Efficient Trajectory Retrieval

By Hasan Pourmahmood

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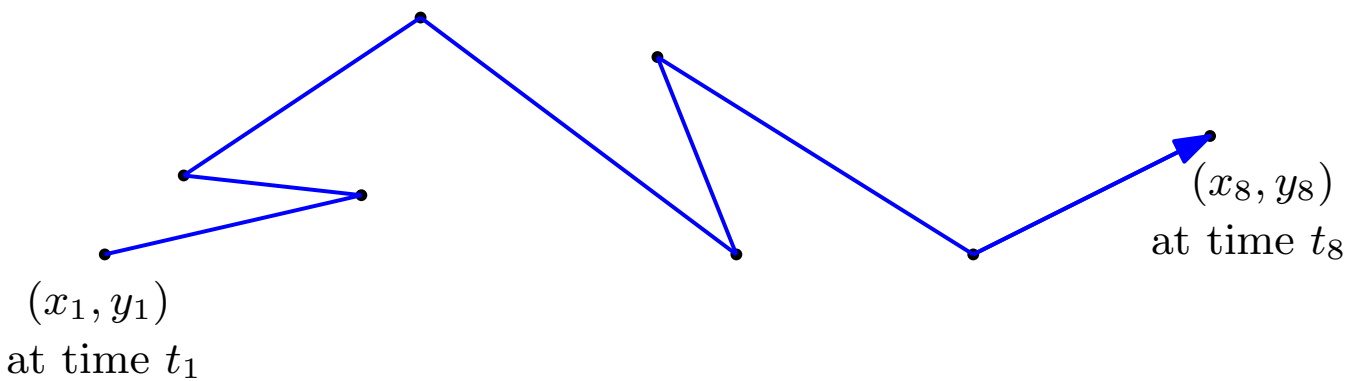
Trajectory

A spatial trajectory (also called a piece-wise linear curve) is a sequence of waypoints along with a time stamp, i.e.

$$T = \{(x_1, y_1, t_1), \ldots, (x_n, y_n, t_n)\}.$$



Popular Distances

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Applications

Route extraction, route recommendation, ...

Approach and Related Work

Related work

We are going to apply a similar method introduced in the following paper:

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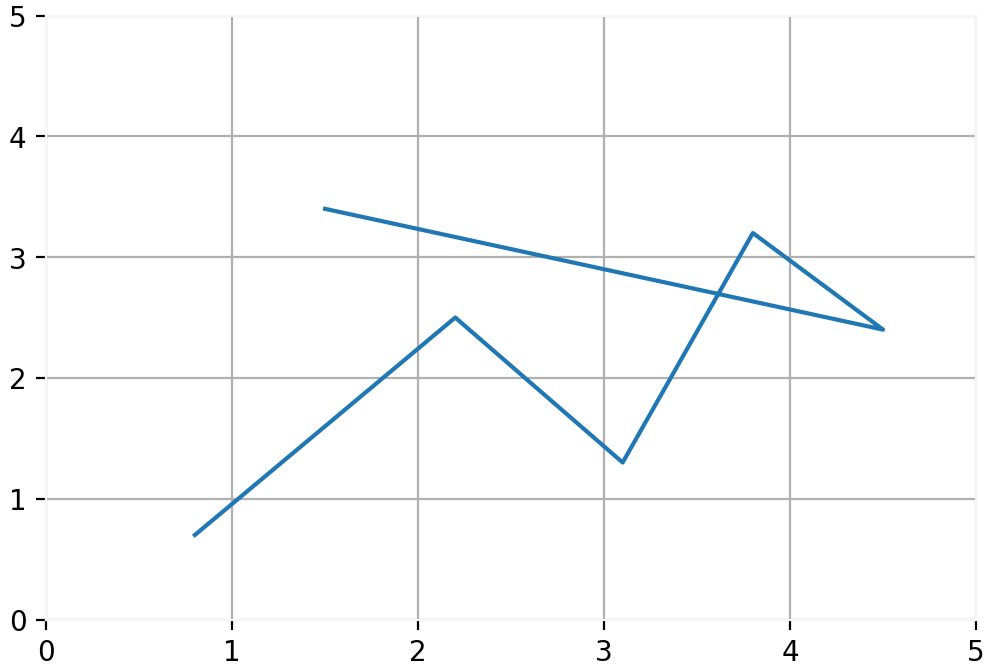
Novelty

However, among other things, we will do some modifications and apply novel vectorization techniques. Moreover, we will evaluate our approach on 3 real world datasets not only on Precision and Recall but also based on MAP and nDCG which will be adapted from their definitions in IR on text data.

Vector space model using grid structure



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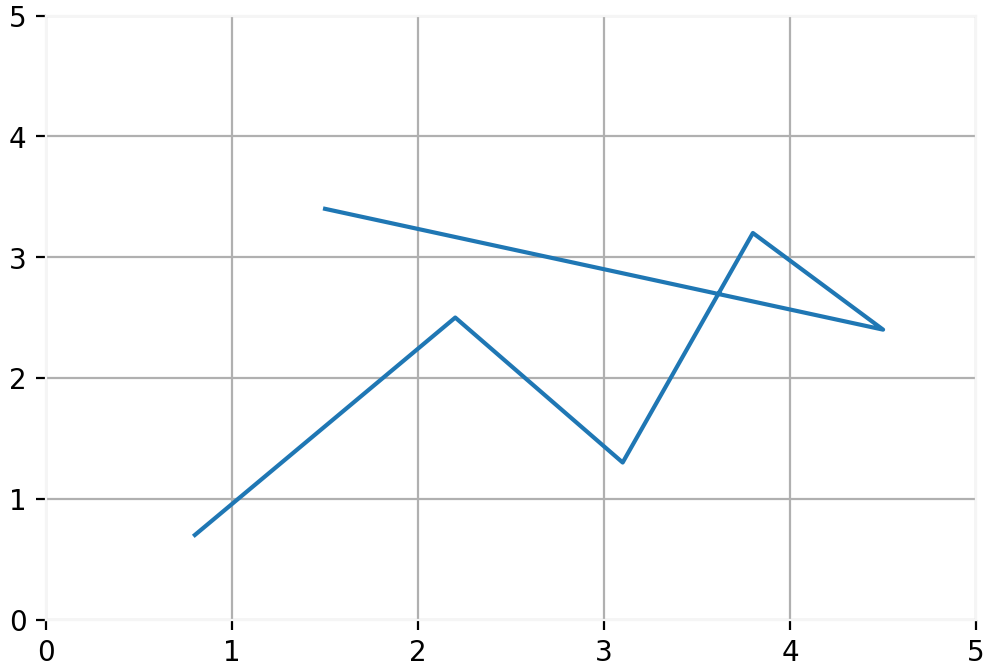

- **Vocabulary:** the whole set of grids $\{c_1, \ldots, c_n\}$
- **Document:** trajectory
- **Term:** a grid

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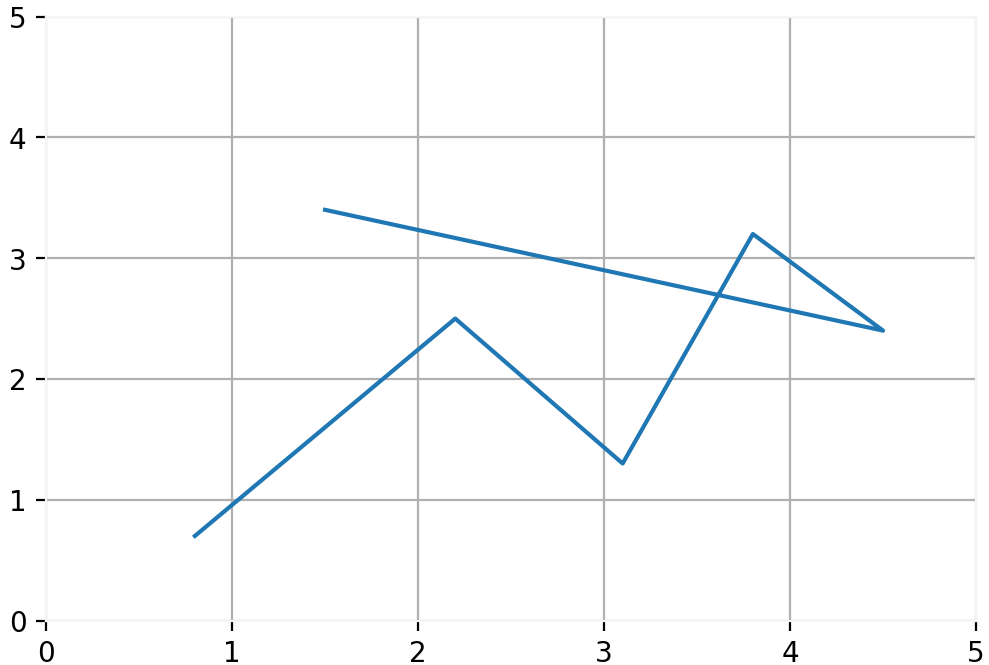

Binary representation:

$$\tilde{T} = [1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0]$$

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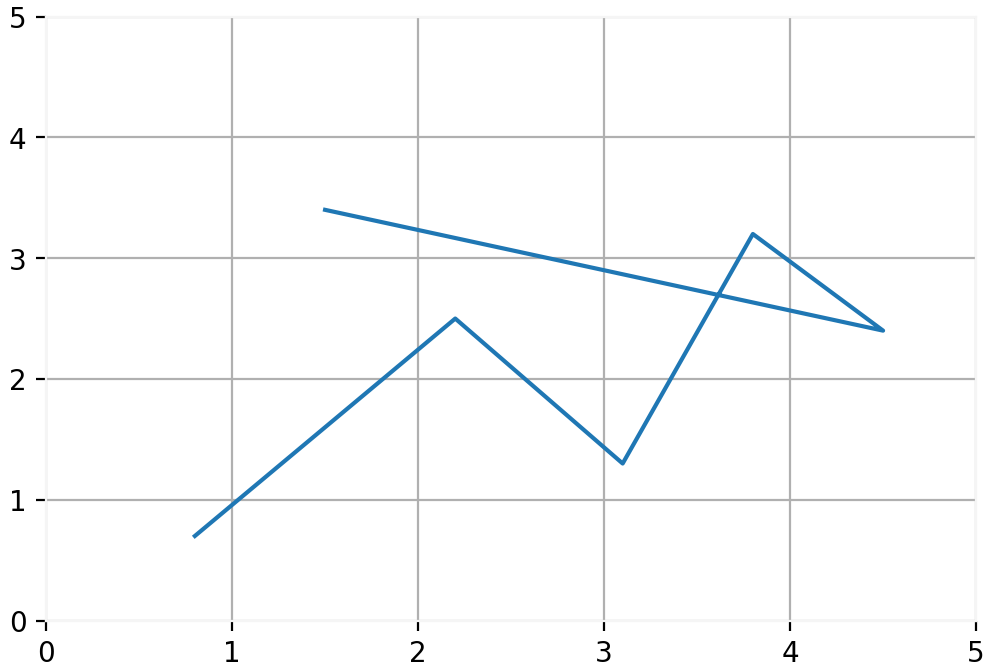

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$$\tilde{T} = [1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0]$$

Frequency representation:

$$\tilde{T} = [1, 1, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, \underbrace{2, 2}_{13,14}, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0]$$

Grid aggregate distance (GAD)

Consider a cell similarity matrix $C_{n\times n} = (sim(c_i, c_j))_{i,j=1}^{n}$ (c_i is the center of cell i), like

$$sim(c_i, c_j) = e^{-\|c_i - c_j\|^2/\sigma^2}$$

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How to address this problem?

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- Employ Euclidean distance for new short vectors as an approximation for GAD between trajectories,
- When a query comes, do the same and consult the index to retrieve appropriate trajectories.

Data Sets

Data	Size	Cleaned	Sample	Grids	Reduced dim
Car-Bus	163	105	105	2500	25
Characters	2858	2858	300	2500	25
Geolife	17621	14187	1000	2500	100

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Figure: Car-Bus (left), Geolife (middle), Characters: letters n, u (right).

kNN Queries (Geolife)

k	Precision/Recall	MAP	nDCG@5	nDCG@10
5	0.8676	0.8632	0.9182	0.9443
10	0.8921	0.8751	0.7822	0.9394
20	0.9074	0.8904	0.5823	0.7989

Table: Performance with binary representation

k	Precision/Recall	MAP	nDCG@5	nDCG@10
5	0.9480	0.9597	0.9822	0.9930
10	0.9590	0.9577	0.8233	0.9886
20	0.9653	0.9604	0.6029	0.8330

Table: Performance with frequency representation

Range Queries (Geolife)

r	Precision	Recall	MAP	nDCG@5	nDCG@10
0.001	0.9275	0.9549	0.8116	0.7260	0.6559
0.005	0.8896	0.9023	0.2025	0.8893	0.8952
0.01	0.8883	0.8892	0.0376	0.9182	0.9390

Table: Performance with binary representation

r	Precision	Recall	MAP	nDCG@5	nDCG@10
0.001	0.9990	1	0.9990	0.4906	0.3367
0.005	0.9894	1	0.9973	0.5291	0.3748
0.01	0.9765	0.9978	0.9582	0.5735	0.4276

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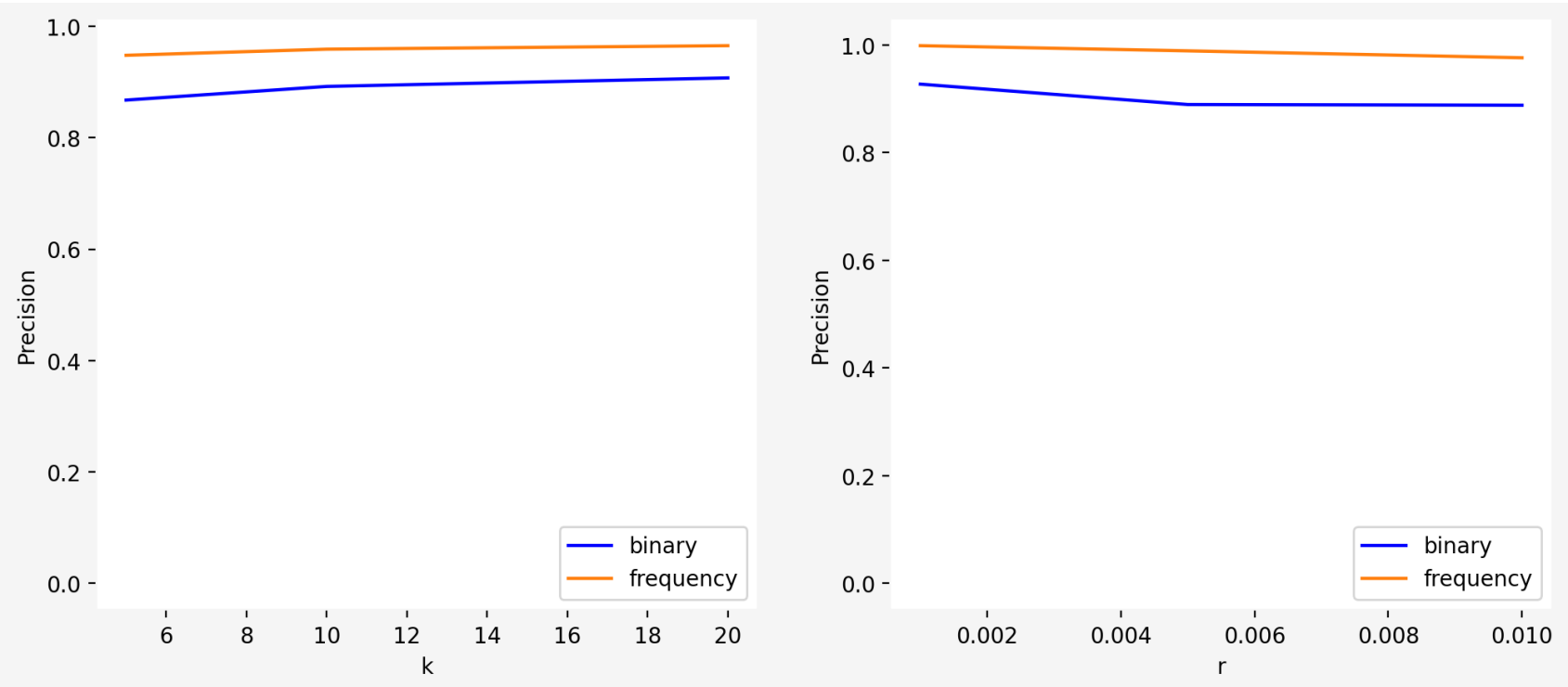
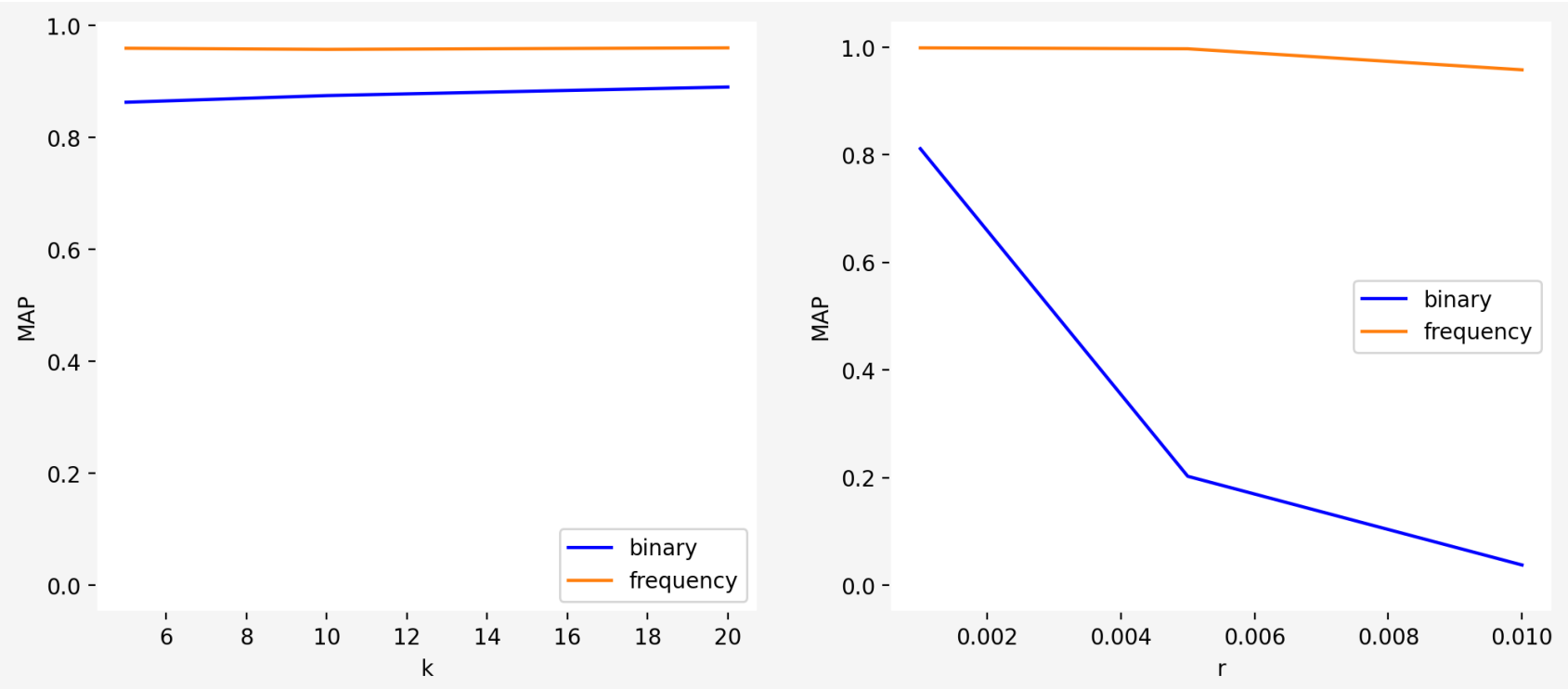
Precision


1.0
0.8
0.6
0.4
0.2
0.0
Precision
6
8
10
12
14
16
18
20
k
binary
frequency
1.0
0.8
0.6
0.4
0.2
0.0
Precision
0.002
0.004
0.006
0.008
0.010
r
binary
frequency

MAP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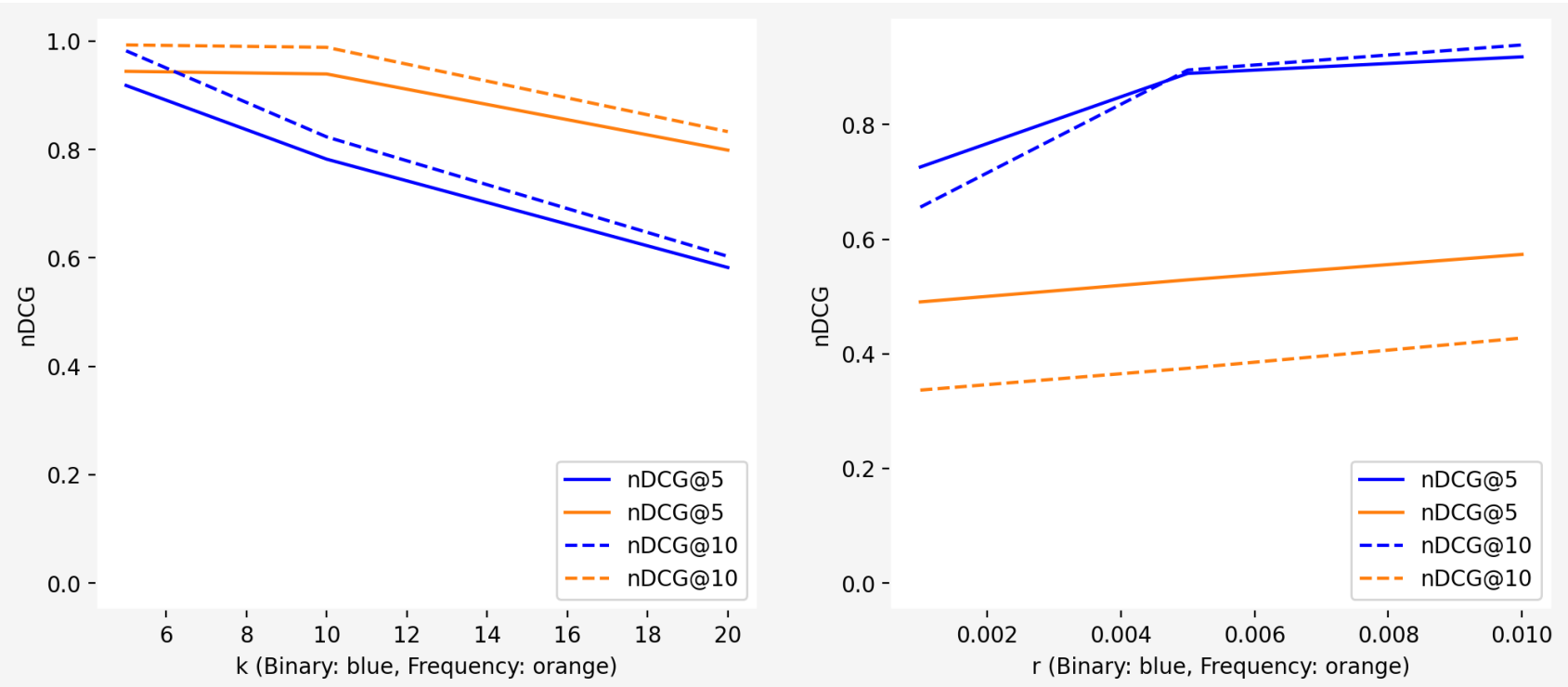
nDCG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k (Binary: blue, Frequency: orange)
nDCG@5
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- The binary representation tended to perform better in terms of nDCG for range queries,
- LSI technique works better with GAD in comparison to DTW distance,
- Increasing the number of grids leads to a better performance,
- Increasing the number of dimensions in dimensionality reduction technique leads to a better performance.

My Contributions

- Applying a kernel-based similarity measure,
- Using 3 different datasets as benchmark including a big data,
- Utlizing binary vectorization of trajectories,
- Using term-frequency type trajectory vectorization,
- Applying DTW as a ground truth distance,
- Adjusting the definition of Precision, Recall, MAP and nDCG for trajectories,
- Creating a user-friendly interactive search tool for kNN and range queries of trajectories.

Questions?