SRM Institute of Science and Technology **Set A**
College of Engineering and Technology
School of Computing
SRM Nagar, Kattankulathur – 603203, Chengalpattu District, Tamilnadu
Academic Year: 2021-22 (Even)

Test: CLA-T2 **Date: 27-05-2022**
Course Code & Title: 18CSC204J Design and Analysis of Algorithms **Duration:** 100 min
Year & Sem: II Year / IV Sem **Max. Marks:** 50

Course Articulation Matrix:

Course Outcome	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	2	3	-	-	-	-	-	-	-	-	-	-
CO2	-	3	2	-	-	-	-	-	-	-	-	-
CO3	-	3	3	-	-	-	-	-	-	-	-	-
CO4	3	2	3	-	-	-	-	-	-	-	-	-
CO5	2	3	-	-	-	-	-	-	-	-	-	-
CO6	-	2	3	-	-	-	-	-	-	-	-	-

Part - A
(10 x 1 = 10 Marks)

Instructions: Answer all

Q. No	Question	Marks	BL	CO	PO	PI Code
1	In divide-and-conquer, to solve a problem recursively by applying three steps at each level of the recursion are ______________ a. Divide, Collide and Conquer b. Divide, Conquer and Combine c. Divide, Collect and Conquer d. Divide, Combination and Conquer	1	1	CO2	PO2	2.1.1
2	Find the Maximum Subarray Sum for the following array elements in array A A= { -15, -3, -1, -2, -4, -8, -9} a. -15 b. -1 c. -42 d. -43	1	3	CO2	PO2	2.4.1
3	The time complexities of binary search is given as a. Best Case : ϴ(n) , Average Case : ϴ(nlogn) and Worst Case: ϴ(n logn) b. Best Case : ϴ(nlogn) , Average Case : ϴ(n) and Worst Case: ϴ(nlogn) c. Best Case : ϴ(1) , Average Case : ϴ(logn) and Worst Case: ϴ(logn)	1	4	CO2	PO3	3.1.1

	d. Best Case : **ϴ**(n) , Average Case : **ϴ**(n) and Worst Case: **ϴ**(nlogn)					
4	Quick Sort is also called as _____________ a. Counting based sort b. Partition-exchange sort c. Comparison-exchange sort d. Grouping based sort	1	4	CO2	PO2	2.1.1
5	A subset S of the plane is called convex if and only if a. For any pair of points p,q in S, the line segments pq is partially contained in S b. For any pair of points p,q in S, the line segments pq is contained outside S c. For any pair of points p,q in S, the line segments pq is not contained in S d. For any pair of points p,q in S, the line segments pq is completely contained in S	1	4	CO2	PO2	2.1.1
6	In greedy method, _______________ requires a minimum or maximum result. a. Average Time Problem b. Optimization Problem c. Performance Problem d. Sorting	1	4	CO3	PO2	3.1.1
7	Which of the following satisfies prefix code property? //1 mark may be awarded if no corrections done in a/b/c/d a. {0,1,10,01} b. {0,01,11,111} c. {01,00,010,000} d. {11,10,110,1111}	1	3	CO3	PO2	2.1.2
8	In 0/1 Knapsack, the items are _________ can be solved by ___________ a. Indivisible & greedy Approach b. Indivisible & Dynamic Approach c. Divisible & greedy Approach d. Divisible & Dynamic Approach	1	4	CO3	PO3	3.1.6
9	Traverse left subtree, Visit the root and Traverse right subtree is ____________ a. Inorder Traversal	1	2	CO3	PO2	2.1.1

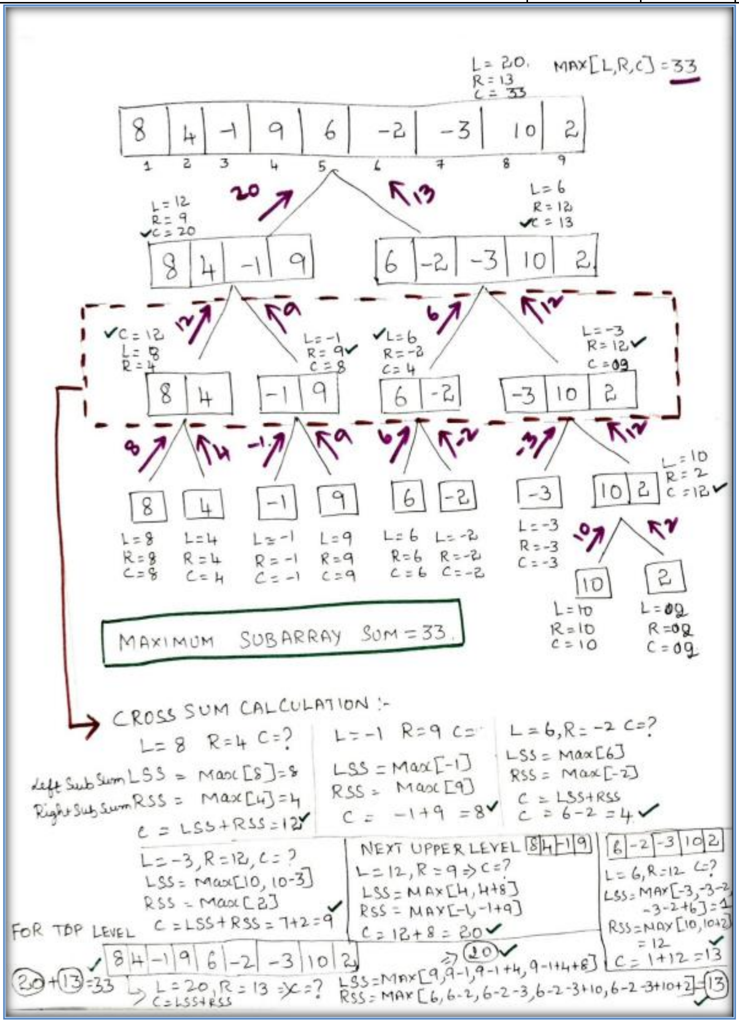
	b. Preorder Traversal c. Postorder Traversal d. Open Traversal					
10	Find the length of the longest common subsequence of the given two strings, S1= Phones & S2=Stone a. 4 b. 3 c. 2 d. 1	1	3	CO3	PO2	2.1.2

Part – B

(4 x 10 Marks = 40 Marks)

Instructions: Answer any 4 Questions

11	**Illustrate the Maximum Subarray Sum problem for the following array elements 8, 4, -1, 9, 6, -2, -3, 10, 2 using Divide and Conquer Method.**	10	4	CO2	PO2	2.4.1



12	**Consider any two square matrices A and B and compute matrix multiplication using Strassen's matrix multiplication method. Compare its time complexity analysis with brute force method.**	10	3	CO2	PO2	2.3.2

Basic Matrix Multiplication

Suppose we want to multiply two matrices of size N x N: for example A x B = C.

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix}$$

$C_{11} = a_{11}b_{11} + a_{12}b_{21}$

$C_{12} = a_{11}b_{12} + a_{12}b_{22}$

$C_{21} = a_{21}b_{11} + a_{22}b_{21}$

$C_{22} = a_{21}b_{12} + a_{22}b_{22}$

2x2 matrix multiplication can be accomplished in 8 multiplication.($2^{\log_2 8} = 2^3$)

Strassens's Matrix Multiplication

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix}$$

$P_1 = (A_{11} + A_{22})(B_{11} + B_{22})$
$P_2 = (A_{21} + A_{22}) * B_{11}$
$P_3 = A_{11} * (B_{12} - B_{22})$
$P_4 = A_{22} * (B_{21} - B_{11})$
$P_5 = (A_{11} + A_{12}) * B_{22}$
$P_6 = (A_{21} - A_{11}) * (B_{11} + B_{12})$
$P_7 = (A_{12} - A_{22}) * (B_{21} + B_{22})$

$C_{11} = P_1 + P_4 - P_5 + P_7$
$C_{12} = P_3 + P_5$
$C_{21} = P_2 + P_4$
$C_{22} = P_1 + P_3 - P_2 + P_6$

Time Analysis

$T(1) = 1$ (assume $N = 2^k$)
$T(N) = 7T(N/2)$
$T(N) = 7^k T(N/2^k) = 7^k$
$T(N) = 7^{\log N} = N^{\log 7} = N^{2.81}$

```
Algorithm Strassen(n, a, b, d)

begin
    If n = threshold then compute
        C = a * b is a conventional matrix.
    Else
        Partition a into four sub matrices a11, a12, a21, a22.
        Partition b into four sub matrices b11, b12, b21, b22.
        Strassen ( n/2, a11 + a22, b11 + b22, d1)
        Strassen ( n/2, a21 + a22, b11, d2)
        Strassen ( n/2, a11, b12 – b22, d3)
        Strassen ( n/2, a22, b21 – b11, d4)
        Strassen ( n/2, a11 + a12, b22, d5)
        Strassen (n/2, a21 – a11, b11 + b22, d6)
        Strassen (n/2, a12 – a22, b21 + b22, d7)

        C = d1+d4-d5+d7    d3+d5
            d2+d4          d1+d3-d2-d6

    end if

      return (C)
end.
```

13	**Demonstrate Multiplication of a sequence of n matrices A1, A2,…,An, Find the optimal parenthesization of the n matrices that have minimal number of multiplication using dynamic programming with an example where n >=3**	10	3	CO3	PO3	3.2.1

Here n=4…but students may have it from 3 also

MATRIX CHAIN MULTIPLICATION

$A_1 \cdot A_2 \cdot A_3 \cdot A_4$
5x4 4x6 6x2 2x7

AIM :- Optimal Paranthesization must be found

formula :-

$$m[i,j] = \begin{cases} 0 & \text{if } i=j \\ \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + P_{i-1}P_kP_j\} \end{cases}$$

m	1	2	3	4
1	0	120	88	158
2		0	48	104
3			0	84
4				0

Multiplication table.

r	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

Root table

$\boxed{\text{Step 1:-}}$ when $i=j$; $m[i,j]=0$

$\therefore$ $m[1,1] \Rightarrow 0$ $m[2,2] \Rightarrow 0$ $m[3,3] \Rightarrow 0$ $m[4,4] \Rightarrow 0$

$\boxed{\text{Step 2:-}}$ Two Paired Matrices

$A_1 \cdot A_2 \cdot A_3 \cdot A_4$

$A_1 \cdot A_2 \Rightarrow m[1,2]$ $A_2 \cdot A_3$ $m[2,3]$ $A_3 \cdot A_4$ $m[3,4]$

$\boxed{\text{Step 2.1}}$:- Calculation for $m[1,2]$

$$\boxed{\text{formula :- } m[i,j] = \min\limits_{i \le k < j} \{m[i,k] + m[k+1,j] + P_{i-1}P_kP_j\}}$$

K lies between i and less than j
so it takes Only one Value here at $m[1,2]$
$\boxed{k=1}$

$m[1,2] = \min\{m[1,1] + m[2,2] + P_0P_1P_2\} \rightarrow$ (A)

K can take 1 but not 2 since it is less than 2. $i \le k < j$

from table $m[1,1]$ & $m[2,2] = 0$. $\rightarrow$ (1)

$$\boxed{\begin{array}{l} P_0P_1P_2 \Rightarrow \begin{array}{cccc} A_1 \cdot & A_2 \cdot & A_3 \cdot & A_4 \\ 5\times4 & 4\times6 & 6\times2 & 2\times7 \\ P_0\ P_1 & P_1\ P_2 & P_2\ P_3 & P_3\ P_4 \end{array} \end{array}}$$

$\therefore$ $P_0P_1P_2 \Rightarrow 5 \times 4 \times 6 = 120 \rightarrow$ (2)

Apply (1) & (2) in (A)

$m[1,2] = \min\{0 + 0 + 120\}$

$\boxed{m[1,2] = 120}$ FILL IN TABLE NOW M & R

120 Obtained when $k=1$ so root will be filled as 1 in $r[1,2]$

Step 2.2 :- m[2,3]

$m[2,3] = \{ \min \{ m[i,k] + m[k+1,j] + P_{i-1} P_k P_j \}$ ($i \le k < j$)

$\boxed{k=2}$ $= \{ \min \{ m[2,2] + m[3,3] + P_1 P_2 P_3 \} \rightarrow (B)$

from table $m[2,2]$ & $m[3,3] = 0 \rightarrow (3)$

$P_1 P_2 P_3 = 4 \times 6 \times 2 = 48 \rightarrow (4)$

Apply 3 & 4 in B.

$= \{0 + 0 + 48\}$

$\boxed{m[2,3] = 48}$ FILL IN M[2,3] & r[2,3]

k = 3 Cannot be done here. k must be less than 3.

Step 2.3 :- m[3,4]

$m[3,4] \Rightarrow \boxed{k=3}$

$\Rightarrow \{ \min \{ m[3,3] + m[4,4] + P_2 P_3 P_4 \} \rightarrow (C)$

$m[3,3]$ & $m[4,4] = 0 \rightarrow (5)$

$P_2 P_3 P_4 \Rightarrow 6 \times 2 \times 7 \Rightarrow 84 \rightarrow (6)$

Apply 5 & 6 in C

$\Rightarrow 0 + 0 + 84 \Rightarrow 84$

$\boxed{m[3,4] = 84}$ FILL IN M[3,4] & r[3,4]

Step 3 :- 3 Paranthesization Matrices

$A_1 \cdot A_2 \cdot A_3 \cdot A_4$

$A_1 \cdot A_2 \cdot A_3$ — m[1,3] — ∴ k values can be k=1, k=2

$A_2 \cdot A_3 \cdot A_4$ — m[2,4] — k values can be k=2, k=3

Step 3.1 :- m[1,3]

$\boxed{\text{When } k=1}$

$m[1,3] \Rightarrow \min \{ m[1,1] + m[2,3] + P_0 P_1 P_3 \} \rightarrow (D)$

$m[1,1] = 0,\ m[2,3] = 48 \rightarrow (7)$

$P_0 P_1 P_3 \Rightarrow 5 \times 4 \times 2 = 40 \rightarrow (8)$

Apply 7 & 8 in (D)

$\Rightarrow 0 + 48 + 40$

$m[1,3] \Rightarrow 88 \rightarrow (9)$ ✓

When k = 2

$m[1,3] = \min \{ m[1,2] + m[3,3] + P_0 P_2 P_3 \}$

$= \{120 + 0 + 5 \times 6 \times 2\} = 180 \rightarrow (10)$

finding m[1,3] minimum b/w when k = 1 & k = 2

ie min among (9) & (10)

$\min \{ 88, 180 \}$

$\boxed{m[1,3] = 88}$ when k = 1

FILL IN M[1,3] & r[1,3]

Step 3.2 :- m[2,3]

When k = 2

$m[2,4] = \min \{ m[2,2] + m[3,4] + P_1 P_2 P_4 \}$

$= \{ 0 + 84 + 4 \times 6 \times 7 \}$

$m[2,4] = 252 \rightarrow (11)$

$\boxed{\text{When } k=3}$

$m[2,4] = \min \{ m[2,3] + m[4,4] + P_1 P_3 P_4 \}$

$= \{ 48 + 0 + 4 \times 2 \times 7 \}$

$m[2,4] = 48 + 56 = 104 \rightarrow (12)$ ✓

finding m[2,4] minimum b/w when k=2 & k=3

min b/w (11) & (12) $\Rightarrow \min \{252, 104\}$

$\boxed{m[2,4] = 104}$ when k = 3.

FILL IN M[2,4] & r[2,4]

Step 4 :- 4 Paranthesization Matrices

$A_1 \cdot A_2 \cdot A_3 \cdot A_4$

$(A_1 \cdot A_2) \cdot (A_3 \cdot A_4)$ $(A_1 \cdot A_2 \cdot A_3) \cdot A_4$ $A_1 \cdot (A_2 \cdot A_3 \cdot A_4)$

Step 4.1 :- m[1,4], k will be k=1, k=2, k=3

When k = 1

$m[1,4] = \min \{ m[1,1] + m[2,4] + P_0 P_1 P_4 \}$

$= 0 + 104 + 5 \times 4 \times 7$

$m[1,4] = 244 \rightarrow (13)$

When k = 2

$m[1,4] = \min \{ m[1,2] + m[3,4] + P_0 P_2 P_4 \}$

$= 120 + 84 + 5 \times 6 \times 7$

$m[1,4] = 414 \rightarrow (14)$

$\boxed{\text{When } k=3}$

$m[1,4] = \min \{ m[1,3] + m[4,4] + P_0 P_3 P_4 \}$

$= 88 + 0 + 5 \times 2 \times 7$

$= 88 + 70$

$m[1,4] = 158 \rightarrow (15)$ ✓

find the minimum of (13), (14) & (15)

$\min \{ 244, 414, 158 \}$

$\boxed{m[1,4] = 158}$ when k = 3.

FILL IN M[1,4] & r[1,4]

Now consider r table & trace backward from bottom to find Optimal Solution

ie OPTIMAL PARANTHESIZATION

r	1	2	3	4
1		1	1	3
2			2	3
3				3
4				

[1,4] = 3

Seperate the pair after A_3.

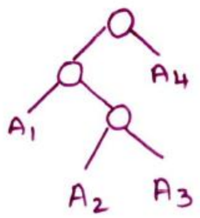
ie $(A_1 \cdot A_2 \cdot A_3) \cdot A_4$

Now consider 1 to 3

r[1,3] = 1

Seperate the pair after A_1

ie $\boxed{(A_1 \cdot (A_2 \cdot A_3)) \cdot A_4}$



Optimal paranthesization of matrices with minimum Number of Multiplication.

14	**Explain in detail about greedy knapsack problem. Find an optimal solution to the knapsack instances where in n and m are the number of items and capacity of the knapsack. n=7, m=15,** **(P1, P2, P3, P4, P5, P6, P7) = (10,5,15,7,6,18,3) and** **(W1, W2, W3, W4, W5, W6, W7) = (2,3,5,7,1,4,1)**	**10**	**4**	**CO3**	**PO2**	**2.4.1**

GREEDY KNAPSACK

14 $n=7$, $m=15$

Obj	1	2	3	4	5	6	7
P	10	5	15	7	6	18	3
ω	2	3	5	7	1	4	1
P/ω	5	1.67	3	1	6	4.5	3

Constraints :- 1. $\sum x_i w_i$ Should not exceed 15 ie $\sum x_i w_i \leq 15$

2. max $\sum x_i P_i$ must be Calculated.

Fully loaded = 1, Not Loaded = 0, Partially loaded means then it x. loaded item's weight

Fully / Partial / Not Loaded	P/ω Frm: MAX wt below	Obj	ω	Remaining Sack wt.
$x_5 = 1$	6	5	1	15 - 1 = 14
$x_1 = 1$	5	1	2	14 - 2 = 12
$x_6 = 1$	4.5	6	4	12 - 4 = 08
$x_3 = 1$	3	3	5	8 - 5 = 3
$x_7 = 1$	3	7	1	3 - 1 = 2
$x_2 = 2/3$	1.67	2	3	Only 2 kg can be loaded 2/ω ie 2/3 will be considered 2 - 2 = 0

Constraint 1 :- from above $x_4 = 0$

$$\sum x_i w_i = x_1 w_1 + x_2 w_2 + x_3 w_3 + x_4 w_4 + x_5 w_5 + x_6 w_6 + x_7 w_7$$
$$= 1 \times 2 + \frac{2}{3} \times 3 + 1 \times 1 + 0 \times 7 + 1 \times 1 + 1 \times 4 + 1 \times 1$$
$$= 2 + 2 + 5 + 1 + 4 + 1 = 15$$

∴ $\sum x_i w_i = 15$ which has not exceeded Sack Capacity 15.

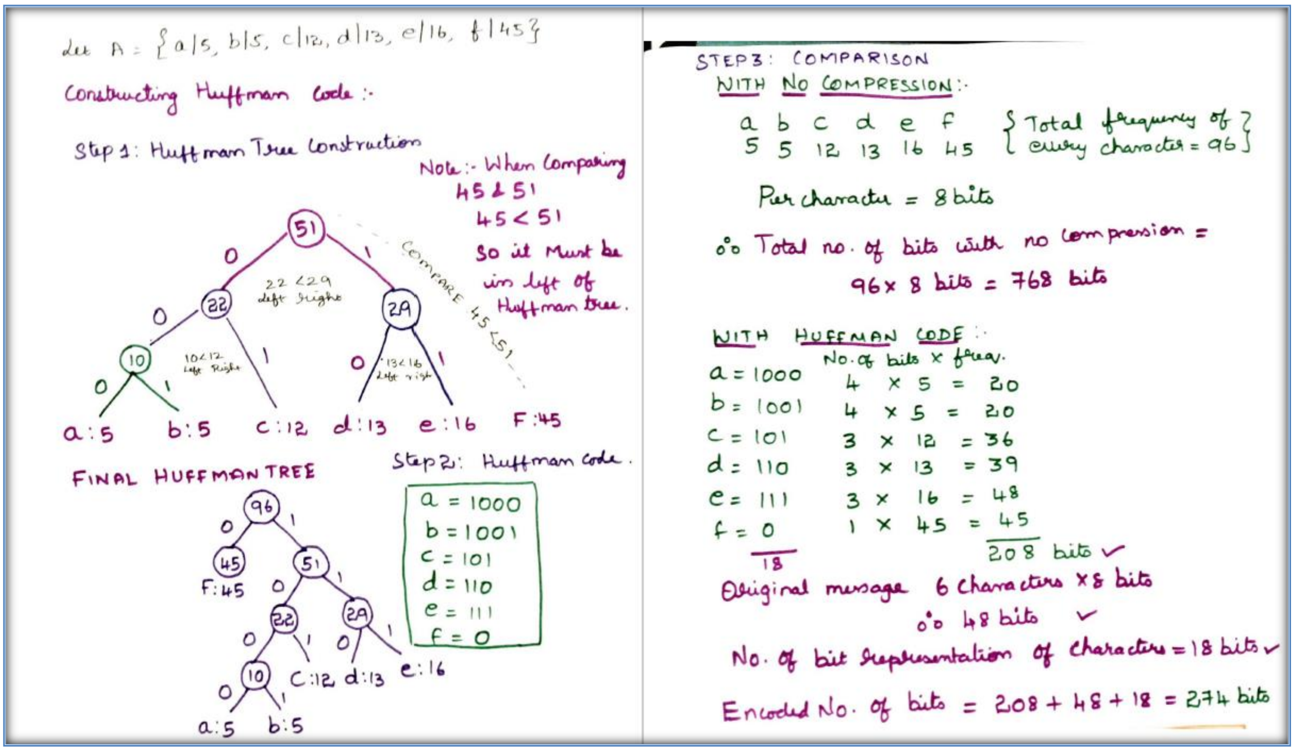
Calculate :-

Constraint 2 :- $x_1 P_1 + x_2 P_2 + x_3 P_3 + x_4 P_4 + x_5 P_5 + x_6 P_6 + x_7 P_7$

$$\sum x_i P_i = 1 \times 10 + \frac{2}{3} \times 5 + 1 \times 15 + 0 \times 7 + 1 \times 6 + 1 \times 18 + 1 \times 3$$
$$= \boxed{10 + 3.33 + 15 + 6 + 18 + 3}$$

$$\sum x_i P_i = 55.33$$

15	**Explain in detail about Huffman code algorithm. Let A= {a/5, b/5, c/12, d/13, e/16, f /45} be the letters and its frequency distribution in a text file. Compute a suitable Huffman coding to compress the data effectively and also compute optimal cost.**	**10**	**4**	**CO3**	**PO2**	**2.2.1**



Let A = {a|5, b|5, c|12, d|13, e|16, f|45}

Constructing Huffman Code :-

Step 1: Huffman Tree Construction

Note :- When Comparing 45 & 51
45 < 51
So it Must be in left of Huffman tree.

FINAL HUFFMAN TREE

Step 2: Huffman code.

a = 1000
b = 1001
c = 101
d = 110
e = 111
f = 0

STEP 3: COMPARISON

WITH NO COMPRESSION :-

a	b	c	d	e	f
5	5	12	13	16	45

{Total frequency of every character = 96}

Per character = 8 bits

∴ Total no. of bits with no compression = 96 × 8 bits = 768 bits

WITH HUFFMAN CODE :-

	No. of bits × freq.
a = 1000	4 × 5 = 20
b = 1001	4 × 5 = 20
c = 101	3 × 12 = 36
d = 110	3 × 13 = 39
e = 111	3 × 16 = 48
f = 0	1 × 45 = 45
18	208 bits ✓

Original message 6 characters × 8 bits
∴ 48 bits ✓

No. of bit representation of characters = 18 bits ✓

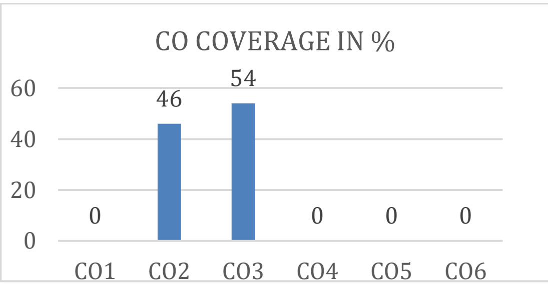
Encoded No. of bits = 208 + 48 + 18 = 274 bits

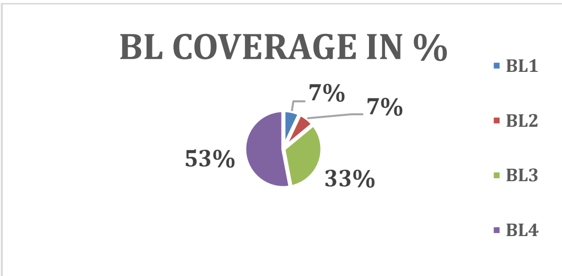
Cost of 768 bits would be greater than the Compressed [Huffman code] 274 bits

∴ Optimal cost will be Obtained by the use of Huffman Code only.

***Program Indicators are available separately for Computer Science and Engineering in AICTE examination reforms policy.**

Course Outcome (CO) and Bloom's level (BL) Coverage in Questions





Approved by the Audit Professor/Course Coordinator