Lecture 12 – Quantum simulation 2 (Phase estimation)

$$H(t) = \sum_{i=1}^{m} a_i H_i \rightarrow \text{acts on } n \text{ qubits}$$

H_i: acts non-trivially on k qubits

$k = O(1)$

$$\underbrace{e^{-iHt}}_{U}$$

$$e^{A+B} \neq e^{A} \cdot e^{B}$$

$$e^{A+B} = \lim_{r \to \infty} \left(e^{A/r} \cdot e^{B/r} \right)^{r} \rightarrow \text{TS of 1st order}$$

$$|\psi(0)\rangle \xrightarrow[\underbrace{\quad}_{U}]{H,\,t} |\psi(t)\rangle$$

$Z \rightarrow |0\rangle, |1\rangle$ estates

$\qquad -1 \quad +1$ evalues

H: $Z \rightarrow X$

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

In: $H = \sum_{i=1}^{m} a_i H_i$ on n qubits, k-local

Out: $|\Psi_{GS}\rangle$ $\quad H \cdot |\Psi_{GS}\rangle = \lambda_{GS} |\Psi_{GS}\rangle$

$$\lambda_{GS} = \min \mathrm{Evals}(H)$$

k-local Ham problem $\left(Z^{\otimes n} \quad |0^n\rangle \right)$

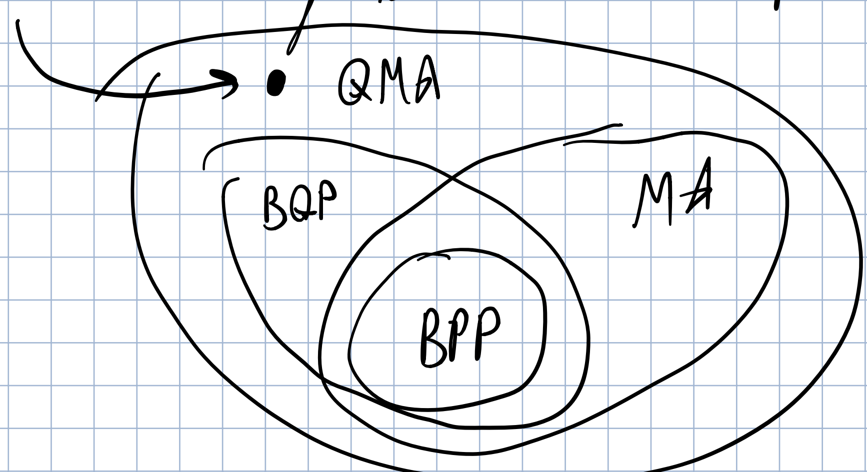
BPP - poly-time classically | BQP - q poly-time

NP - poly-time classically verify yes instances

(MA)

QMA - q poly-time verif of yes instances

k-local Ham problem is QMA-complete



Phase estimation

$$U, \quad |u\rangle$$

$$U|u\rangle = e^{2\pi i \cdot \varphi_u}|u\rangle \qquad \varphi_u \to [0, 1]$$

Quantum Fourier Transf QFT$_n$

$$QFT_n |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} \omega^{x \cdot y} |y\rangle$$

$$\omega = e^{\frac{2\pi i}{2^n}}$$

$$QFT_n |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} e^{\frac{2\pi i}{2^n} \cdot x \cdot y} |y\rangle$$

$$y = y_1 y_2 \cdots y_n$$

$$y = \sum_{j=1}^{n} 2^{n-j} \cdot y_j$$

$$QFT(x) = \frac{1}{\sqrt{2^n}} \sum_{y_1=0}^{1} \sum_{y_2=0}^{1} \dots \sum_{y_n=0}^{1} e^{2\pi i \cdot x \cdot \sum_{j=1}^{n} \frac{y_j}{2^j}} |y_1 y_2 \dots y_n\rangle$$

$$\prod_{j=1}^{n} e^{2\pi i x \cdot \frac{y_j}{2^j}} \qquad \bigotimes_{j=1}^{n} |y_j\rangle$$

$$= \frac{1}{\sqrt{2^n}} \sum_{y_1=0}^{1} \dots \sum_{y_n=0}^{1} \bigotimes_{j=1}^{n} e^{2\pi i x \cdot \frac{y_j}{2^j}} |y_j\rangle$$

$$= \frac{1}{\sqrt{2^n}} \bigotimes_{j=1}^{n} \sum_{y_j=0}^{1} e^{2\pi i x \cdot \frac{y_j}{2^j}} |y_j\rangle$$

$$\left(|0\rangle + e^{2\pi i x \cdot \frac{y_j}{2^j}} \right)$$

$$0.b_1 b_2 \dots b_n = 2^{-1} \cdot b_1 + 2^{-2} \cdot b_2 + \dots + 2^{-n} \cdot b_n$$

$$\frac{1}{2} x_n \rightarrow 0.x_n$$

$$= \frac{1}{\sqrt{2^n}} \left(|0\rangle + e^{2\pi i \cdot \frac{1}{2} x_n} |1\rangle \right) \otimes \left(|0\rangle + e^{2\pi i \cdot 0.x_{n-1} x_n} |1\rangle \right) \dots \left(|0\rangle + e^{2\pi i \, 0.x_1 x_2 \dots x_n} |1\rangle \right)$$

$$x = 2^{n-1} \cdot x_1 + 2^{n-2} \cdot x_2 + \dots + 2^0 \cdot x_n$$

$$\frac{x}{2} = \underbrace{2^{n-2} x_1 + \dots + 2^0 \cdot x_{n-1}}_{W} + \frac{1}{2} \cdot x_n$$

$U, \ |u\rangle$

$0.\varphi_1\varphi_2\ldots\varphi_n$

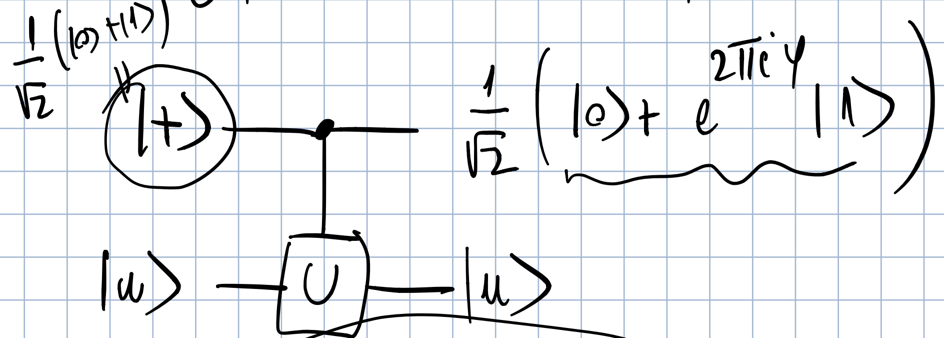
$$U|u\rangle = e^{2\pi i \cdot \varphi} \cdot |u\rangle$$

$\varphi = 0.75$

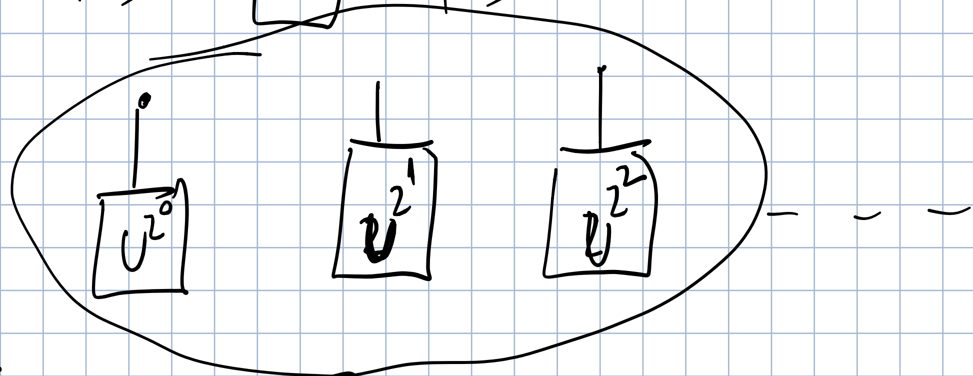
$= 0.5 + 0.25$

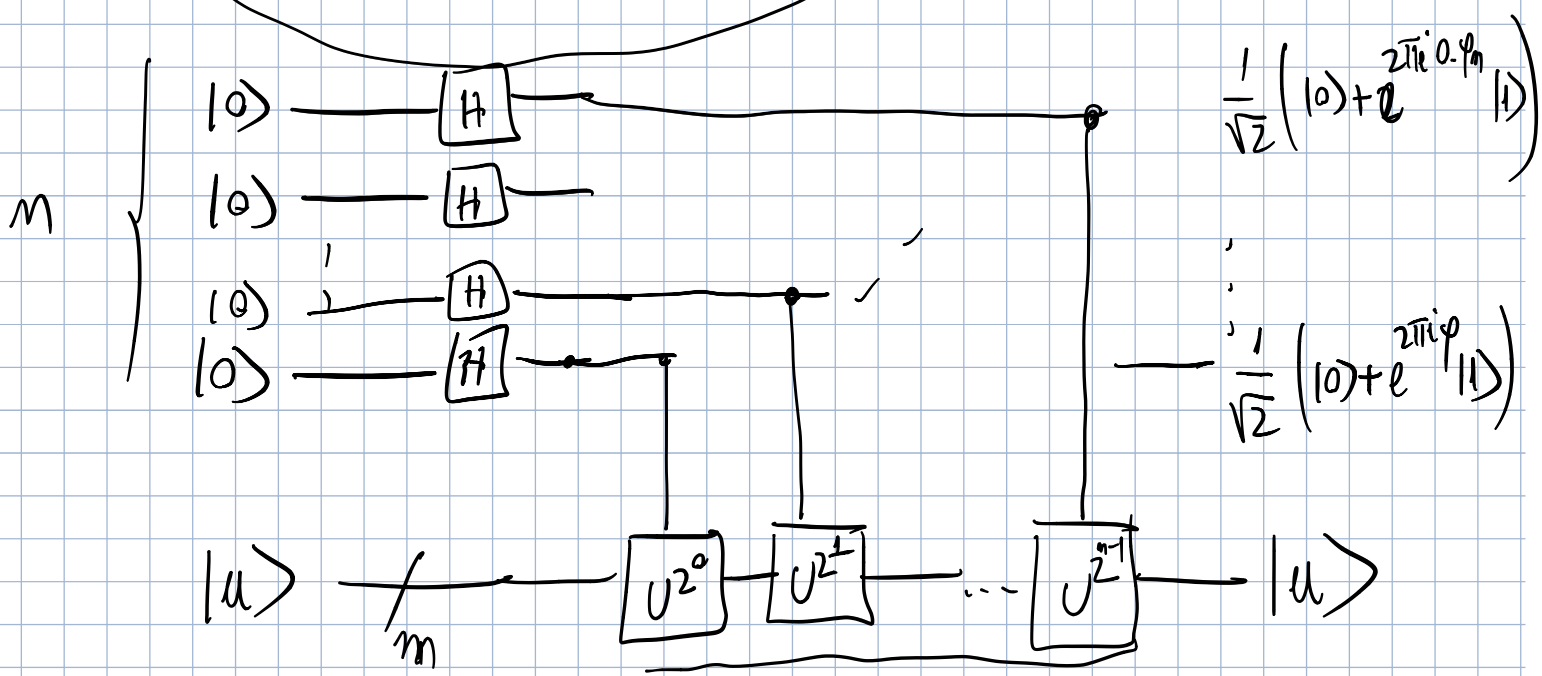
$\downarrow$

$0.1100\ldots$

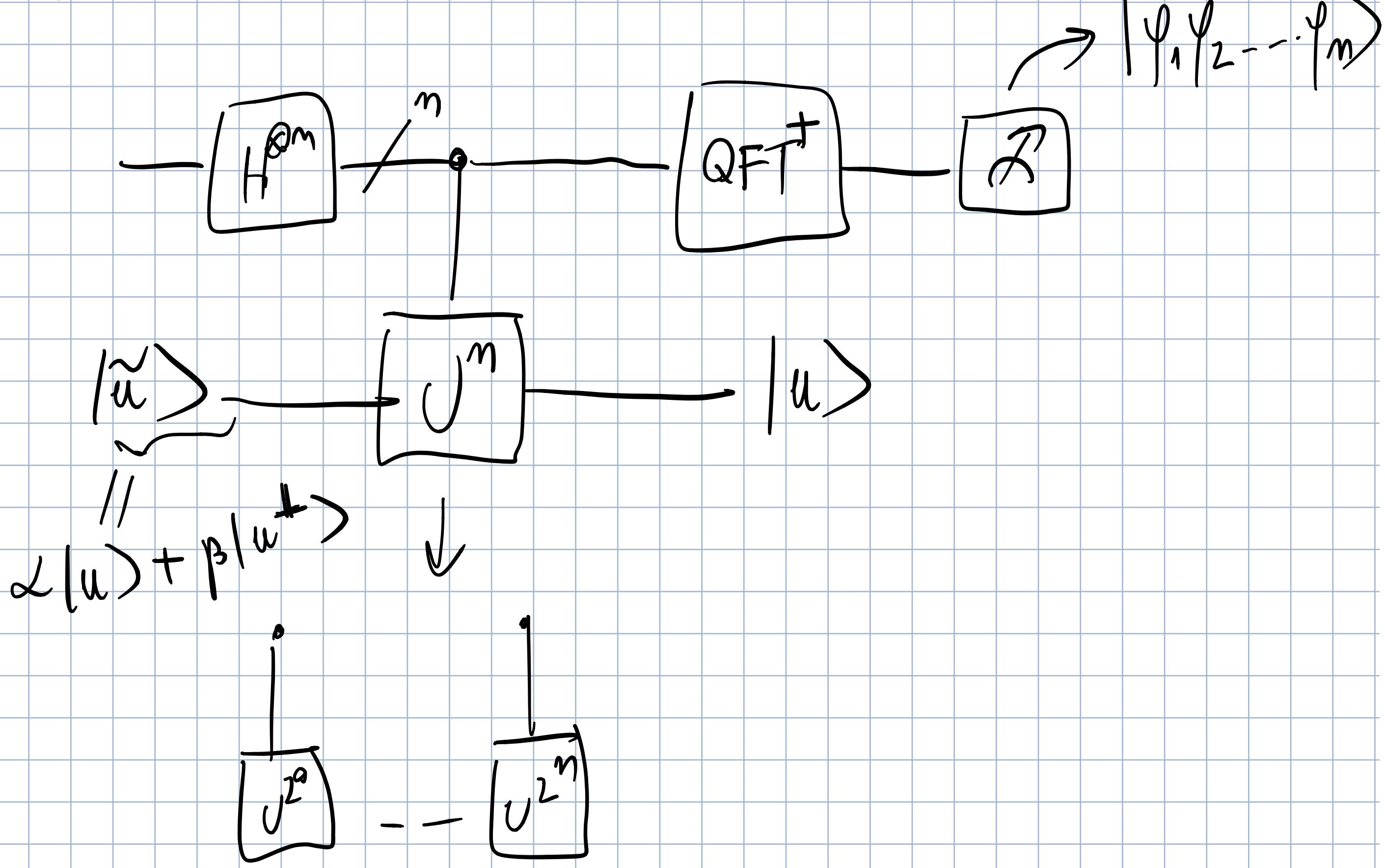


$\boxed{0.\varphi_2\ldots\varphi_n}$





Phase est



$$H \xrightarrow{\text{Trotter-Suzuki}} e^{2\pi i H} \rightarrow \text{same eigenstates as } H$$

$$H \downarrow \quad \lambda_1 \cdots \lambda_n$$

$$e^{2\pi i H} \downarrow \quad e^{2\pi i \lambda_1} \cdots e^{2\pi i \lambda_n}$$

- Guess (ansatz)
- DMRG
- Tensor network

$$-\theta \quad +\theta$$

$$\theta z \to |0\rangle, |1\rangle$$

$$e^{2\pi i z \cdot \theta} \to |0\rangle, |1\rangle$$

$$e^{2\pi i \cdot (-\theta)} \quad e^{2\pi i \theta}$$

$$\downarrow$$

$$1 - \theta$$

Phase est for $U_g |y\rangle = |\underline{g} \cdot y \mod N\rangle$

$$\downarrow$$

$$|u_s\rangle = \frac{1}{\sqrt{r}} \sum_{k=0}^{r-1} e^{\frac{2\pi i k \cdot s}{r}} |g^k \cdot y \mod N\rangle$$

r is order of g

$$g^r \mod N = 1$$

$$U_g \cdot |u_s\rangle = e^{\frac{2\pi i s}{r}} |u_s\rangle$$

$$\sum_s |u_s\rangle = |1\rangle$$

$$\hookrightarrow |0 \ldots 1\rangle$$

n qubits