KERNEL SMOOTHING FOR JAGGED EDGE REDUCTION

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ABSTRACT

In this paper, we consider the problem of removing jaggy artifacts from images. We consider the kernel regression framework and propose a reduced-rank quadratic adaptive method that adapts to the local gradient direction. The proposed technique is effective in shrinking isophote fluctuations, and the result is smooth edges. We observe that it is critical to differentiate jaggy artifacts from texture, junctions and corners, so that meaningful image structure is preserved. Here, we demonstrate that the spectrum of the local covariance matrix of gradients, also known as the structure tensor, is well suited for differentiation of jaggy artifacts from image structure, and we incorporate this into the kernel regression framework. Results show the efficacy of the approach. Namely, that the method is effective in reducing jaggy artifacts without blurring meaningful image structure.

***Index Terms*—** image, video upscaling, jagged edge reduction, kernel regression.

1. INTRODUCTION

Jaggy artifacts describe a "stair case" or aliasing artifact that appears along straight lines in an image. For synthetic content, the artifacts may be introduced by poor rendering algorithms. Alternatively, for all content, jaggies may be introduced during resolution conversion. For example, jaggies can be created in the down-sampling process with insufficient pre-filtering. Jaggy artifacts can also be introduced during the up-sampling process, typically when filters with small support are used due to complexity restrictions. No matter the cause though, jaggies are visually noticeable and associated with poor quality somewhere within the acquisition to display chain.

A direct approach to reduce jaggy artifacts is to employ improved rendering, down-sampling or upscaling technologies within a system. For example, anti-aliasing algorithms can be employed during the rendering process to reduce the contrast at the edge and attenuate the jaggy artifact [1]. Alternatively, improved pre-filtering can be employed prior to down sampling [2]. Finally, edge adaptive (directional) upscaling methods can provide smooth renditions of edges and prevent the creation of jaggy artifacts [3].

Unfortunately, our scenario of interest does not allow the use of the methods above. Here, we are concerned with a display application, where the display receives image content already containing jaggy artifacts. The creation of these artifacts is due to design decisions in other components of the capture to display chain and not under the control of the display algorithm design. Given this scenario, the goal of our work is to identify and remove jaggy artifacts in order to provide the ultimate image quality to the viewer. This requires attenuation of the artifact without degrading meaningful image structure.

In the rest of the paper, we describe a novel approach that incorporates the structure tensor within a kernel regression framework [4]. We show the method to be well suited for both jaggy artifact reduction and texture preservation. The paper is organized as follows: In Section 1.1, we discuss prior work on the problem of jaggy artifact reduction. In Section 2, we review kernel regression and describe the recent advances in this field. The proposed method is explained in Section 3, which focuses on adopting kernel regression for smoothing of jagged edges. In Section 4, we describe efficient methods for computing kernels and explain the setting under which experiments were conducted. Finally, we conclude this work in Section 5.

1.1. Prior Work

Generally, the task of jagged edge reduction falls under the broad class of video enhancement techniques; particularly, edge enhancement. Our purpose here is not to enhance the cross-sectional gradient attributes of edges; rather, edges are enhanced along their level curves to produce smooth and continuous edges. There are a few approaches that specifically target the same goal and are reviewed in this section.

A large portion of methods that particularly target the reduction of jaggies belong to the PDE-based fields of level set motion [6] and anisotropic diffusion [7]. These methods iteratively reduce the curvature of level curves using the general propagation of fronts and anisotropic diffusion equations. However, in their basic form, they can result in loss of texture

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without proper regularization and stopping criterions.

More optimized approaches to level set motion and diffusion are proposed through the framework of bounded variation optimization [8, 9]. Similar to anisotropic diffusion, most bounded variation methods employ step-wise and iterative optimization with convergence guarantees. Total variation minimization has been extensively used for image restoration and reconstruction. However, it often results in staircasing artifacts [9].

Another class of approaches stem from the kernel regression framework. Specifically, normalized convolution [5] and steering kernel regression [4] are premiers of this field and have been utilized extensively in the past few years. In these approaches, the regression kernel is adapted to the image and is proved to be robust with respect to noise. In this paper, we intend to build a new framework upon adaptive kernel regression with the specific goal of jagged edge reduction. The main challenge of this task is preserving the sharpness of edges, texture and critical points such as junctions and corners. We propose a rank-reduced regression that adapts to the local gradient attributes that are computed using robust measures of structure.

2. KERNEL SMOOTHING

In this section, we describe basic concepts from regression-based smoothing. In quadratic local polynomial regression, the pixel values in the neighborhood of the pixel (i, j), defined by the local window, are estimated as follows:

$$f(i', j') = b_0 + \begin{bmatrix} i'-i \\ j'-j \end{bmatrix}^T \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} i'-i \\ j'-j \end{bmatrix}^T \begin{bmatrix} b_3 & b_4 \\ b_5 & b_6 \end{bmatrix} \begin{bmatrix} i'-i \\ j'-j \end{bmatrix} + e_q \quad (1)$$

where e_q denotes the estimation error associated with quadratic regression. Define $\beta_{ij} = [b_1, \ldots, b_5]$, the vector of parameters and $\hat{f}_{\beta_{ij}}(i', j')$ the estimation of $f(i', j')$ using β_{ij}.

The shape and size of regression windows play an important role in piecewise regression of images. Regression windows can either have clear boundaries or decay smoothly according to a function known as the kernel. A popular choice of kernel function is the Gaussian function with a diagonal covariance matrix $\sigma^2 I$. Furthermore, it is possible to tailor the kernel to the structure of the underlying image, as we describe later in this section.

Kernel regression is performed using weighted least squares estimation as it is expressed below:

$$\beta_{ij} = \min_{\beta_{ij}} \sum_{i'j'} K_{ij}(i', j')[f(i', j') - \hat{f}_{\beta_{ij}}(i', j')]^2 \quad (2)$$

where K_{ij} is the window kernel centered at (i, j).

It is desired that the kernel function decays quickly across the edges, to preserve the sharpness of edges, and smoothly along the edges to create smooth and visually pleasing edges. Also, it is desired that the kernel support vanishes when texture is detected to avoid the difficulties in dealing with texture. A well known approach to adaptation of kernels is through *structure tensor field*. For images, structure tensor at pixel location (i, j) is the 2×2 weighted covariance matrix of local gradients in horizontal and vertical directions:

$$S_{ij} = \sum_{kl} w_{ij}(k, l) \begin{bmatrix} f_x(l) \\ f_y(l) \end{bmatrix} \begin{bmatrix} f_x(k) & f_y(k) \end{bmatrix} \quad (3)$$

f_x and f_y are gradients in horizontal and vertical directions and $w_{ij}(k, l)$ is usually a bounded Gaussian function centered at (i, j). A critical property of S_{ij} is its positive-definiteness except at regions with a fixed gradient, like flat surfaces. Such singular cases are excluded from further processing since edges are the main target of this work.

Well known methods such as adaptive normalized convolution [5] and steering kernel regression [4] employ the structure tensor field to extract piecewise linear structures like edges. This information is utilized in a kernel function of the following form:

$$K_{ij}(i', j') \propto \exp(-[i'-i, j'-j] S_{ij} \begin{bmatrix} i'-i \\ j'-j \end{bmatrix} / (2\alpha_{ij})) \quad (4)$$

which is basically a Gaussian function with $\mu = (i, j)$ and $\Sigma^{-1} = S_{ij}/\alpha_{ij}$. α_{ij} is a scalar scaling parameter that is used to further tune the kernel for our specific application as it is explained in the following section.

3. MODIFYING KERNEL SMOOTHING FOR JAGGED EDGE REDUCTION

In this paper, kernel smoothing is utilized to reduce jaggedness of edges. This is accomplished by reducing the excessive curvature of the edge's level curves or isophotes. The amount of smoothing that is desired can be achieved by adjusting the order of the polynomial regression. However, due to the bias-variance trade-off, desired suppression of jaggies cannot be attained without sacrificing edge sharpness. To overcome this issue, we propose an adaptive regression method that consists of quadratic regression across edges (isophotes) and zero-order regression along edges (isophotes). The piecewise orientation of isophotes can be reliably extracted from the structure tensor that is computed for the regression kernel. After the orientation of the gradient (normal to isophote) at pixel (i, j) is computed as $(\hat{f}_x(i, j), \hat{f}_y(i, j))$, the new regression can be written as:

$$f(i', j') \approx a_0 + a_1 \begin{bmatrix} i'-i \\ j'-j \end{bmatrix}^T \begin{bmatrix} \hat{f}_x(i, j) \\ \hat{f}_y(i, j) \end{bmatrix} + a_2 \left(\begin{bmatrix} i'-i \\ j'-j \end{bmatrix}^T \begin{bmatrix} \hat{f}_x(i, j) \\ \hat{f}_y(i, j) \end{bmatrix} \right)^2 \quad (5)$$

The regression of (5) is basically a one-dimensional quadratic regression in the direction of the gradient vector. It is not hard to show that the regression of (5) forces the isophote curvature [6] at pixel (i, j) towards zero. Meanwhile, large-scale curves, related to the shape of objects, are preserved since regression coefficients are computed locally through a kernel that adapts to the underlying structure. In Figs. 1 and 2 we have plotted the independent variables, also known as covariates, associated with regression problems of (1) and (5) on 2-D image grids.

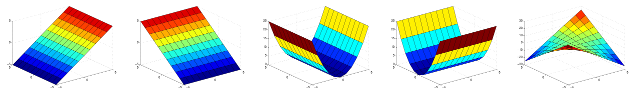

Fig. 1. From left to right: regression covariates $i' - i$, $j' - j$, $(i' - i)^2$, $(j' - j)^2$ and $(i' - i)(j' - j)$ corresponding to (1).

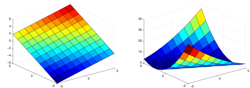

Fig. 2. Regression covariates of (5) that adapt to the structure.

Note that covariates in Fig. 2 that adapt to the structure of edges preserve sharpness of edges and at the same time reduce variations along the edges. The non-adaptive covariates of Fig. 1 allow large variations both across and along the edges that is the main reason for ineffectiveness of these filters for smoothing jaggies.

3.1. Selection of the scaling parameter

Complexity of natural textures cannot be modeled by simple linear functions. Hence, in this work, we avoid smoothing of fine texture, corners and junctions by reducing the size of regression window over such structures. The scaling parameter α_{ij} in (4), controls the width of the Gaussian kernel for each pixel and is computed using the measure of coherency [10]:

$$c_{ij} = \frac{(\lambda_1 - \lambda_2)^2}{(\lambda_1 + \lambda_2)^2}$$

where λ_1 and λ_2 are respectively the larger and smaller eigenvalues of S_{ij}, the structure tensor. λ_1 corresponds to the variance of pixel values in the direction of the gradient and λ_2 is the variance in the normal direction. Hence, coherency measures the normalized anisotropy of neighborhood which represents the edge confidence. Near zero coherency corresponds to the absence of a single dominant direction and can be associated with smooth areas, texture, corners or edge junctions. Coherency values close to one correspond to strict directionality and can be confidently associated with edges that are the subject of this paper.

An important factor in the visibility and energy of edges is the presence of contrast that is not a factor in the definition of normalized coherency. Contrast can be incorporated by inserting the variance term $\sqrt{\lambda_1 \lambda_2}$ into the definition of α_{ij}:

$$\alpha_{ij} = \sqrt{\lambda_1 \lambda_2} c_{ij} = \sqrt{\lambda_1 \lambda_2} \frac{(\lambda_1 - \lambda_2)^2}{(\lambda_1 + \lambda_2)^2}$$

To summarize, α_{ij} measures the degree of anisotropy and visual appeal of a local region.

4. RESULTS

The algorithmic implementation of proposed method is carried out in two stages. In the first stage, the structure tensor field and its derivatives such as its eigenvectors and eigenvalues are computed. In the second stage least-square based kernel regression is applied to smooth jagged edges. Before showing the results, we describe an efficient method for computation of structure tensor field.

4.1. Fast calculation of the structure tensor field

First, gradients are computed. In order to have a robust estimation of gradients, the Gaussian derivative mask was used. The structure tensor field and its eigenvalue decomposition are obtained from f_x and f_y planes using the following operations:

$$\begin{aligned} \mathbf{S}_{11} = G * [f_x \odot f_x], \mathbf{S}_{22} = G * [f_y \odot f_y] \\ \mathbf{S}_{12} = \mathbf{S}_{21} = G * [f_x \odot f_y] \end{aligned} \tag{6}$$

Here, $\mathbf{S}_{ij}$ denotes the plane of S_{ij} values for all pixels, G is a finite support Gaussian filter with tunable width, $*$ and $\odot$ respectively denote the convolution and the element-wise multiplication operators. This formulation presents a more efficient per-frame basis for computing the structure tensor for pixels. Before the regression is performed, eigenvalues and eigenvectors are computed using basic algebraic expressions.

4.2. Experiment setting

To simulate jagged edges, we first downscale and then upscale a set of real-world videos using classic upscaling methods such as bilinear interpolator. Bilinear interpolator was chosen to simulate the worst case scenario. Also the downsampling is done without anti-aliased prefiltering to produce significant jaggies for easier inspection.

A comprehensive comparison of our proposed method with the published works in this field is clearly out of the scope of this paper. However, it is reasonable to compare our method with the state-of-the-art in the kernel smoothing domain, namely the Steering Kernel Regression (SKR) [4] which has attracted much attention recently. For a fair comparison of SKR and our method, we have tuned the two methods to similar levels of smoothing.

4.3. Simulation Results

For demonstration, we use a frame from each of the two sequences. The first sequence is a recorded basketball match that contains a clear combination of edges and texture on the ground and audience areas plus it has compression artifacts that further increase the amount of edge jaggedness. The second one is a high-definition video containing complex patterns of plantal bodies that overlap and do not conform to

Fig. 3. First row: input images (downsampled and upscaled using bilinear interpolator). Second row: SKR with quadratic regression. Third row: SKR with zero-order regression. Fourth row: The proposed kernel regression.

most idealistic models and therefore, is of our interest in this work. Cropped and zoomed patches from these videos are shown in the first row of Fig. 3 after they were downsampled and upscaled as explained in the previous subsection.

The result of applying SKR is shown in Fig. 3, the second row. SKR is proved to work well for reduction of noise and blocking artifacts. However, direct application of SKR, as can be seen in the figure, results in texture blur while jaggies are moderately corrected. We have marked several areas in Fig. 3 where the original SKR either fails to reduce the jaggedness or blurs the texture. Also, we implemented SKR with zero-order regression to test the maximum smoothing power of SKR in correcting significant jaggies that is shown in the third row of Fig. 3.

The results of the proposed kernel smoothing is presented in the fourth row of Fig. 3. We note to the two advantages of these results: a) jagged edges have become smooth without significant loss of sharpness and b) textures, corners and junctions are mostly preserved. The marked regions in Fig. 3 visualize and ascertain these points. The reduction in jaggies comes as a natural consequence of the proposed reduced-rank regression. The preservation of texture is due to the contraction of window kernel over isotropic regions and is directly related to the definition of α_{ij} parameter in Section 3.1. To conclude, the new kernel regression improves smoothing of jaggies, even compared to zero-order SKR, and introduces less blurring which alleviates the need for post-deblurring.

We implemented the proposed method on an Intel Core 2 Due CPU 2.93GHz using the Matlab platform. The average processing time for each frame of the basketball HD sequence was measured around 7.2 seconds.

5. CONCLUSION

Recently, it has been shown that regression windows can be well adapted to the underlying structure using the notion of structure tensor field. However, there has not been enough work on the selection of different regression functions along with the adaptive kernel based smoothing. In this work, we proposed a novel reduced-rank quadratic estimation that reduces variance associated with isophote oscillations while preserving texture. The proposed method leads to smooth edges with sharp cross-section gradient profiles.

6. REFERENCES

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