PL-FSI Model Simulation for matrices in Graph-Laplacians:

Let 'm' be the dimension of the connectivity matrix.

$\mathcal{L}_m$ be the space of graph-laplacians, defined as,

$$\mathcal{L}_m = \{L = (l_{ij}) : L = L', \, L 1_m = 0_m ; \, -W \leq l_{ij} \leq 0 \text{ for } i \neq j\}$$

Define, the embedding space,

$$\mathcal{M}_m = \{ S \in S_m^+ : \lambda_1(S) \leq D^\alpha \} \quad *$$

where $\lambda_1(S)$ is the largest eigenvalue of S.

for every $L \in \mathcal{L}_m$; the graph-Laplacian matrix;

$vech(L)$ is the vectorization of the upper (or lower) triangular part.

Then $vech(L)$ is of length $d = m(m-1)/2$.

Let $i = 1, 2, \dots, n$ be index for the observations, n = total number of observations in the study.

We want to fit the model;

$$E(Y_i | Z_i, X_i) = \alpha_0 + \beta_0 Z_i + g(\theta' X_i)$$

where $\underset{q\times 1}{Z_i}, \underset{p\times 1}{X_i}$ are the covariates for the linear and non-linear parts of covariates for the i-th participant in the study.

$\alpha_0, \beta_0, \underset{p\times 1}{\theta}$ are the parameters of regression, g is a uniformly continuous function; twice differentiable.

Let $\theta = (0.45,\ 0.893)$ for $p=2$

$\theta = (0.3,\ -0.6,\ 0.742)$ for $p=3$

Generate $x \in$ Unif $(0,1)$ beta?? for each i

$g = x^2 + 3x$ for all x; (Choose other functions)

$\beta_0 = -2.5$

$\alpha_0 = 1.5$

Then get $X = 1.5 \times (-2.5) Z_i + g(\theta' X_i)$

then generate $(\beta_1, \beta_2, \ldots, \beta_d)$ a random sample of size d from Beta$(X, 1-X)$. (Change to $\frac{X}{\|X\|}$?)

then $\mathrm{Vech}^{-1}(-\beta_1, -\beta_2, \ldots, -\beta_d) = L$

Estimation of the parameter θ:

Run the Global Network regression on the $L(\theta)$

Create an equidistant grid over the range of θ;

Let $q = 1, 2, \ldots, Q$ be the index for the simulations;

then $\hat{Y}^{(i)}_q(\theta)$ be the predicted Graph-Laplacian matrix for the parameter θ for the simulation q; for the i-th participant (observation).

then we compute,

$$W(\theta) = \frac{1}{Q} \sum_{q=1}^{Q} \left(\frac{1}{n} \sum_{i=1}^{n} d^2 \left(L_i, \hat{y}_q^{(i)}(\theta) \right) \right)$$

then we estimate $\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmin}} \; W(\theta)$