Maths

Unit-2

Permutation :-
↳ arrangement

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Combination :-
↳ Selection

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

Q. In how many ways 6 boys and 4 girls sit in a row?

$\Rightarrow \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$\Rightarrow 10!$

Q. In how many ways 6 boys and 4 girls sit in a row if boys sit together and girls sit together.

$\Rightarrow \underline{6 \text{ boys}} \times \underline{4 \text{ girls}} + \underline{4 \text{ girls}} \times \underline{6 \text{ boys}}$

$\Rightarrow 6! \times 4! + 4! \times 6!$

$\Rightarrow 2 \times 6! \times 4!$

Q. In how many ways can they sit in a row if all girls are to sit together.

$\Rightarrow \underbrace{\frac{4!}{4 \text{ girls}}\ \_\ \_\ \_\ \_\ \_\ \_}_{7!}$

$\Rightarrow 4! \times 7!$

Q. From a club consisting of 6 men and 7 women, in how many ways we can select a committee of

(a) 3 men & 4 women

$\Rightarrow$ 3 men can be selected in ${}^6C_3$ ways

4 women can be selected in ${}^7C_4$ ways

$\Rightarrow$ Total No. of ways = ${}^6C_3 \times {}^7C_4 = 700$

(b) 4 person which has atleast one women.

$\Rightarrow {}^7C_1\,{}^6C_3 + {}^7C_2\,{}^6C_2 + {}^7C_3\,{}^6C_1 + {}^7C_4\,{}^6C_0$

$= 700$ ways.

(c) 4 person that has atmost one man.

$\Rightarrow {}^6C_0\,{}^7C_4 + {}^6C_1\,{}^7C_3 = 245$ ways

Q. In how many ways can 20 student out of class of 30 be selected for an activity if

(a) Rama refuses to be selected.

$\Rightarrow {}^{29}C_{20}$

(b) Raja insist to be selected.

$\Rightarrow {}^{29}C_{19}$

(c) Gopal & Govind insist on being selected.

$\Rightarrow {}^{28}C_{18}$.

Pigeon Hole Principle :-

If n pigeons are accomodated in m pigeonholes and $n > m$ then atleast one of the pigeonhole will must contain atleast $\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1$ pigeons.

where $\lfloor n \rfloor$ denotes greatest integer less than or equal to n.

Q. Show that if 9 colors are used to paint 100 houses then atleast 12 houses will be of same color.

Sol → Here, $n = 100$,
$m = 9$

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1$$

$$= \left\lfloor \frac{99}{9} \right\rfloor + 1 = 11 + 1$$

$$= 12$$

⇒ 12 houses will be of same color.

Q. A man hiked for 10 hr and covered a total distance of 45 km. It is known that he hiked 6 km in first hour and only 3 km in last hour. Show that he must have hiked atleast 9 km within a certain period of 2 consecutive hours.

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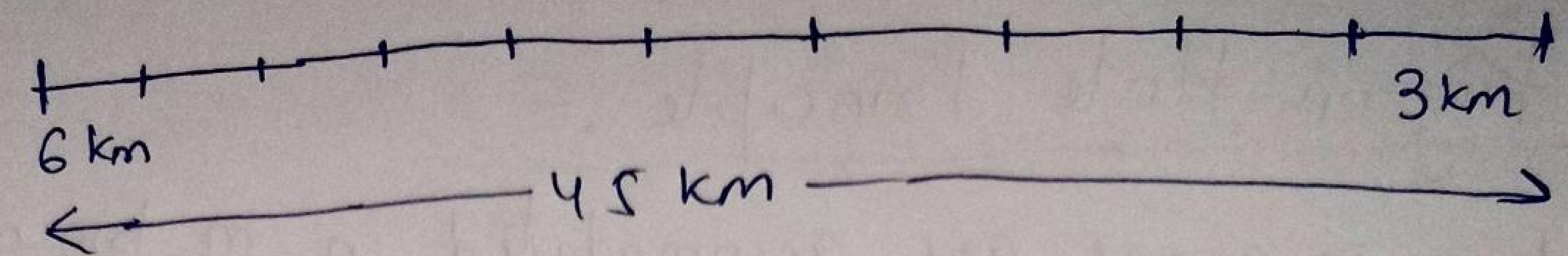
Sol →



Since he hiked 6 km in 1st & 3 km in last hours, he must have hiked $(45-6-3)$ km in 2nd to 9th.
If we consider two consecutive hour block. There will be total of 4 block.

$\Rightarrow m = 4 \qquad n \Rightarrow 36$

$$\left\lfloor \frac{36-1}{4} \right\rfloor + 1$$

$$= \left\lfloor \frac{35}{4} \right\rfloor + 1 = \lfloor 8.75 \rfloor + 1$$

$$= 8 + 1$$

$$= 9$$

So, he must have hiked atleast 9 km in 2 consecutive hours.

Principle of Inclusion & Exclusion :-

If A & B are 2 finite subset of universal set, then
$|A \cup B| = |A| + |B| - |A \cap B|$.

$\boxed{* \ |A| \rightarrow \text{No. of element in } A}$

or $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

$$|A_i \cup A_j \cup A_k \cup \cdots \cup A_n| = |A_i| + |A_j| + \cdots |A_n| - |A_i \cap A_j| + \cdots + (-1)^n |A_i \cap A_j \cap \cdots A_n|$$

$$\Rightarrow |A_i \cup A_j \cdots A_n| = \sum |A_i| - \sum |A_i \cap A_j| + \cdots + \cdots (-1)^{n+1} \sum |A_i \cap A_j \cdots A_n|$$

Q. There are 250 student in an Engineering college of these 188 have taken course in fortain, 100 in C. 35 in Java, 88 in both C and Fortain, 23 in both C and Java, 29 in both Fortain and Java and 19 in all three. How many of them have not taken any course.

Sol → Let F, C and J denotes student who have taken course in Fortain, C & Java respectively.

a/q. $|F| = 188$ $\quad |C| = 100$ $\quad |J| = 35$

$|F \cap C| = 88$ $\quad |F \cap J| = 29$ $\quad |J \cap C| = 23$

$|F \cap C \cap J| = 19$

~~IFA~~ Student not in any course = Total − $|F \cup C \cup J|$

$|F \cup C \cup J| = 188 + 100 + 35 - 88 - 29 - 23 + 19$

$= 202$

$\to 250 - 202 = 48$ Ans

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Q. Find out no. of integers between 1 & 250 both inclusive that are not divisible by any of the integer 2, 3, 5, 7.

Sol → $n(2) = \left\lfloor \frac{250}{2} \right\rfloor = 125$ $\quad n(2 \cap 3) = \left\lfloor \frac{250}{6} \right\rfloor = 41$

$n(3) = \left\lfloor \frac{250}{3} \right\rfloor = 83$ $\quad n(2 \cap 5) = \left\lfloor \frac{250}{10} \right\rfloor = 25$

$n(5) = \left\lfloor \frac{250}{5} \right\rfloor = 50$ $\quad n(2 \cap 7) = \left\lfloor \frac{250}{14} \right\rfloor = 17$

$n(7) = \left\lfloor \frac{250}{7} \right\rfloor = 35$ $\quad n(3 \cap 5) = \left\lfloor \frac{250}{15} \right\rfloor = 16$

$n(3 \cap 7) = \left\lfloor \frac{250}{21} \right\rfloor = 11$ $\quad n(5 \cap 7) = \left\lfloor \frac{250}{35} \right\rfloor = 7$

$n(2 \cap 3 \cap 5) = \left\lfloor \frac{250}{30} \right\rfloor = 8$

$n(2 \cap 3 \cap 7) = \left\lfloor \frac{250}{42} \right\rfloor = 5$

$n(3 \cap 5 \cap 7) = \left\lfloor \frac{250}{105} \right\rfloor = 2$

$n(2 \cap 5 \cap 7) = \left\lfloor \frac{250}{70} \right\rfloor = 3$

$n(2 \cap 5 \cap 3 \cap 7) = \left\lfloor \frac{250}{210} \right\rfloor = 1$

$n(2 \cup 3 \cup 5 \cup 7) = 125 + 83 + 50 + 35 - 41 - 25 - 17$
$- 16 - 7 - 11 + 8 + 5 + 2 + 3 - 1$
$= 193$

Not divisible by $2 \cup 3 \cup 5 \cup 7 \Rightarrow 250 - 193$
$= 57$ Ans

Divisibility :-

a/b or $b|a$ → both have same meaning.

a/b → a is divisible by b

$b|a$ → b divides a

$b|a$

b → divisor, a → dividend

$$\boxed{\begin{gathered} b = mq + r \\ \text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder} \end{gathered}}$$

Note :-

1) If $a|b$ & $a|c$ then $a|(b+c)$.
2) If $a|b$ and $b|c$ then $a|c$.
3) If $a|b$ then $a|mb$ where $m \to$ +ve integer.
4) If $a|b$ and $a|c$ then $a|(mb+nc)$ for any integer m & n

this comes by combining (i) & (iii).

Prime Number :- A positive integer $p>1$ is called prime if the only positive factor of p are 1 and p.

If p is not prime ⇒ p is composite.

1) 1 is neither prime nor composite
2) n is composite if there exist positive integer a and b such that $n = ab$ where $1<a, b<n$.

Fundamental Theorem of Arithmetic :-

→ Every integer $n>1$ can be written as a product of prime numbers.

E.g. $12 = 2 \times 2 \times 3$ $\quad 14 = 2 \times 7$

$= 2^2 \times 3^1$ $\quad = 2^1 \times 7^1$

→ If there are k_i prime factor of n equal equal to p_i where $1 \le i \le r$ then n can be written as

$$n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdots p_r^{k_r}$$

→ Expression for integer $n \geq 1$ as product of prime is called as prime factorization or prime decomposition.

Q Find prime factorization of (i) 6647 (ii) 10!

(i) $6647 = 23 \times 7^2$ Ans

(ii) $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 2 \times 5 \times 3 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 2$

$= 2^8 \times 3^4 \times 5^2 \times 7^1$ Ans

GCD :- (Greatest Common Divisor)

↳ Also known as HCF (Highest Common Factor)

If a and b are non-zero integers then the integer $d (\neq 0)$ is said to be common divisor if $d|a$ and $d|b$.
If d is largest of all common divisor, then d is called gcd of a and b denoted by $\gcd(a, b)$.
If $\gcd(a, b) = 1$, then a and b are co-prime or relatively prime.

→ $\gcd(a, b) = p_1^{\min(a,b)} * p_2^{\min(a,b)} \times \cdots p_n^{\min(a_n, b_n)}$

Q. Find GCD of 8571, 1231.

Sol → $8571 = 3 \,\underline{|\,8571}$, $2857\,\underline{|\,2857}$, 1 ; $1231 = 1231\,\underline{|\,1231}$, 1

$= 1231 \times \cancel{0}1$

$= 3 \times 2857 \times \cancel{0}1$

$GCD(8571, 1231) = 3^0 \times 2857^0 \times 1231^0$

$= 1$ Ans

Euclid's Algorithm for finding gcd(a, b) :-

When a and b are two integer $a > b$. If r_1 is remainder when a is divided by b, r_2 is remainder when b is divided by r_1, r_3 is remainder when r_1/r_2 and so on and if $r_{k+1} = 0$, then the last non-zero remainder r_k is gcd(a, b).

Q Find gcd(1575, 231).

Sol → $\underset{(a)}{1575} = \underset{(b)}{231} \times 6 + 189 \; (r_1)$

$\underset{(b)}{231} = \underset{(r_1)}{189} \times 1 + 42 \; (r_2)$

$189 = 42 \times 4 + 21 \; (r_3)$

$42 = 21 \times 2 + 0 \; (r_4)$

r_4 have remainder $= 0$

$\Rightarrow gcd(1575, 231) = r_3 = 21$ Ans

Q find gcd(3587, 1819).

Sol → $3587 = 1819 \times 1 + 1768$

$1819 = 1768 \times 1 + 51$

$1768 = 51 \times 34 + 34$

$51 = 34 \times 1 + 17$

$34 = 17 \times 2 + 0$

⇒ 17 is gcd(3587, 1819).

Theorem :- gcd(a, b) can be expressed as an integral linear combination of a and b.

i.e. $\gcd(a,b) = ma + nb$ where m & n → integers.

Eg. $\gcd(4,14) = 2$

$= (4)(4) + (-1)(14)$

$= m\,a + n\,b$

$m = 4,\ n = -1$.

Q. Express gcd(1575, 231) as a linear combination of 1575 and 231.

Sol → First of all we have to find gcd by Euclid method.

$\gcd = 21$

$= 189 - 42 \times 4$

$= 189 - 4 \times (231 - 189 \times 1)$

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$= 189 - 4\times 231 + 4\times 189$

$= 5\times 189 - 4\times 231$

$= 5\times(1575 - 231\times 6) - 4\times(231)$

$= 5\times(1575) - 30\times 231 - 4\times 231$

$= 5\times(1575) + (-34)\times(231)$

$\Rightarrow m = 5,\ n = \underline{\underline{-34}}$.

Q Express gcd(3587, 1819) as linear combination of 3587, 1819.

Sol → $\gcd(3587, 1819) = 17$

$= 51 - 34\times 1$

$= 51 - (1768 - 51\times 34)$.

$= (1819 - 1768\times 1) + 34(1819 - 1768\times 1) - 1768$

$= 35(1819) - 36(1768)$

$= 35(1819) - 36(3587 - 1819\times 1)$

$= (35+36)(1819) - 36\times(3587)$

$= (-36)\times(3587) + 71\times(1819)$

$m = -36,\ n = 71$ Ans

Some properties of gcd :-

1) If $c|ab$ and a and c are coprime then $c|b$.

2) If a and b are coprime and a and c are coprime then a and bc are coprime.

3) $gcd(ka, kb) = k \cdot gcd(a, b)$.

4) If $gcd(a, b) = d$ then $gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$

5) If $gcd(a, b) = 1$, then for any integer c,
$gcd(ac, b) = gcd(c, b)$.

6) If each of $a_1, a_2, a_3 \cdots a_n$ is coprime to b, then product of $a_1 a_2 a_3 \cdots a_n$ is also coprime to b.

LCM (Lowest Common Multiple) :-

→ $LCM = p_1^{max(a_1, b_1)} \cdot p_2^{max(a_2, b_2)} \cdots p_n^{max(a_n, b_n)}$.

* $\boxed{gcd(a, b) \times LCM(a, b) = a \cdot b}$