TRAVELLING SALESPERSON PROBLEM.

$G = \langle V, E \rangle$

$V \Rightarrow N$ cities / Vertices

$E \Rightarrow$ Edges between cities.

$C_{ij} \Rightarrow$ Cost of the edge (i, j) and it can be observed that $C_{ij} = \infty$ as the travelling person cannot visit the city itself (cannot be both source and destination)

⇒ Travelling salesperson starts the tour from a city, say vertex 1, visit all other vertices only once and ends the tour at the city of origin.

3 bounds associated with the problem

① Bound $C(i) \Rightarrow$ length of the path.

* In case of leaf node, this represents the length from the root to given node i.
* In case of non leaf node i, this represents the minimum cost node of the state-space tree.

② Bound $l(i) \Rightarrow$ lower bound that indicates the length of the path.

⇒ This is obtained from the reduced cost matrix.

(2) To find $l(i)$ for the node i, distance matrix should be reduced

⇒ It can be recollected that a row or column is said to be reduced if it has at least one zero.

⇒ A matrix is said to be reduced if every row and column of the matrix is reduced ⇒ Row or column should have atleast one zero.

(1) ⇒ To reduce the distance matrix, subtract a constant p from every entry

(2) ⇒ Choose the minimum amount of every row or column.

(3) Repeat the process as required.

(4) Total amount subtracted from the rows and columns is a low bound l for the root.

(5) Associate the reduced cost matrix with every node of the state-space tree.

(3) Bound $u(i)$ → Upper bound of the node i. UB of all the nodes is ∞. It is the maximum possible effort required for the tour.

Alg:

(1) Let $C \rightarrow$ reduced cost matrix.
Assume that the edge (i, j) is included in the tour. Change all the entries in row i and column j of the distance matrix to ∞. [This is to exclude all the paths leaving the vertex i and reaching the vertex j]

(2) Set $C(j, 1) = \infty$. Exclude all $(j, 1)$ paths

(3) Reduce distance matrix except for the rows and the columns that contain ∞.

(4) Compute lower bound. If the node is a non-leaf, then LB =

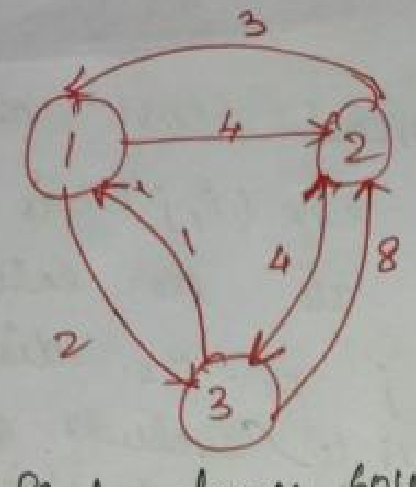
$$l(B) = l(A) + C(i, j) + T.$$

$A \rightarrow$ parent of node B in state space tree.

$T \Rightarrow$ sum of all elements involved in ~~rote~~ row & column reduction.

If node is leaf, then l is the cost itself.

4



$$A = \begin{bmatrix} \infty & 4 & 2 \\ 3 & \infty & 4 \\ 1 & 8 & \infty \end{bmatrix}$$

$\Rightarrow$ find lower bound.

$\Rightarrow$ find row minimum.

Row minimum's are 2, 3, 1.

Subtract row min from the entire row $\Rightarrow$ reduced matrix.

Row reduction before and after

$$\begin{bmatrix} \cancel{\infty} & \cancel{4} & \cancel{2} \\ 3 & \infty & 4 \\ 1 & 8 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 2 & 0 \\ 0 & \infty & 1 \\ 0 & 7 & \infty \end{bmatrix}$$

Column red 0, 2, 0

$$\begin{bmatrix} \infty & 2 & 0 \\ 0 & \infty & 1 \\ 0 & 7 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix}$$

Total reduction $2+3+1+0+2+0 = 8$.

$\Rightarrow$ lower bound.

1, 1, 2, 2

This is min. cost required for any

3, 3, (4, 4) tour.

Similarly reduced matrix for node (1, 2) is computed

Row red. 0, 0, 0

$$\begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix}$$

Col. red 0, 0, 0

$$\begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix} \Rightarrow \begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix}$$

~~Total row & Col. red = 0+0+0+~~

path (1,2)

⇒ set all entries of row1 & col 2 as ∞
is set A[2,1] as ∞ (to prevent path (1,2) from becoming part of tour).

$$\begin{matrix} \infty & 0 & 0 \\ \infty & \infty & 1 \\ 0 & 5 & \infty \end{matrix} \Rightarrow \begin{bmatrix} \infty & 0 & 0 \\ \infty & \infty & 1 \\ 0 & 5 & \infty \end{bmatrix} \begin{bmatrix} \infty & 0 & 0 \\ \infty & \infty & 0 \\ 0 & 5 & \infty \end{bmatrix}$$

Row min ,0,1,0,

Col. red 0,2,0

$$\begin{bmatrix} \infty & 0 & 0 \\ \infty & \infty & 0 \\ 0 & 5 & \infty \end{bmatrix} \quad \begin{matrix} \infty & 0 & 0 \\ \infty & \infty & 0 \\ 0 & 5 & \infty \end{matrix}$$

Total row & col. red. is 0+1+0+0+0
= 1.

Cost involved in traversing (1,2) is 0.

lower bound is 8 + (1,2) + total reduction for this node

= 8 + 4 + 1 = 13.

path (1,3)

row1 & col.3 ⇒ ∞.

0,5

Col. red

0,0

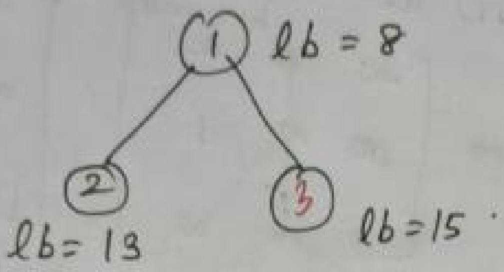
$$\begin{matrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ \infty & 5 & \infty \end{matrix} \xrightarrow[\text{red}]{\text{row}} \begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ \infty & 0 & \infty \end{bmatrix} \xrightarrow[\text{red}]{\text{col}} \begin{bmatrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ \infty & 0 & \infty \end{bmatrix}$$

5

6

8 + A(1,3) + total row & col. red

= 8 + 2 + 5 = 15.



⇒ only remaining node is C.

Lower bound of node B is < node C.

Explored it further

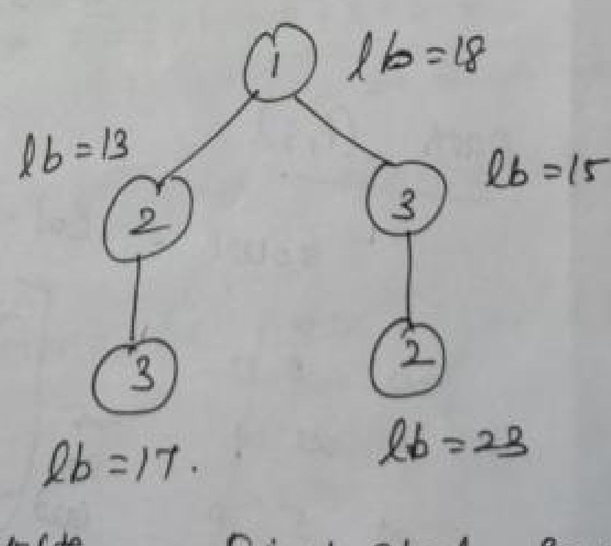
pat (2,3)

$$\begin{matrix} \infty & 0 & 0 \\ 0 & \infty & 1 \\ \infty & 0 & \infty \end{matrix} \Rightarrow \begin{matrix} \cancel{\infty} & \cancel{0} & \cancel{0} \\ \cancel{\infty} & \cancel{\infty} & \cancel{0} \\ 0 & \infty & \cancel{\infty} \end{matrix} \Rightarrow \begin{matrix} \infty & 0 & 0 \\ \infty & \infty & 0 \\ 0 & \infty & \infty \end{matrix}$$

13 + A(2,3) + 0

13 + 4 + 0 = 17.

(3,2) 23.



Optimal path

1 − 2 − 3 − 1 holds Final state space tr.
min cost = 17.

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 10 & \infty \end{bmatrix}$$

Solve the travelling salesman problem using branch and bound technique.

6, 7, 9, 12, 13, 14, 21, 26, 30, 33, 36, 37, 38, 39, 44, 45, 47, 50, 52, 55, ~~45~~

FLOYD WARSHALL ALGORITHM ⇒ All pair ~~weighted~~ shortest path alg.

⇒ Transitive closure of a directed graph G is a matrix P, whose value is 1 if there exists a path b/n any 2 given vertices and 0 if there is a path b/n them

P → connectivity matrix / transitive cl.

closure, as it a transitive relation than encloses the binary relation E.

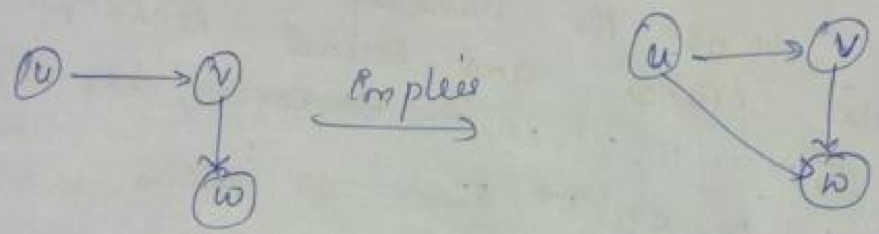
Binary relation

If a binary relation R exists b/n 2 vertices say u and v then mathematically it can be represented as uRv.

⇒ Binary relation considered here is a path relation.

⇒ A relation uRv indicates that there is a path from u ~~and~~ to v.

⇒ Transitive relation states that if binary relationships exist b/n u and v and b/n v and w, then there exists a relation b/n u & w.



Transitive relation.

BRUTE FORCE (involves multiple multiplications).

A → Adjacency matrix of graph G.

Binary matrix

$$B = A + A^2 + A^3 + \dots A^n.$$

n ⇒ no. of vertices that are present in the graph.

A^2 ⇒ Adjacency matrix that represents the edge that connects 2 vertices that are 2 hops away.

A^3 ⇒ " " " 3 vertices

⇒ Path matrix depends on k

$$p_{ij} = \begin{cases} 1 & \text{if } ij^{th} \text{ entry of matrix B is not 0} \\ 0 & \text{otherwise} \end{cases}$$

Path matrix

$p_{ij} = 1 \rightarrow$ trivial path from i to j

$p_{ij} = 0 \rightarrow$ cannot reach from i to j

Procedure

① Compute the adjacency matrix of a given directed graph $R^{(0)}$

② Compute $R^{(1)}$ in which 1st vertex is used as intermediate vertex for computation

③ Similarly construct $R^{(k)}$ & $R^{(k-1)}$ by vertices as including k intermediate vertices as per (1) & (2).

④ Finally $R^{(n)}$ is computed. In this n vertices are used as intermediate vertices. The Resultant is the transitive closure of a given digraph.

(10)

$$D^0 = \begin{matrix} 0 & \infty & 3 & \infty \\ 2 & 0 & \infty & \infty \\ \infty & 7 & 0 & 1 \\ 6 & \infty & \infty & 0 \end{matrix}$$

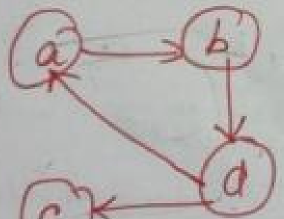
Unweighted graph.

$$\boxed{R^{(k)} = r_{ij}^{(k)} = r_{ij}^{(k-1)} \text{ or } r_{ik}^{(k-1)} \text{ and } r_{kj}^{(k-1)}}$$

logical

0 or (1=0)

If $r_{ij}^{(k)} = 1$ then either $r_{ij}^{(k-1)} = 1$ [A path from vertex i to j with $k-1$ intermediate vertices] both $r_{ik}^{(k-1)} = 1$ & $r_{kj}^{(k-1)} = 1$ [A path from vertex i to k with $k-1$ intermediate vertices & a path from k to j with $k-1$ intermediate vertices).



$$R^{(0)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

1 or (0)

, 1

$R^{(0)}(d,a) = 1$ & $r^{(0)}(a,b) = 1$, $\therefore$ path exists from d to b with a as intermediate vertex.

$$d_{ij} = \min\{d_{ij}^{k-1}, d_{ik}^{(k-1)} + d_{kj}^{k-1}\}$$

weighted graph.

11

$$R^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

ones reflect the existence of paths with intermediate vertices numbered not higher than 1, i.e just vertex a (note a path from d to b). boxed row & column.

min {i, j or (i, k or k, j)}

$$R^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

ones reflect the existence of paths with intermediate vertices numbered not higher than 2. i.e a and b (note 2 new paths.) boxed row & column.

$$R^{(3)} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

ones reflect the existence of paths with intermediate vertices not higher than 3.

$$R^{(4)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$r(2,3) = r(2,3)$ or $(0 \& 0)$

$= 0$ or $0 = 0$

Analysis

$r(2,2) = r(2,2)$ or $(r_{ij}$ and $r_{kj})$

$= 0$ or $(1 \& 0)$

$= 0$ or $0 = 0.$

Alg. Warshal [A [1, n, 1,..n]

// Implements Warshall for computing transitive closure

// I/p : Adj matri A of a digraph.

// o/p : Transitive closure of digraph.

$R^{(0)} \leftarrow A$.

for $k \leftarrow 1$ to n do.

for $i \leftarrow 1$ to n do

for $j \leftarrow 1$ to n do

$R^{(k)}[i, j] \leftarrow R^{(k-1)}[i, j]$ or $R^{(k-1)}[i, k]$ and $R^{(k-1)}[k, j]$

return $R^{(n)}$.

Analysis.

B. O of this alg. is computation of $R^{(k)}[i, j]$. It is located in 3 nested loops. $C(n) = \sum_{k=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} 1$

$$= \sum_{k=1}^{n} \sum_{i=1}^{n} (n-1+1)$$

$$= n \sum_{k=1}^{n} \sum_{i=1}^{n} 1 = n \sum_{k=1}^{n} (n-1+1).$$

cost matrixe.