Lecture 9 – Complexity theory 2

Last time

- complexity classes
- P, EXP, NP, BPP, BQP, PSPACE
- reductions, hardness, completeness
- oracles

This lecture

- the polynomial hierarchy PH
- counting classes #P, GaP functions, PP
- BPP, NP from GaP and counting functions
- AWPP and interference computation
- post-selection PostBPP, PostBQP

The polynomial hierarchy (PH)

Using oracles, we can define a hierarchy of complexity classes.

NP^{NP} — $L \in NP^{NP}$ if $\exists$ det poly-time alg A s.t

$x \in L$, $\exists w_1 \in \{0,1\}^{poly(|x|)}$ s.t

$\forall w_2 \in \{0,1\}^{poly(|x|)}$ $A(x, w_1, w_2) = 1$

$x \notin L$, $\forall w_1$, $\exists w_2$ $A(x, w_1, w_2) = 0$

$NP^{NP^{NP}}$ and so on

$$PH = P \cup NP \cup NP^{NP} \cup NP^{NP^{NP}} \cup \ldots$$

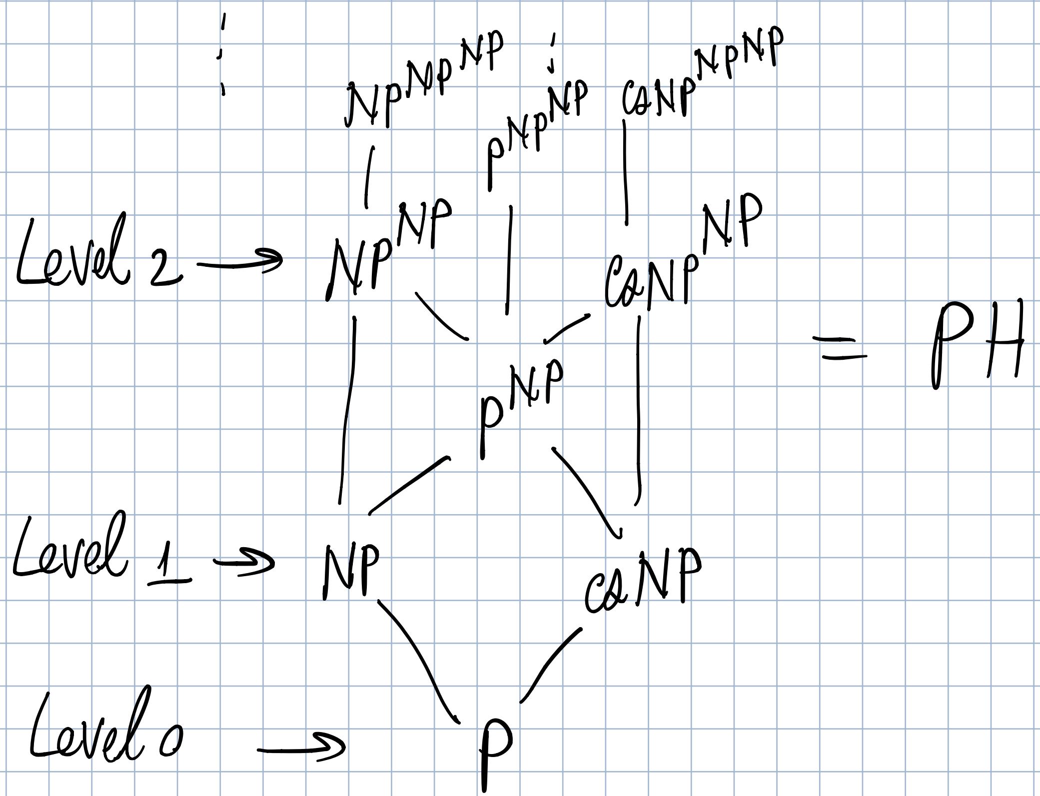
Recall that languages have complements.

$$\overline{L} = \{x \mid x \in \{0,1\}^* \setminus L\}$$

if $L \in NP$, $\overline{L} \in coNP$

coNP — complement of NP

Conjectured that $NP \neq coNP$



If classes on level k are equal then all classes at higher levels are equal to the ones on level k.

= hierarchy collapses at level k.

0 level collapse $\iff P = NP$

Conjectured that PH does not collapse at all.

We will see that we can prove results conditioned on PH not collapsing at same level.

Counting classes

We've mostly talked about decision problems, but there's another interesting kind of problems known as counting problems.

Recall that for an NP problem we required the existence of a witness for Yes instances. But how many witnesses can there be? This is a counting problem.

Let $L \in NP$ and A an algorithm s.t

$$x \in L, \exists w \text{ s.t } A(x,w) = 1$$

$$x \notin L, \nexists w \text{ s.t } A(x,w) = 1$$

Define $f_A : \{0,1\}^* \to \mathbb{N}$ such that

$$f_A(x) = \# \text{ of } w \text{ s.t } A(x,w) = 1$$

E.g. for 3-coloring, f counts the number of 3-colorings for a graph G.