. This has implications for the relative size of the exiters. With random c_f, even large firms have a non-zero probability of exiting and very small firms have non-zero probability of continuing. This affects the average size of exiting firms (See Table 2 row 7).[2]

2. I assume no fixed component of the adjustment costs i.e. here I consider convex capital adjustment costs. Given the calibrated value in Table 2 of Clementi and Palazzo (2016), this has a smaller effect on overall results except for few firm-level statistics (See Table 2 row 4).[3]

1.2.2 Entrants:

- There is a constant mass M of *potential* entrants each with a noisy signal s_e about the future idiosyncratic productivity

[1] In Clementi and Palazzo (2016), operating costs are random. I assume these to be constant here.

[2] This is easy to incorporate, simply requires an extra expectation operator over c_f for the incumbents' problem (3a).

[3] To incorporate the fixed component of the adjustment costs, we need to add another discrete choice stage with taste shocks.

- The distribution of the signals is Pareto such that $H(s_e) = 1 - \left(\frac{\underline{s_e}}{s_e}\right)^{\xi}$ with $\xi > 1$
- Entry entails a fixed cost c_e
- Entrant's problem is:

$$V_t^{entrant}(s_e) = \max_k -k + \frac{1}{R_t}\mathbb{E}_{s|s_e}\left[V_{t+1}(k,s) \mid s_e\right] \tag{6}$$

- They will enter iff $V_t^{entrant}(s_e) \geq c_e$
- Further, assume that the realization of productivity conditional on the signal follows:

$$\log s \quad = \quad \rho_s \log s_e + \sigma_s \eta, \qquad \eta \sim \mathcal{N}(0,1)$$

- s is the realization of productivity with which entrants produce

Inputs. (R_t, V_{t+1}) and parameters $\left(\xi, \underline{s_e}, c_e, \rho_s, \sigma_s\right)$ to solve for entrants' policies and M for s.s. distribution.

Note. The signal distribution of the entrants matters a lot for the dynamics of entry and for the relative size of the entrants. If the signal is drawn from the stationary distribution of productivity (which is log-normal), entrants and continuing firms tend to be very similar. As a result, the entry *rate* doesn't budge a lot upon a productivity shock. This is an important point in the paper.

1.3 Equilibrium

- Labor market clearing:

$$\underbrace{w_t^{\gamma}}_{\text{Supply}} = \underbrace{\int l(k_-, s)\, d\Gamma_t(k_-, s)}_{\text{Labor Demand}}$$

determines equilibrium wage w_t. Above $l(k_-, s)$ is the the labor demand of incumbents given by (5) and Γ_t is the cross-sectional distribution of incumbents. [See the exact equilibrium definition in the paper.]

2 Calibration and Results

2.1 Calibration

- There are 14 parameters: $\left(\gamma, \alpha_k, \alpha_l, \theta, \rho_s, \sigma_s, R, c_1, \delta, c_f, c_e, \xi, \underline{s_e}, M\right)$
- Table 1 reports the parameter values and the calibration targets are listed in Table 2.
- Relative to the paper, following are the differences in calibration methodology:
    1. I fix $\gamma = 2$. It is otherwise calibrated to hit the standard deviation of employment growth

2. I assume $\rho_s = 0.55$. It is otherwise calibrated to hit the weighted average of the mean and standard deviation of the investment rate in the data
3. I assume $\sigma_{c_f} = 0$. It is otherwise calibrated to hit the exiters' relative size.
4. I assume $c_0 = 0$. It is otherwise calibrated to hit the investment inaction rate
5. I assume $c_1 = 0.03141$. It is otherwise calibrated to hit the autocorrelation of investment

- Rest is the same:

1. M is calibrated to hit s.s. wage rate of $w = 3$
2. μ_{c_f} is calibrated to hit the s.s. exit (=entry) rate
3. ξ is calibrated to hit the entrants' relative size
4. c_e is set to be equal to mean operating costs c_f

Table 1: Parameter Values

Description	Symbol	CP		Rishabh	
		Value	Target	Value	Target
Capital Share	α_k	0.3	Assigned	0.3	Assigned
Labor Share	α_l	0.7	Assigned	0.7	Assigned
Span of control	θ	0.8	Assigned	0.8	Assigned
Depreciation rate	δ	0.1	Assigned	0.1	Assigned
Interest rate	R	1.04	Assigned	1.04	Assigned
Labor supply elasticity	γ	2	sd(employment growth)	2	Assigned
Mass of potential entrants	M	161.1	$w = 3$	343.85	$w = 3$
Persistence idio. shock	ρ_s	0.55	Mean and sd of investment rate	0.55	Assigned
Variance idio. shock	σ_s	0.22	Assigned	0.22	Assigned
Operating cost - mean	μ_{c_f}	-5.63872	Exit rate=0.062	-5.27049	Exit rate = 0.062
Operating cost - var	σ_{c_f}	0.90277	Exiters' relative size	0	Assigned
Mean operating cost	c_f	0.005347	$\exp\left(\mu_{c_f} + 0.5\sigma^2_{c_f}\right)$	0.005141	$\exp\left(\mu_{c_f} + 0.5\sigma^2_{c_f}\right)$
Fixed cost of investment	c_0	0.00011	Inaction rate	0	Assigned
Variable cost of investment	c_1	0.03141	Investment autocorrelation	0.03141	Assigned
Pareto exponent	ξ	2.69	Entrants' relative size = 0.6	3.78	Entrants' relative size = 0.6
Entry cost	c_e	0.005347	$= c_f$	0.005141	$= c_f$
Minimum support of signal	$\underline{s_e}$	Lowest grid point that comes out of discretization			

Notes:
(a) *There is a typo in the Table 1 of the paper.* $M = 1,766.29$ *is actually the total mass of firms in s.s.*
(b) *Orange shaded rows represent the differences in calibration methodology*
(c) *Blue shaded rows represent the calibrated parameters*

- Table 2 reports the model moments.
  - Because I assumed no fixed adjustment costs, the inaction rate is too low.
  - because I assumed constant operating costs, the exiters' relative size is too low (with constant c_f, large firms exit with zero probability and exiters only include small firms).

2.2 Results

Below I reproduce some of the main results from the paper.

2.2.1 Steady State Firm Dynamics

Figures A.1, A.2 and A.3.

Table 2: Calibration Targets

	Statistic	Data	Model: CP Table 2	Model: Rishabh	Note
1.	Mean investment rate	0.122	0.153	0.152	
2.	SD investment rate	0.337	0.325	0.323	
3.	Investment autocorrelation	0.058	0.059	-	
4.	Inaction rate	0.081	0.067	0.028	$c_0 = 0$
5.	Entry (=exit) rate	0.062	0.062	0.062	
6.	Entrants' relative size	0.60	0.58	0.61	
7.	Exiters' relative size	0.49	0.47	0.212	$\sigma_{c_f} = 0$

Note: Blue shaded rows represent the targeted moments for calibration. Other rows report the moments not targeted (but targeted by CP). See differences in methodology in Table 1

2.2.2 Transition Dynamics to a TFP shock

Figures A.4 and A.5

2.2.3 Correlation with Output

Table 3: Correlation With output

	Entry rate	Exit Rate	Entrants' size	Exiters' size
CP	0.402	-0.779	-0.725	-0.892
Rishabh	0.275	-0.575	-0.607	-0.736

3 Computation

Following section describes the computation method which uses the stage-block concept. [See NBER Workshop Lecture]

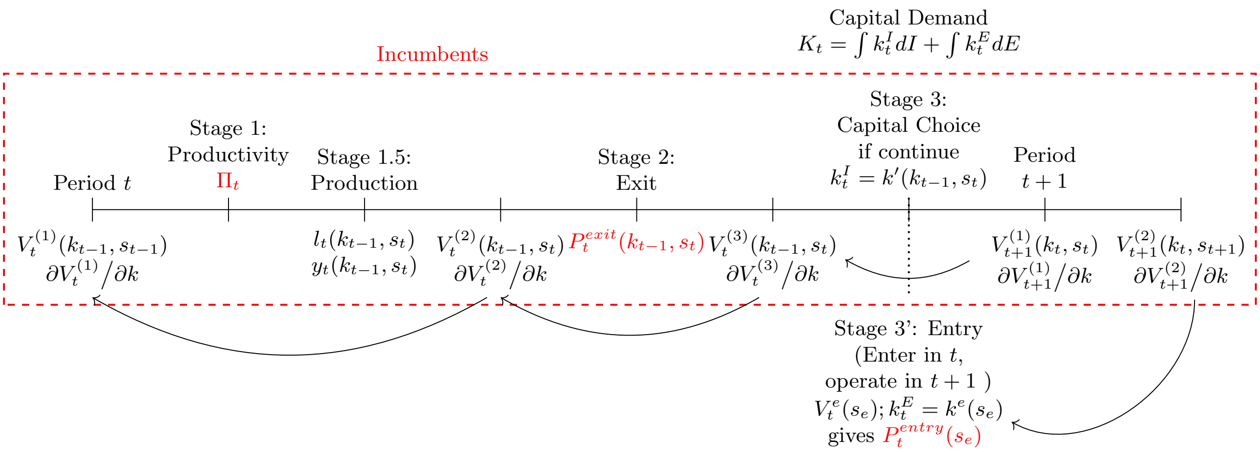
3.1 Policy and value functions

3.1.1 Incumbents (Achievable with StageBlock)

Break the problem of the incumbent (3) into stages.

- **Stage 0.** Incumbent enters a new period with state variables inherited from last period (k_-, s_-)
- **Stage 1.** Idiosyncratic productivity shock: $(k_-, s_-) \to (k_-, s)$. They hire labor l_t per (5) and produce y_t giving $\pi_t\,(k_-, s)$ per (4)
- **Stage 2.** Realize taste shock ϵ. Makes exit choice: $e\,(k_-, s) \in [0, 1]$ (e is the probability of exiting using logsum formula)

Figure 1: Backward Iteration



- **Stage 3.** If continue to operate, make investment choice $(k_-, s) \to (k, s)$. If exit, $(k_-, s) \to (0, .)$

Write it backwards: Inputs for single backward iteration are $V_{t+1}^{(2)}(k, s')$ and $\frac{\partial}{\partial k} V_{t+1}^{(2)}(k, s')$. Note that, we could use $V_{t+1}^{(1)}(k, s)$ and $\frac{\partial}{\partial k} V_{t+1}^{(1)}(k, s)$ as the input to backward iteration. But given the timing assumptions for the entrants, it is convenient to use $V_{t+1}^{(2)}(k, s')$ and $\frac{\partial}{\partial k} V_{t+1}^{(2)}(k, s')$. It does not matter as long as we are consistent with the way stages are written and all the inputs are correctly specified.

- **Stage 1 of Period** $t+1$**.** Productivity shock

$$\begin{aligned} V_{t+1}^{(1)}(k, s) &= \Pi V_{t+1}^{(2)}(k, s') \\ \frac{\partial}{\partial k} V_{t+1}^{(1)}(k, s) &= \Pi \frac{\partial}{\partial k} V_{t+1}^{(2)}(k, s') \end{aligned}$$

  where Π is the transition matrix for productivity process.

- **Stage 3 of Period** t**.** Discrete choice-specific investment choice and value function:

  - For exiting firms, $e(k_-, s) = 0$

$$\begin{aligned} k(e = 0, k_-, s) &= 0 \\ V_t^{(3)}(e = 0, k_-, s) &= \pi(k_-, s) + (1 - \delta) k_- - g(-(1 - \delta) k_-, k_-) \\ \frac{\partial}{\partial k_-} V^{(3)}(e = 0, k_-, s) &= \pi_1(k_-, s) + (1 - \delta) - g_{k_-}(-(1 - \delta) k_-, k_-) \end{aligned}$$

    which are all in closed form

  - For continuing firms, $e(k_-, s) = 1$

$$V_t^{(3)}(e = 1, k_-, s) = \max_k \pi(k_-, s) - k + (1 - \delta) k_- - g(k - (1 - \delta) k_-, k_-) - c_f + \frac{1}{R_t} V_{t+1}^{(1)}(k, s) \tag{7}$$

    * Use EGM + upper envelope + envelope condition to obtain $V_t^{(3)}(e, k_-, s)$, $\frac{\partial}{\partial k} V_t^{(3)}(e, k_-, s)$, $k_t(e, k_-, s)$ for $e \in \{0, 1\}$
    * Note that there is no expectation operator for the continuation value. The productivity stage 1 takes care of it.

- **Stage 2 of Period t.** Realize taste shock ϵ. Makes exit choice: $e_t\left(k_-,s\right) \in [0,1]$

$$V_t^{(2)}\left(k_-,s\right) = \max_{e\in\{0,1\}} V_t^{(3)}\left(e,k_-,s\right) + \underbrace{\epsilon\left(e\right)}_{\text{Taste Shock}}$$

giving

$$\begin{aligned}
V_t^{(2)}\left(k_-,s\right) &= \sigma_\epsilon \ln\left(\sum_{e\in\{Cont,Exit\}} \exp\left(\frac{V_t^{(3)}\left(e,k_-,s\right)}{\sigma_\epsilon}\right)\right) \\
\frac{\partial}{\partial k}V_t^{(2)}\left(k_-,s\right) &= \frac{\sum_{e\in\{Cont,Exit\}} \exp\left(\frac{V_t^{(3)}(e,k_-,s)}{\sigma_\epsilon}\right)\cdot \frac{\partial}{\partial k}V_t^{(3)}\left(e,k_-,s\right)}{\sum_{e\in\{Cont,Exit\}} \exp\left(\frac{V_t^{(3)}(e,k_-,s)}{\sigma_\epsilon}\right)} \\
Pr_t\left(e\mid k_-,s\right) &= \frac{\exp\left(\frac{V_t^{(3)}(e,k_-,s)}{\sigma_\epsilon}\right)}{\sum_{e\in\{Cont,Exit\}} \exp\left(\frac{V_t^{(3)}(e,k_-,s)}{\sigma_\epsilon}\right)} \qquad (8)
\end{aligned}$$

- **Stage 1 of Period t.** Productivity shock

$$\begin{aligned}
V_t^{(1)}\left(k_-,s_-\right) &= \Pi V_t^{(2)}\left(k_-,s\right) \\
\frac{\partial}{\partial k}V_t^{(1)}\left(k_-,s_-\right) &= \Pi\frac{\partial}{\partial k}V_t^{(2)}\left(k_-,s\right)
\end{aligned}$$

where Π is the transition matrix for productivity process.

Summary of single backward iteration for incumbents:

$$\left(V_{t+1}^{(2)}\left(k,s'\right), \frac{\partial}{\partial k}V_{t+1}^{(2)}\left(k,s'\right)\right) \xrightarrow[\text{of period } t+1]{\text{Productivity (S1)}} \left(V_{t+1}^{(1)}\left(k,s\right), \frac{\partial}{\partial k}V_{t+1}^{(1)}\left(k,s\right)\right)$$

$$\big\downarrow \text{Capital (S3) of period } t$$

$$\left(V_t^{(2)}\left(k_-,s\right), \frac{\partial}{\partial k}V_t^{(2)}\left(k_-,s_-\right), e_t\left(k_-,s\right), Pr_t\left(e_t\left(k_-,s\right)\right)\right) \xleftarrow[\text{of period } t]{\text{Exit Choice (S2)}} \left(k_t\left(e,k_-,s\right), V_t^{(3)}\left(e,k_-,s\right), \frac{\partial}{\partial k}V_t^{(3)}\left(e,k_-,s\right)\right)$$

How is it implemented:

- The three stages are written as separate functions
- Then another function simply calls the stages one-by-one. This takes $V_{t+1}^{(2)}\left(k,s'\right)$ as input and spit out $V_t^{(2)}\left(k_-,s\right)$ as output (along with other stuff). The implementation is equivalent to “StageBlock”

3.1.2 Entrants

Break the problem of the entrant (6) into stages. (Entrants start producing with lag of one-period)

- **Stage 0.** Potential entrant draws a productivity signal $s_e \sim H\left(\cdot\right)$

- **Stage 1.** Realize taste shock ϵ. Makes entry choice: $m(s_e) \in [0,1]$ (m is the probability of entry using logsum formula)
- **Stage 2.** If enter, investment choice $k_t = k_e(s_e)$. If stay-out, $k_t = 0$

Write it backwards: Inputs for single backward iteration is $V_{t+1}^{(2)}(k, s')$ (Ref. Figure 1).[4]

- **Stage 2 Period t.** Discrete choice-specific investment choice and value function:

$$
\begin{aligned}
V_t^{entrant(2)}(Enter, s_e) &= \max_k -k - c_e + \frac{1}{R}\mathbb{E}_{s'|s_e}\left[V_{t+1}^{(2)}(k, s') \mid s_e\right] \\
k_t^{entrant}(Enter, s_e) &= \arg\max_k -k - c_e + \frac{1}{R}\mathbb{E}_{s'|s_e}\left[V_{t+1}^{(2)}(k, s') \mid s_e\right] \qquad (9) \\
V_t^{entrant(2)}(StayOut, s_e) &= 0 \\
k_t^{entrant(2)}(StayOut, s_e) &= 0 \qquad (10)
\end{aligned}
$$

  where

$$\log s' = \rho_s \log s_e + \sigma_s \eta$$

- **Stage 1 Period t.** Realize taste shocks. Makes entry decision $m_t(s_e) \in [0,1]$ where m is the probability of entry

$$V_t^{entrant(1)}(s_e) = \max_{m \in \{Enter, StayOut\}} V_t^{entrant(2)}(m, s_e) + \underbrace{\epsilon(m)}_{\text{Taste Shock}}$$

  giving probability of entry as

$$Pr_t(Enter \mid s_e) = \frac{\exp\left(\frac{V_t^{entrant(2)}(Enter, s_e)}{\sigma_\epsilon}\right)}{\sum_{m \in \{Enter, StayOut\}} \exp\left(\frac{V^{(2)}(m, s_e)}{\sigma_\epsilon}\right)} \qquad (11)$$

Summary of single backward iteration for entrants:

$$
\begin{array}{ccc}
\left(V_{t+1}^{(2)}(k, s')\right) & \xrightarrow{\text{Capital}} & \left(k_t^{entrant}(m, s_e), V_t^{(2)}(m, s_e)\right) \\
 & & \downarrow \text{Entry Choice} \\
 & & \left(m_t(k_-, s_e), Pr_t(Enter \mid s_e)\right)
\end{array}
$$

This is achievable outside the Incumbent StageBlock.

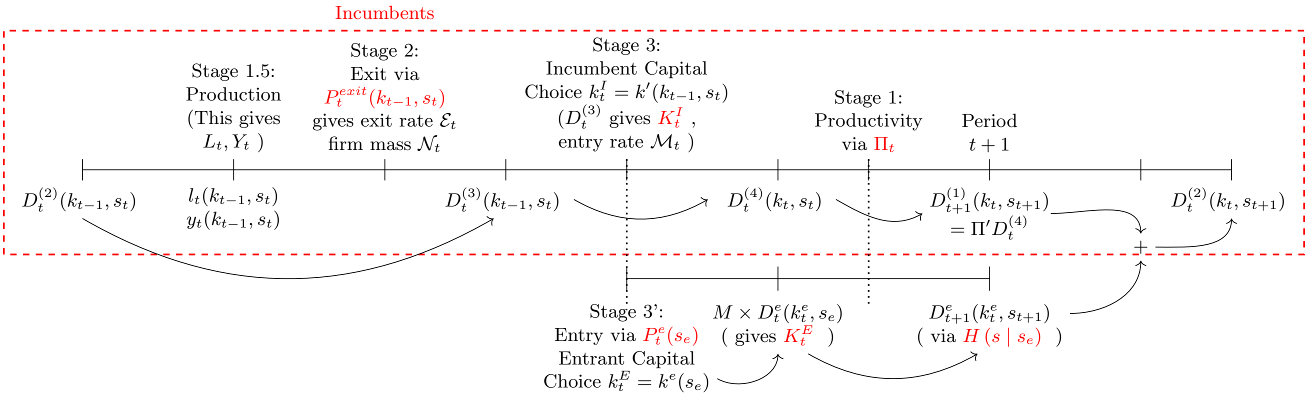
How is it implemented:

- The two stages are written as separate functions
- Then another function simply calls the stages one-by-one. This takes $V_{t+1}^{(2)}(k, s')$ as input and spit out policy functions as output.
  - ► The beginning of stage value functions for the entrants are not required.

[4] We want to write it in a way that entrants consider the value function with which they start producing. This is $V^{(2)}$

3.2 Distributions

Figure 2: Forward Iteration



Start with the cross-sectional distribution $D_t^{(2)}(k_-, s)$ which consists of firms that produce. We should end with $D_{t+1}^{(2)}(k, s')$

3.2.1 Incumbents:

Given the stages of incumbents, forward iteration along the stages is:

- **Stage 2.** Exit choice with $P_t^{exit}(k_-, s)$ from (8).
  - Since exiters are no more in the economy, they do not proceed to the next stage. Therefore, we have:

$$D_t^{(3)}(e = 0, k_-, s) = \left(1 - P_t^{exit}(k_-, s)\right) D_t^{(2)}(k_-, s) \tag{12}$$

$$D_t^{(3)}(e = 1, k_-, s) = 0$$

  Hence, we get

$$\sum_{(k_-, s)} D_t^{(3)}(k_-, s) < \sum_{(k_-, s)} D_t^{(2)}(k_-, s)$$

  since a mass equal to $\sum_{(k_-, s)} P_t^{exit}(k_-, s) D_t^{(2)}(k_-, s)$ exit

- **Stage 3.** If continue to operate, make investment choice $(k_-, s) \to (k, s)$ - *forward_policy* giving

$$D_t^{(4)}(\mathcal{K}, s) = D_t^{(3)}\left(k_-^{-1}(\mathcal{K}, s), s\right)$$

  where $k_-^{-1}(\mathcal{K}, s)$ is the inverse of $k_t(\cdot, s) \in \mathcal{K}$

- **Stage 1.** Idiosyncratic productivity shock:

$$D_{t+1}^{(1)}(k, s') = \Pi_s^{\top} D_t^{(4)}(k, s) \tag{13}$$

  where $D_{t+1}^{(1)}(k, s')$ is the distribution of survivors.

3.2.2 Entrants:

- Given $P_t\left(enter \mid s_e\right)$ from (11) and $k_t^{entrant}\left(Enter, s_e\right)$ from (9), entrants distribution is

$$D_{t+1}^{e}\left(k, s'\right)=\sum_{s_e} M \cdot \underbrace{Pr\left(s_e\right)}_{\substack{\text{Unconditional} \\ \text{mass of signal}}} \cdot \underbrace{P_t\left(enter \mid s_e\right)}_{\substack{\text{entry} \\ \text{probability}}} \cdot \underbrace{Pr\left(s' \mid s_e\right)}_{\substack{\text{transition from} \\ \text{signal to prod.}}} \cdot \underbrace{\mathbf{1}\left\{k=k_t^{entrant}\left(Enter, s_e\right)\right\}}_{\substack{\text{capital} \\ \text{choice}}}$$

- Hence, end-of-stage distribution is

$$D_{t+1}^{(2)}\left(k, s'\right)=\underbrace{D_{t+1}^{(1)}\left(k, s'\right)}_{\substack{\text{survivors from} \\ \text{period } t}}+\underbrace{D_{t+1}^{e}\left(k, s'\right)}_{\substack{\text{new} \\ \text{entrants}}} \tag{14}$$

4 Incorporating into SSJ

Below I lay out the reasons why the code is not written directly using SSJ. The key problem is that of computing distributions.

4.1 Why can't we directly adopt above method into SSJ

There are two issues related to computing distributions: i) need to kick out exiters and ii) need to add entrants.

4.1.1 w.r.t Incumbents:

Stage 2 of Section 3.1.1 is the discrete exit choice stage where incumbents choose whether to continue operating or exit. For firms that choose to exit, the mass of these firms should *not* be carried forward to the next stage. Currently, in the logic choice stage, SSJ does the following:

1. Computes $P_t\left(e \mid k_{-}, s\right)$ with $e \in \{\text{Exit}, \text{Cont}\}$ using log-sum formula such that $\sum_e P_t\left(e \mid k_{-}, s\right)=1$
2. Iterate forward from Stage 2 to Stage 3 using choice probabilities as

$$D_t^{(3)}\left(e, k_{-}, s\right)=P_t\left(e \mid k_{-}, s\right) D_t^{(2)}\left(k_{-}, s\right) \tag{15}$$

3. This then leads to incorrect distribution in the subsequent stages $D_t^{(4)}(e, k, s)$ because $D_t^{(4)}(\text{Exit}, k, s)$ contains a positive mass (at the lowest grid point in this case since $k_t\left(\text{Exit}, k_{-}, s\right)=0$.)
4. Then, in each iteration, this extra mass of exiting firms keeps getting added and the distribution won't converge.

How is it implemented:

- I simply add another line where the mass of exiters is set to 0 i.e.

$$D_t^{(4)}(\text{Exit}, k, s)=0$$

  for all (k, s)

4.1.2 w.r.t Entrants:

- In forward iteration, adding the mass of entrants amounts to

$$D_{t+1}^{(2)} = D_{t+1}^{(1)} + D_{t+1}^{e} \tag{16}$$

  where D_{t+1}^{e} is computed using the $k_t^e(s_e)$, $P_t^{entry}(s_e)$ and $s' \mid s_e$.

- SSJ does not directly have a stage to achieve this *addition*.

How is it implemented:

- I perform this addition manually

References

Auclert, Adrien, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub, "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, 2021, *89* (5), 2375–2408.

Clementi, Gian Luca and Berardino Palazzo, "Entry, Exit, Firm Dynamics, and Aggregate Fluctuations," *American Economic Journal: Macroeconomics*, July 2016, *8* (3), 1–41.

A Results

Note: The upper panel in each figure is a screenshot from the paper and bottom panels are replicated figures.

Figure A.1: Signal Distribution

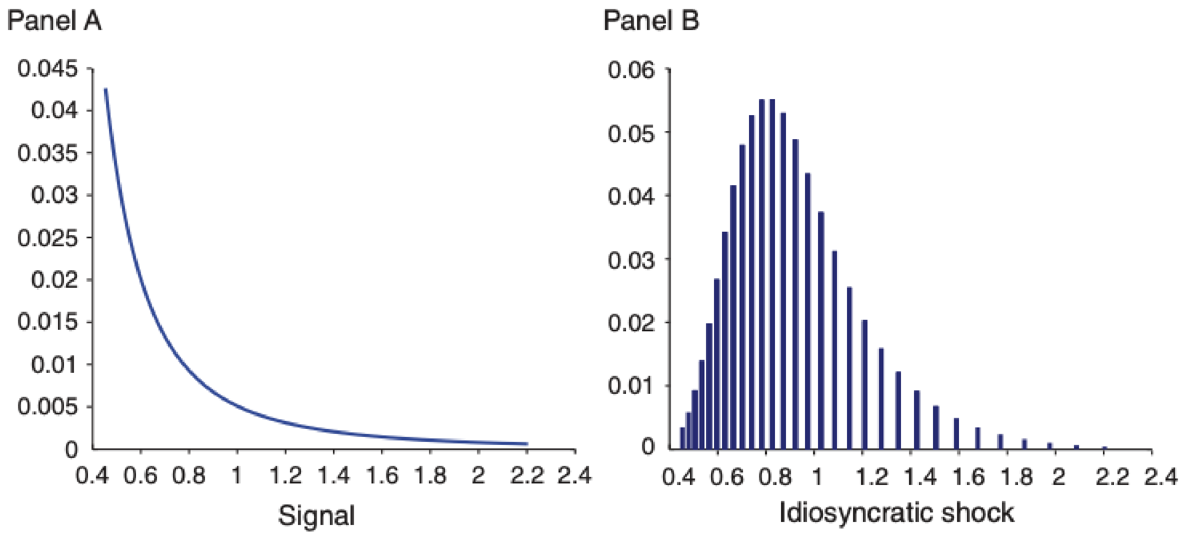
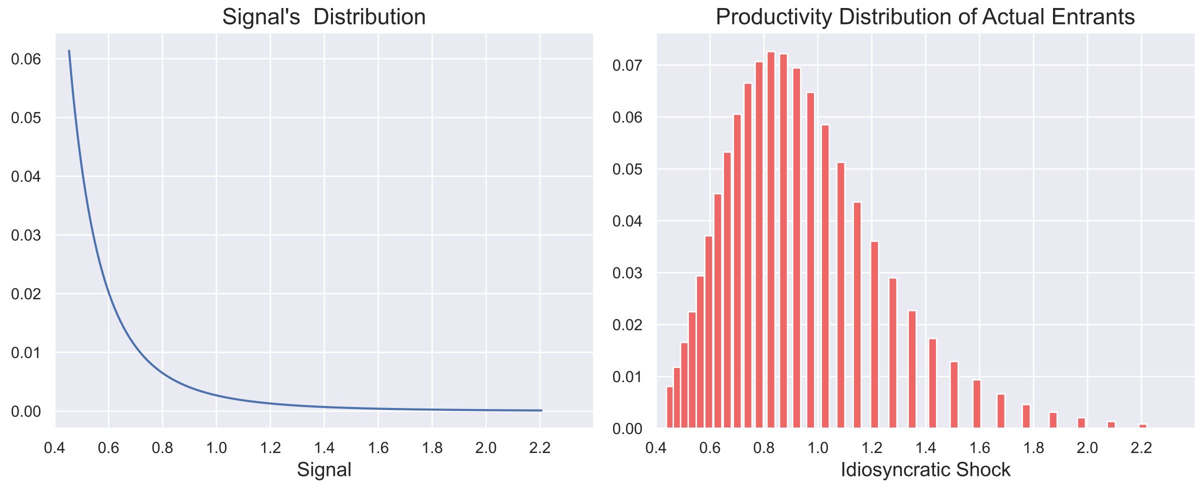


FIGURE 2. SIGNAL'S DISTRIBUTION (*left*) AND PRODUCTIVITY DISTRIBUTION OF ACTUAL ENTRANTS



Back to beginning of Section 2.2.1

Figure A.2: Exit Hazard Rate

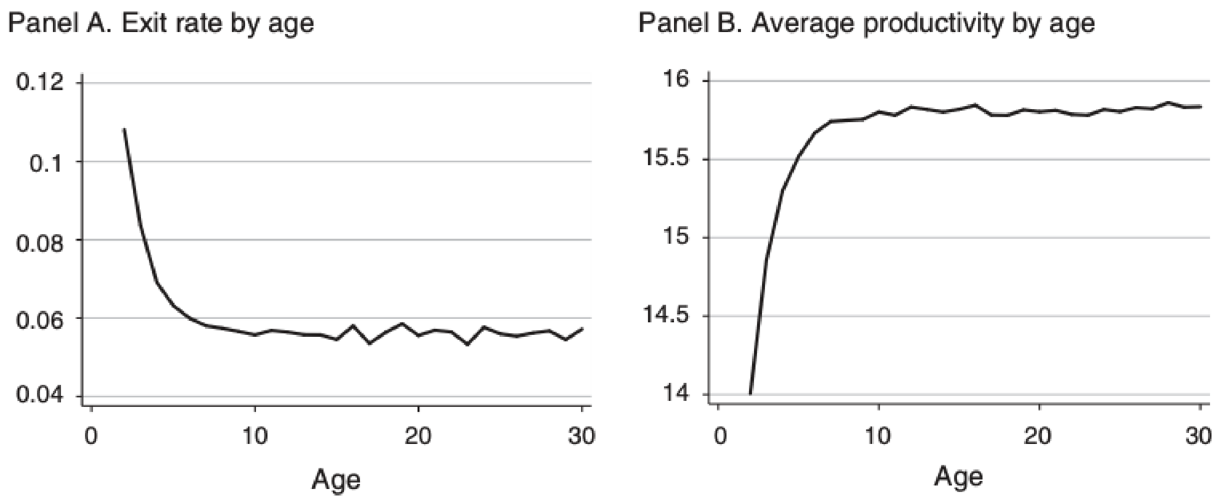
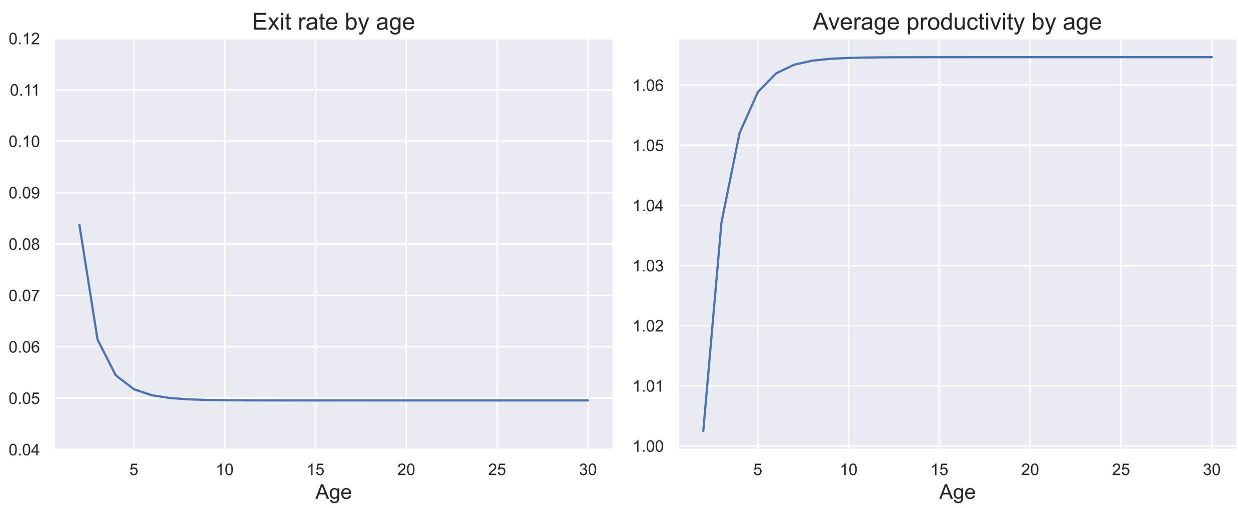


FIGURE 4. THE EXIT HAZARD RATE



Note: Panel B of the top figure erroneously plots the <u>*index*</u> *of the grid points of the productivity shocks rather than the actual productivity.*

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Figure A.3: Employment

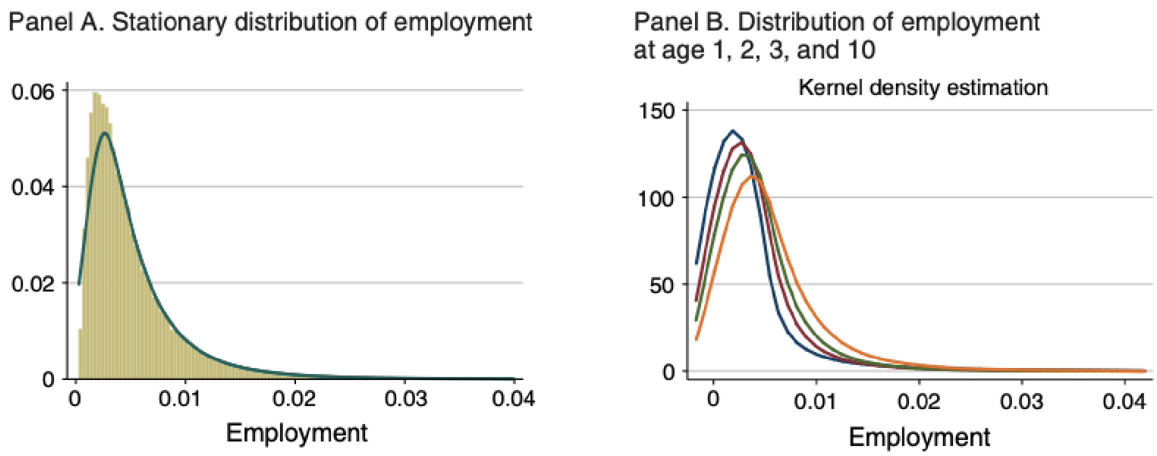
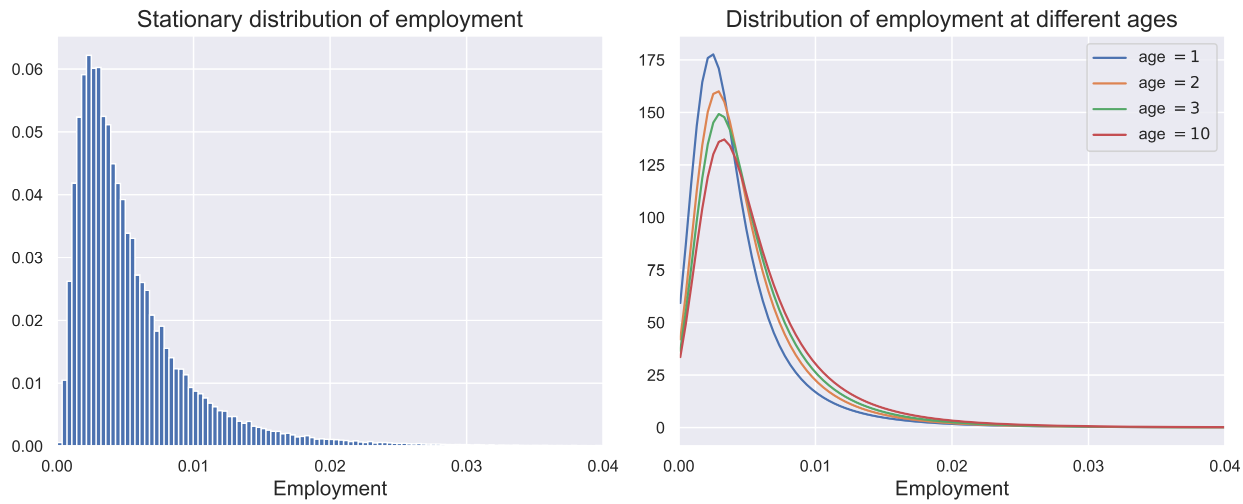


FIGURE 6. EVOLUTION OF A COHORT'S SIZE DISTRIBUTION



Note: Panel B of top figure does not have a legend, but seems like blue=1, red=2, green=3, orange=10 (trend is similar in both figs - dist. shifts towards right with age)

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Figure A.4: Impulse Responses I

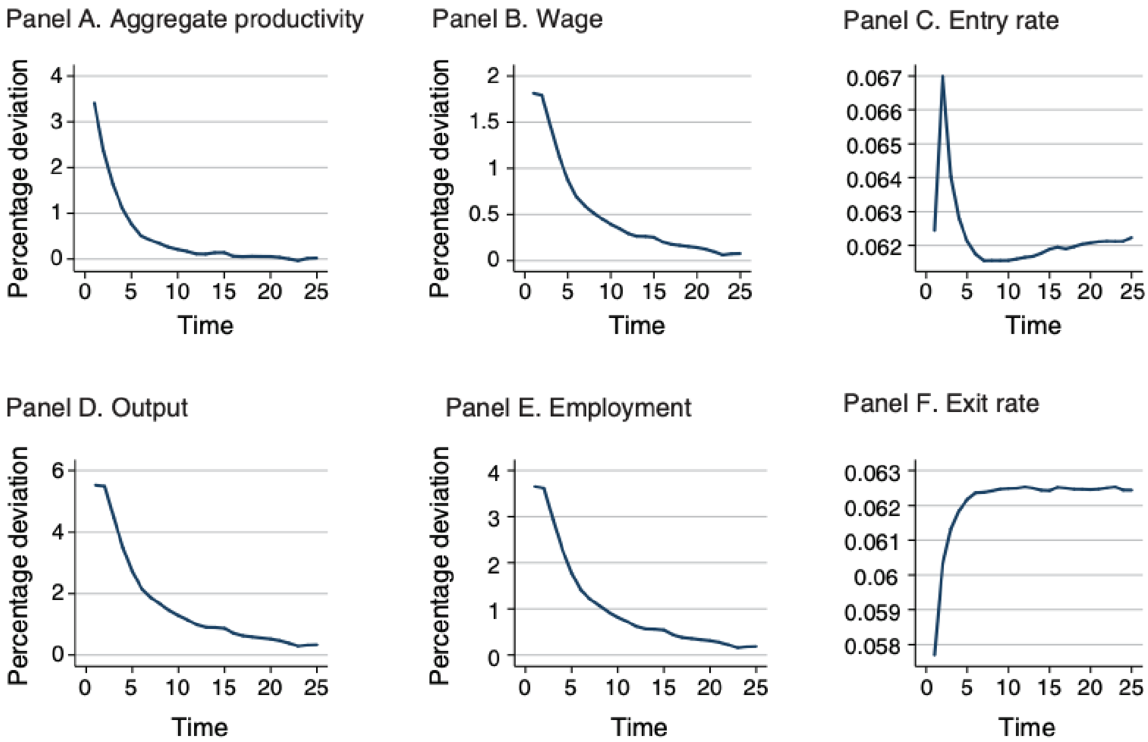
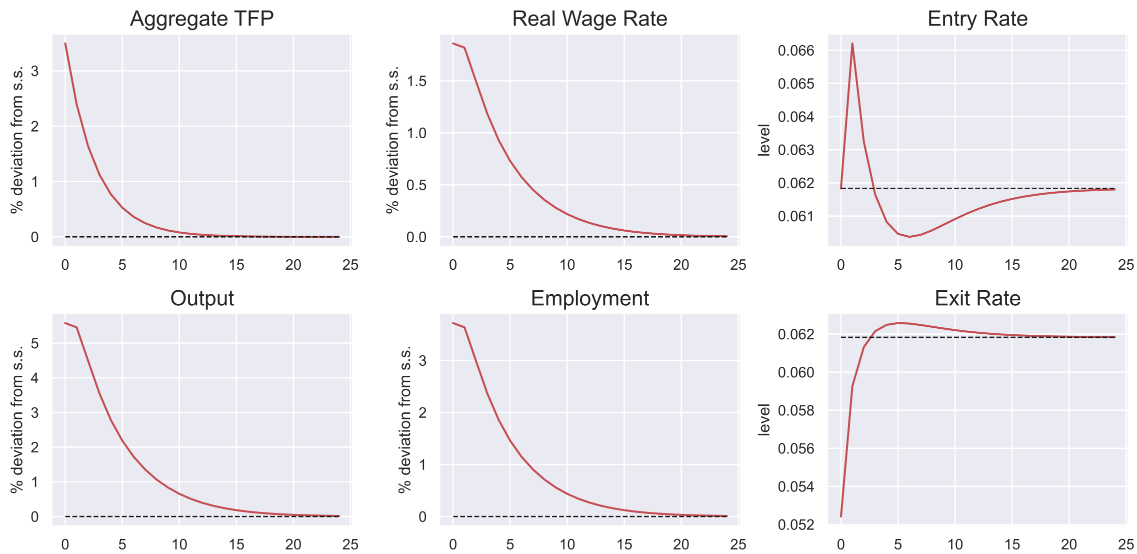


Figure 9. Response to a Positive Productivity Shock



Note: SS entry and exit rate = 0.062

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Figure A.5: Impulse Responses II

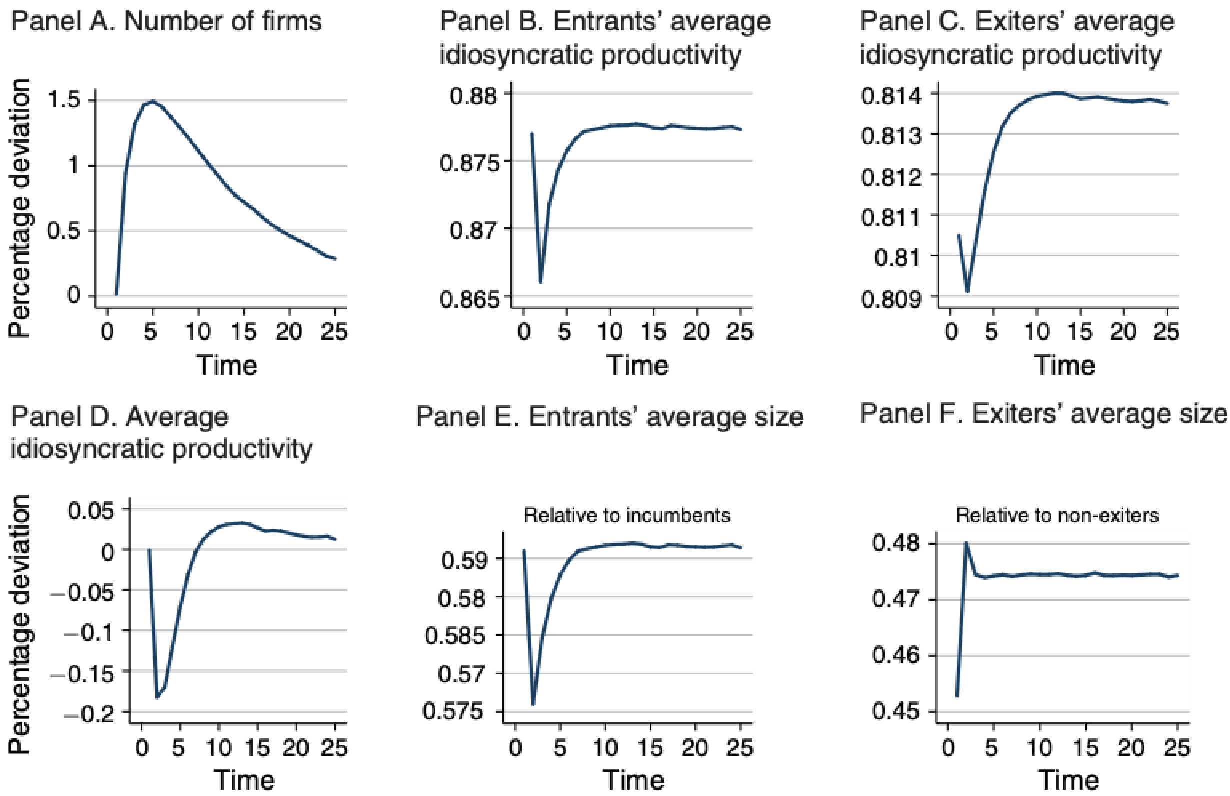
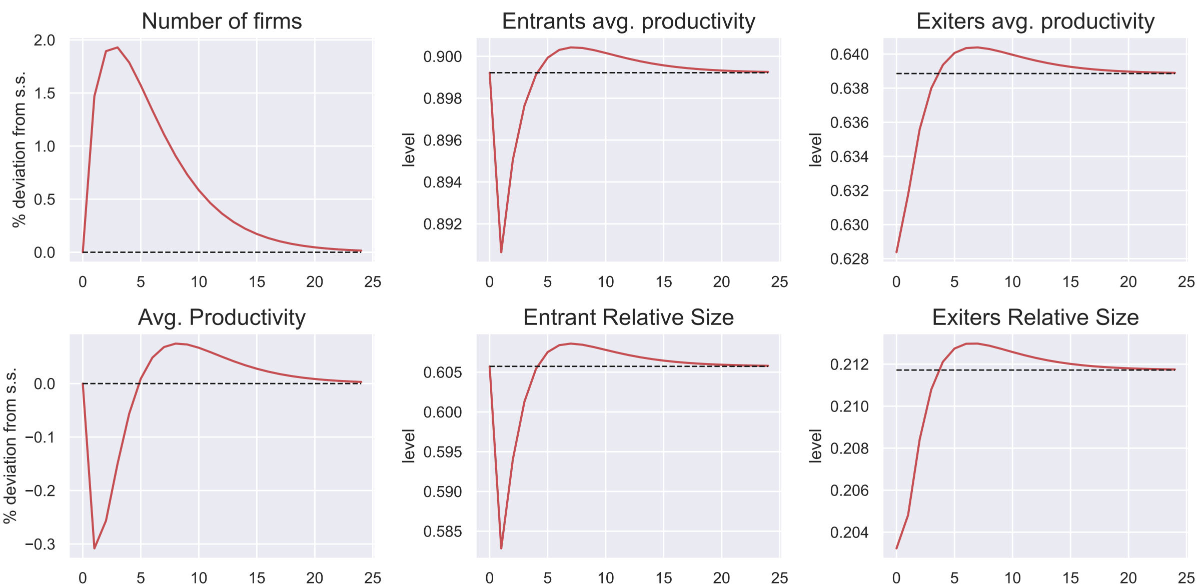


FIGURE 10. RESPONSE TO A POSITIVE PRODUCTIVITY SHOCK



Note: Panel C's and F's levels are different across the two figures because $\sigma_{c_f} = 0$.

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