Maths

Unit - 1

Set Theory

* Set → A set is a well defined collection of object.

* Representation of Set :-
  (a) Roster form :- eg. Set of binary digit = {0, 1}
  (b) Set Builder form :- eg. A = {x : x is a natural no. less than 9}

A = {x : x is +ve natural no. not exceeding 9}

Set Builder form.

Roster form → A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Builder form is Converted into this Roster form.

Types of Sets :-

(1) Universal Sets :- A set which contain all the objects is called Universal Set.

(2) NULL Set or Empty Set :- A set which does not Contain any element is called NULL set.

$$\text{e.g. } x = \{x : x^2 + 1 = 0, x \text{ is real}\}$$

$$= \phi.$$

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(3) Singleton Set :- A set containing one element is called singleton set.

eg. $A = \{0\}$,
$A = \{\phi\}$, etc

(4) Finite & Infinite Set :- A set which contain finite no. of elements is called finite set and a set which contain infinite no. of elements is called infinite set.

eg. $A = \{1, 3, 5\}$
$A = \{n : n^2 < 100, n \in Z^+\}$
$\rightarrow A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

e.g. $A = \{n : n$ is an even +ve integer$\}$
$\rightarrow \{2, 4, 6, 8, 10, 12 - - - - - - \}$

(5) Equal Set :- Two set are said to be equal when elements of one set is equal to the elements of another set.

e.g. $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$
$A = B$

* Not :- Two set is not equal by having equal size. e.g. $A = \{4, 5, 6\} \rightarrow$ Size $= 3$
$B = \{1, 2, 3\} \rightarrow$ Size $= 3$.

$A \neq B$

* Subset :- A set is said to be a subset of another set if and only if every element of that set is also available in that another set.

e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{1, 2, 3\}$

$\Rightarrow B \subset A$

(a) Proper Subset :- $B \subset A$

$\Rightarrow$ B is a subset of A but $A \neq B$.

(b) Improper Subset :- $B \subseteq A$.

Note :-

1. A is not subset of B is represented by $A \not\subseteq B$
2. Every subset A is a subset of itself $A \subseteq A$
3. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
4. If $A \subseteq B$, then A is called subset of B and B is called superset of A.

* Power Set :- The set of all subset of set S is called Power Set of S.

$S = \{a, b, c\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

MCQ

* Total no. of Power Set Element $= 2^n$ ← no. of element in Set S

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* Cartesian Product :-

$A = \{1, 2\}$, $B = \{3, 4\}$.

$A \times B = \{\{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}\}$

$B \times A = \{\{3,1\}, \{3,2\}, \{4,1\}, \{4,2\}\}$

Not same because order matter.

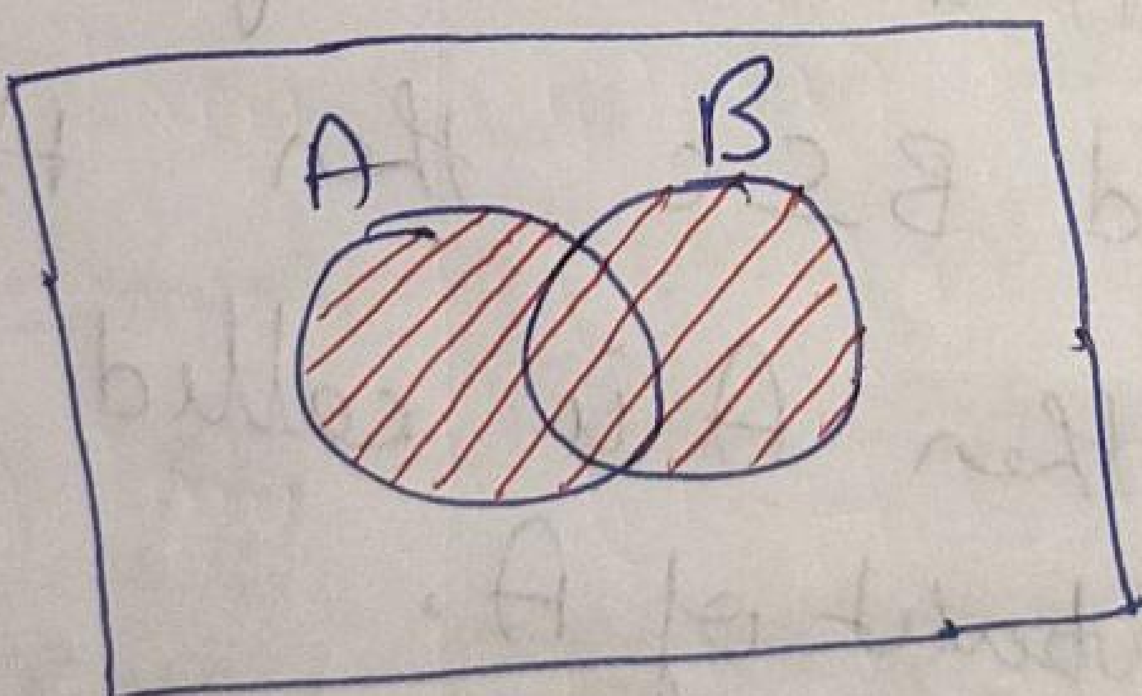
* $(a, b) \neq (b, a)$ unless $a = b$.

* Operations On Set :-

(I) Union :- Union of two set A and B denoted by $A \cup B$ is set of element belonging to A or to B or to both.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

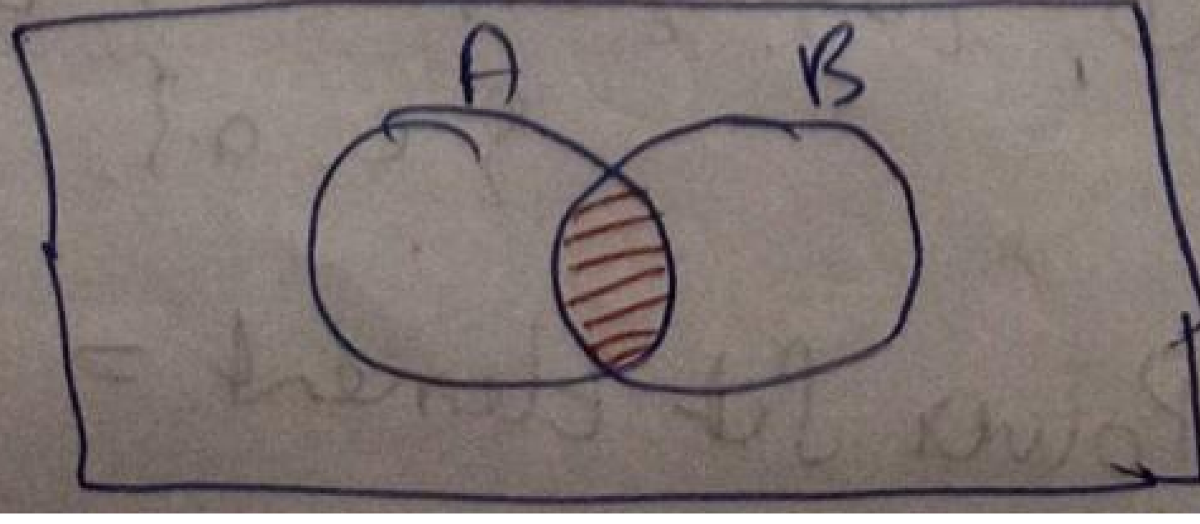
Eg. $A = \{1, 2, 3\}$
$B = \{4, 2\}$
$A \cup B = \{1, 2, 3, 4\}$



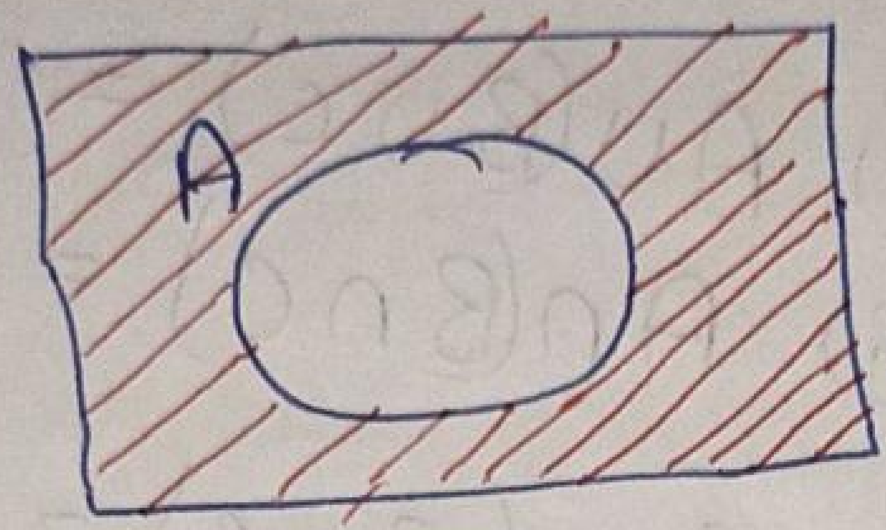
(II) Intersection :- Intersection of two set A and B, denoted by $A \cap B$, is the set of elements that belong to both A and B.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Eg. $A = \{1, 2, 3, 4\}$
$B = \{2, 3, 5, 6\}$
$A \cap B = \{2, 3\}$



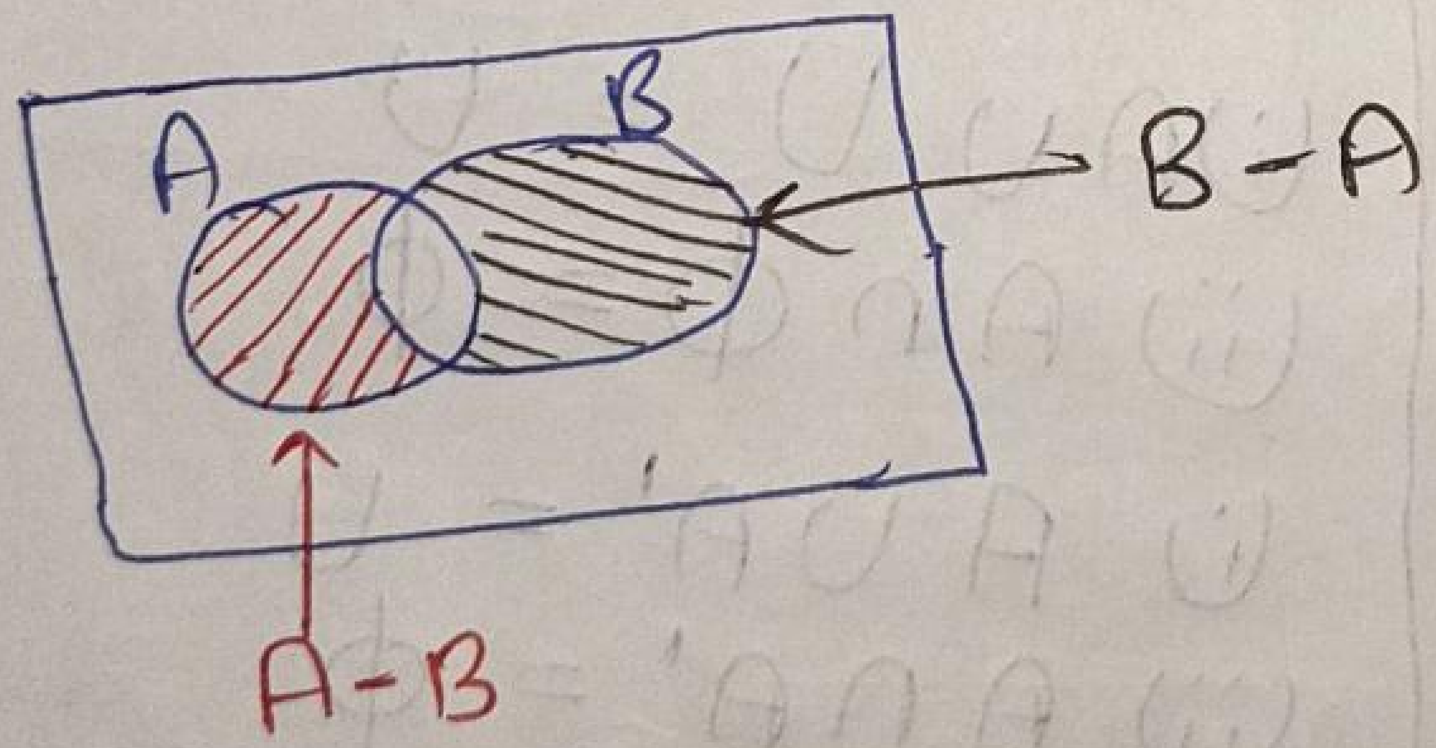
(III) Complement :- If U is Universal set and A is any set, then, Complement of A is the set of element that does not belong to A but belong to U. Denoted by (A', A^- or $\bar{A}$)



$\bar{A} = \{x : x \in U \text{ and } x \notin A\}$

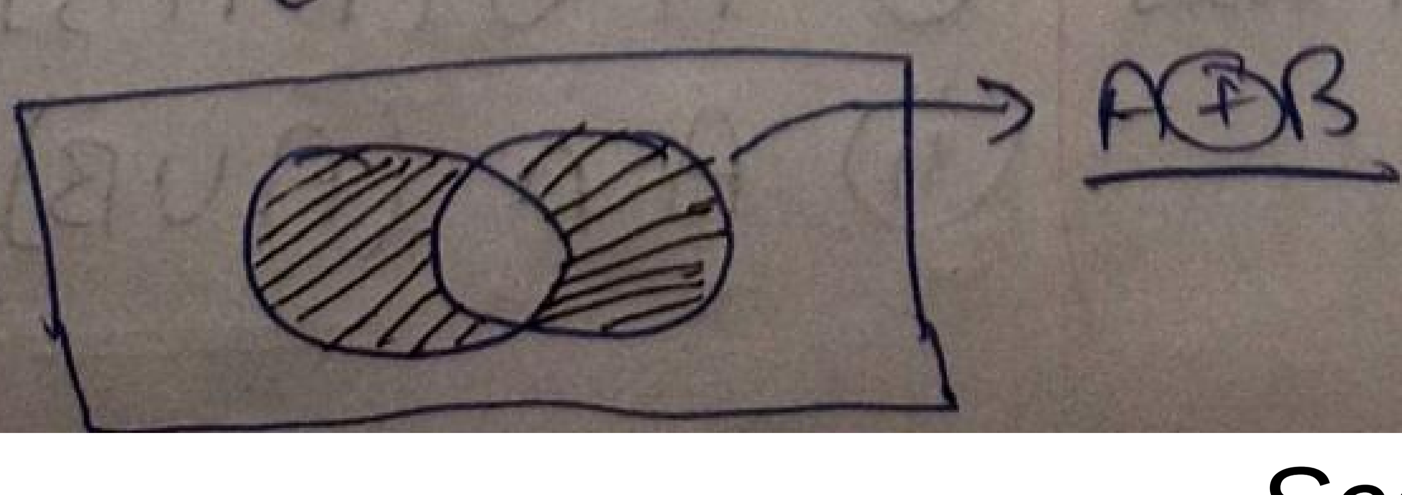
(IV) Difference :- If A and B are any two sets then the set of elements that belong to A but do not belong to B is called difference of A and B

$A - B = \{x : x \in A \text{ and } x \notin B\}$



(V) Symmetric Difference :- If A and B are any two set, the set of element that belong to A or B but not to both is called the symmetric difference of A and B.

$A \oplus B = (A-B) \cup (B-A)$



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Algebric Law of Set Theory :-

Name of Law	
1. Commutative	(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
2. Associative	(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$
3. Distributive	(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Identity Law	(i) $A \cup \phi = A$ (ii) $A \cap U = A$
5. Idempotent Law	(i) $A \cup A = A$ (ii) $A \cap A = A$
6. Dominant Law	(i) $A \cup U = U$ (ii) $A \cap \phi = \phi$
7. Complement Law	(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$
8. Double Complement Law	$\overline{\overline{A}} = A$
9. De Morgan's Law	(i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (ii) $\overline{A \cap B} = \bar{A} \cup \bar{B}$
10. Absorption Law	(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Q. Prove that $(A-C)\cap(C-B)=\phi$ analytically. where A, B & C are set. Verify graphically.

Sol $\Rightarrow$ $(A-C)\cap(C-B) = \{x : x\in(A-C)\cap(C-B)\}$

$= \{x : x\in(A-C) \text{ and } x\in(C-B)\}$

$= \{x : (x\in A \text{ and } x\notin C) \text{ and } (x\in C \text{ and } x\notin B)\}$

$= \{x : x\in A \text{ and } (x\notin C \text{ and } x\in C) \text{ and } x\notin B\}$

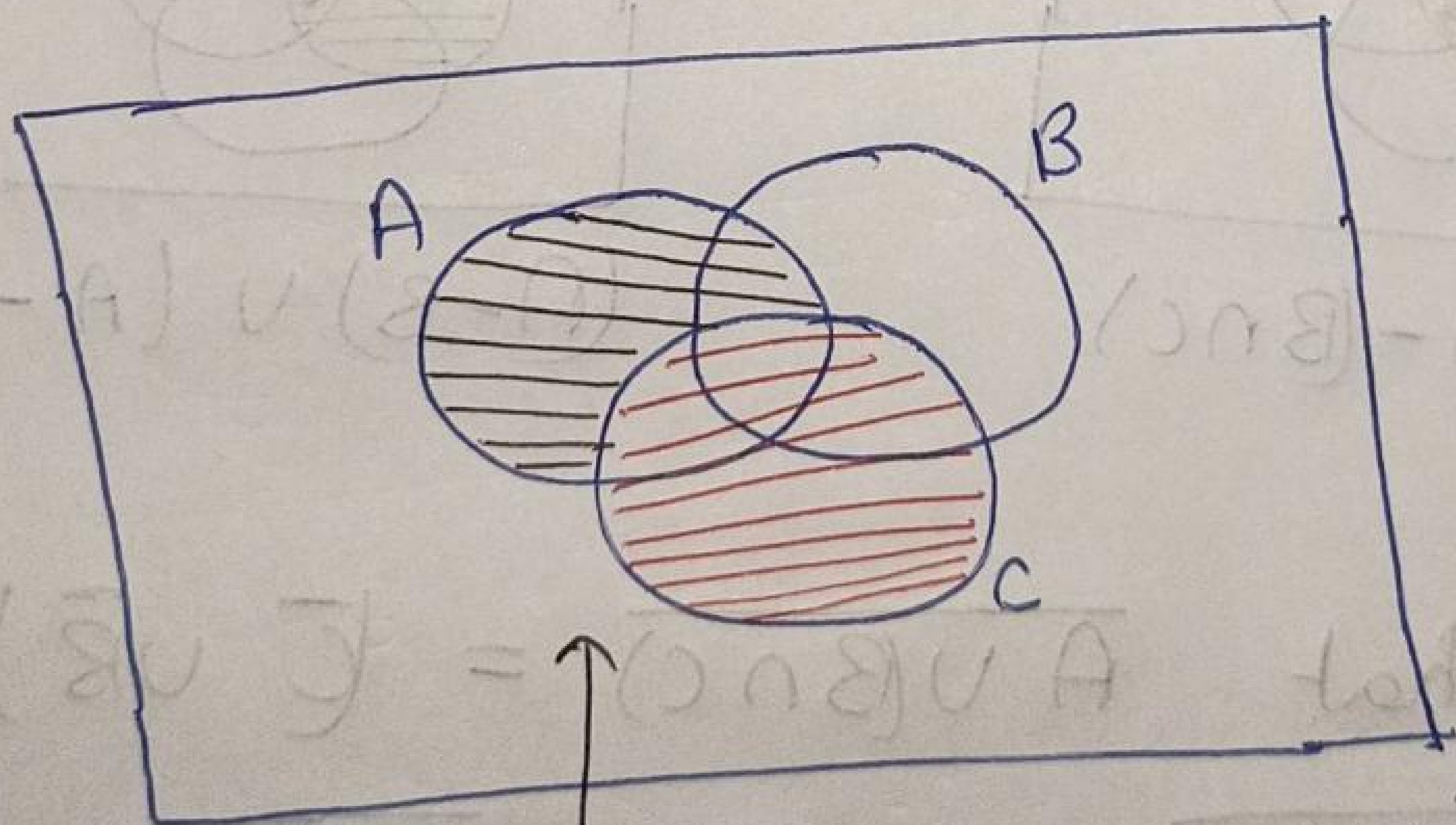
$= \{x : (x\in A \text{ and } x\in\phi) \text{ and } x\notin B\}$

$= \{x : (x\in\phi \text{ and } x\notin B)\}$

$= \{x : x\in\phi\}$

$= \phi$.

LHS = RHS



No common area $\Rightarrow \phi$.

2) Prove $A-(B\cap C) = (A-B)\cup(A-C)$ analytically and then verify graphically. where A, B, C are set

Sol $\Rightarrow$ $A-(B\cap C)$

$= \{x : x\in A-(B\cap C)\}$

~~$= x \notin$~~

$= \{x : x \in A \text{ and } x \notin (B \cap C)\}$

$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C)$

$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C)\}$

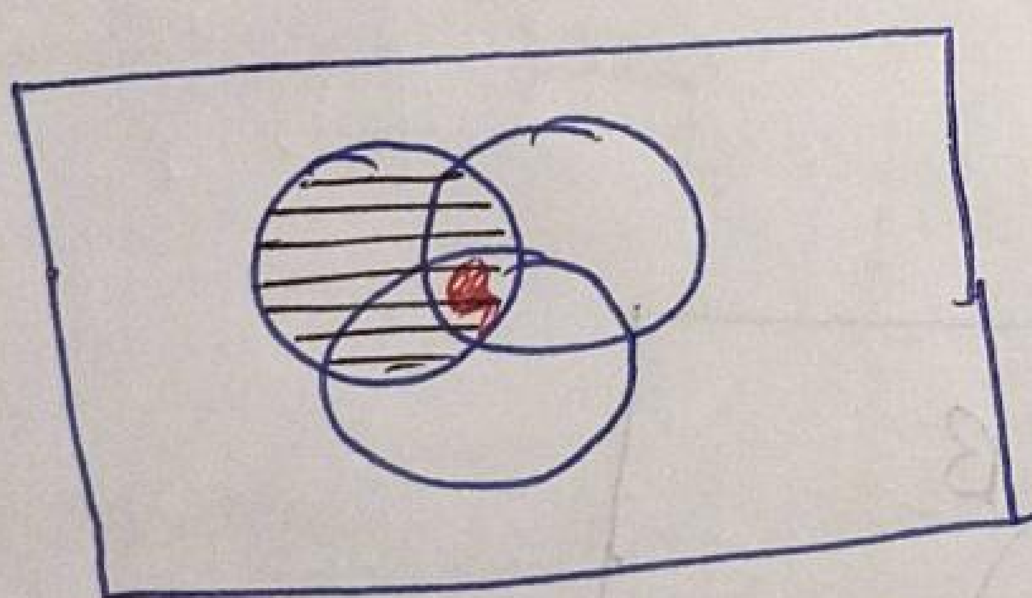
$= \{x : (x \in A \text{ and } x \notin B\} \text{ or } (x \in A \text{ and } x \notin C)\}$

$= \{x : x \in (A-B) \text{ or } x \in (A-C)\}$

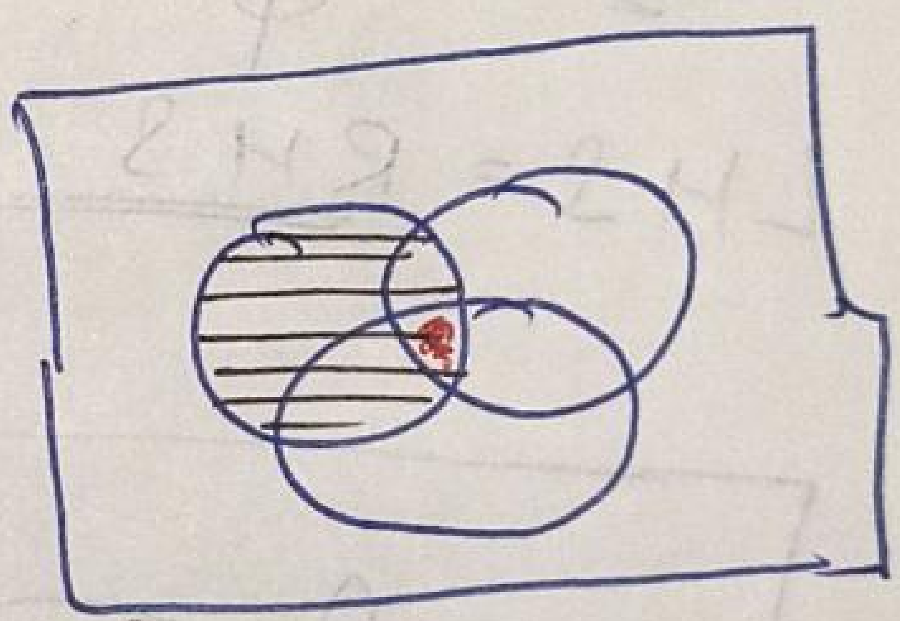
$= \{x : x \in (A-B) \cup (A-C)\}$

$= (A-B) \cup (A-C)$

LHS = RHS



$A-(B\cap C)$



$(A-B) \cup (A-C)$

3) Prove that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

Sol → $\overline{A \cup (B \cap C)} = \bar{A} \cap \overline{(B \cap C)}$

$= \bar{A} \cap (\bar{B} \cup \bar{C})$

$= \bar{A} \cap (\bar{C} \cup \bar{B})$

$= (\bar{C} \cup \bar{B}) \cap \bar{A}$

LHS = RHS

4) Prove $A \cap (B - C) = (A \cap B) - (A \cap C)$ analytically & graphically.

Sol → $A \cap (B - C)$

$= \{x : x \in A \text{ and } x \in (B - C)\}$

$= \{x : x \in A \text{ and } x \in B \text{ and } x \notin C\}$

$= \{x : x \in A \cap B \cap \bar{C}\}$

$= A \cap B \cap \bar{C}$

$(A \cap B) - (A \cap C)$

$= \{x : x \in (A \cap B) \text{ and } x \notin (A \cap C)\}$

$= \{x : x \in A \text{ and } x \in B \text{ and } x \in (\bar{A} \cup \bar{C})\}$

$= \{x : x \in (A \cap B) \text{ and } (x \in \bar{A} \text{ or } x \in \bar{C})\}$

$= \{x : x \in (A \cap B) \text{ and } x \in \bar{A} \text{ or } x \in (A \cap B) \text{ and } x \in \bar{C}\}$

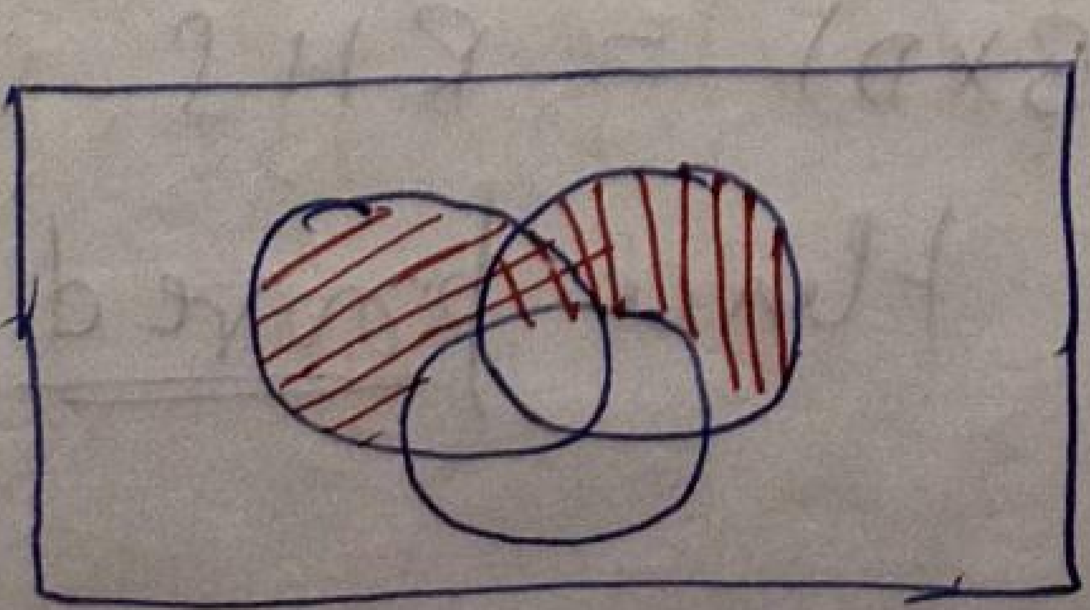
$= \{x : (x \in (A \text{ and } x \in \bar{A}) \text{ and } x \in B) \text{ or } (x \in A \cap B \cap \bar{C})\}$

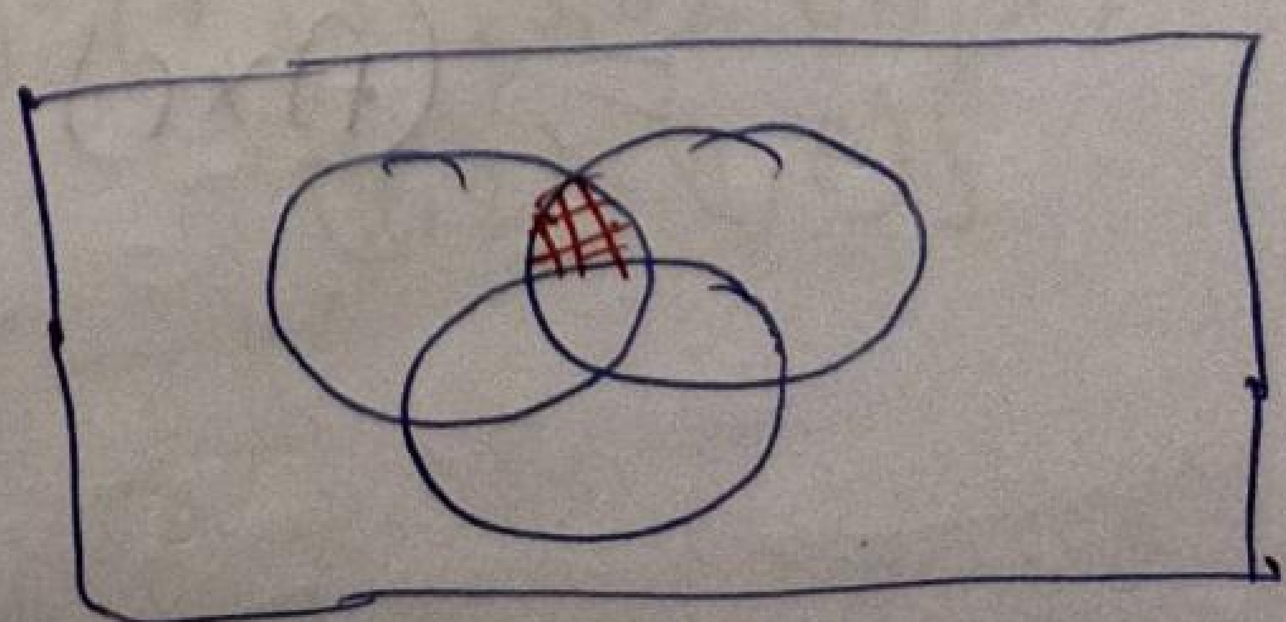
$= \{x : (\phi \text{ and } x \in B) \text{ or } x \in (A \cap B \cap \bar{C})\}$

$= \{x : \phi \text{ or } x \in (A \cap B \cap \bar{C})\}$

$= A \cap B \cap \bar{C} = LHS.$

Hence proved





5) Prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$ analytically.

Sol → $A \times (B \cap C)$

$= \{(n, y) : n \in A \text{ and } y \in (B \cap C)\}$

$= \{(n, y) : n \in A \text{ and } (y \in B \text{ and } y \in C)\}$

$= \{(n, y) : (n \in A \text{ and } y \in B) \text{ and } (n \in A \text{ and } y \in C)\}$

$= \{(n, y) : A \times B \cap A \times C\}$

$= A \times B \cap A \times C = RHS.$

Hence proved.

6) If A, B, C, D are set. Proove $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

Sol → $(A \cap B) \times (C \cap D)$

$= \{(n, y) : n \in A \cap B \text{ and } y \in (C \cap D)\}$

$= \{(n, y) : n \in A \text{ and } n \in B \text{ and } y \in C \text{ and } y \in D\}$

$= \{(n, y) : (n \in A \text{ and } y \in C) \text{ and } (n \in B \text{ and } y \in D)\}$

$= \{n, y : (n, y) \in (A \times C) \text{ and } (n, y) \in (B \cap D)\}$

$= \{n, y : (n, y) \in (A \times C) \cap (B \times D)\}$

$= (A \times C) \cap (B \times D) = RHS$

Hence proved

Relation

$A = \{1, 2\}$
$B = \{3, 4\}$
$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$
$R \subseteq A \times B$

Let A and B are two non-empty sets, then a binary relation from A to B is a subset of $A \times B$ i.e. $R \subseteq A \times B$.

$a \in A$, $b \in B$

$a R b \rightarrow$ a is related to b.

$a \not R b \rightarrow$ a is not related to b.

1) $A = \{1, 2, 3, 4\}$, $B = \{1, 7, 8\}$. R is defined such that "Less than".

Sol → Long Method :-

$A \times B = \{(1,1), (1,7), (1,8), (2,1), (2,7), (2,8), (3,1), (3,7), (3,8), (4,1), (4,7), (4,8)\}$

$aRb = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$

Short Method :-

$R = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$

2. $A = \{0, 1, 2, 3, 4\}$, $B = \{0, 1, 2, 3\}$

$a R b$ iff $a + b = 4$.

Sol → $a R b = \{(1,3), (2,2), (3,1), (4,0)\}$

Domain → Range

Domain : $\{1, 2, 3, 4\}$

Range : $\{3, 2, 1, 0\}$.

Types of Relation :-

1) Universal Relation :- Any relation in which all the relation is included. $A \times B$ is having all the relation.

$\boxed{\text{or}}$

A relation R on set A is called universal relation if $R = A \times A$.

2) Void Relation :- A relation R on set A is called void Relation if R is a null set

Eg. $A = \{3, 4, 5\}$ & $a R b$ iff $a + b > 10$

$R = \phi$.

3) Identity Relation :- A relation R on set A is called Identity Relation iff $a R a$ $a \in R$.

or $R = \{(a, a) ; a \in A\}$.

Eg. $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$

4 Inverse Relation :- Let R be any relation from A to B then the inverse is denoted by R^{-1} is relation from B to A.

i.e. iff $a R b$ then $b R^{-1} a$.

or

$$R = \{(a,b) : a \in A, b \in B\}$$

$$R^{-1} = \{(b,a) : a \in A, b \in B\}$$

Eg. $A = \{2,3,5\}$, $B = \{6,8,10\}$ &

$a R b$ iff a divides b.

$R = \{(2,6), (2,8), (2,10), (3,6), (5,10)\}$

$R^{-1} = \{(6,2), (8,2), (10,2), (6,3), (10,5)\}$

Q. $A = \{0,1,2,3,4\}$, $B = \{0,1,2,3\}$ find R iff $\mathrm{Lcm}(a,b) = 2$.

Sol → $a R b = \{(1,2), (2,1), (2,2)\}$

Q The relation R on set $A = \{1,2,3,4,5\}$ is defined by $((a,b) \in R)$ iff 3 divides $(a-b)$.

(i) Find R & R^{-1}

(ii) Domain and Range of R & R^{-1}

(iii) List the element of the Complement of R.

Sol → (i) $R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (5,2)\}$

$R^{-1} = \{(1,1), (4,1), (2,2), (5,2), (3,3), (1,4), (2,5)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 5\}$ Ans
Range of $R = \{1, 2, 3, 4, 5\}$
Domain of $R^{-1} = \{1, 2, 3, 4, 5\}$ Ans
Range of $R^{-1} = \{1, 2, 3, 4, 5\}$

(iii) Complement of $R = \{(1,2), (1,3), (1,5), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (3,5), (4,2), (4,3), (4,5), (5,1), (5,3), (5,4)\}$. Ans

Q If $R = \{(1,2), (2,4), (3,3)\}$ $S = \{(1,3), (2,4), (4,2)\}$
Find (i) $R \cup S$ (ii) $R \cap S$ (iii) $R - S$ (iv) $S - R$
(v) $R \oplus S$. Also verify $\text{dom}(R \cup S) = \text{dom}(R) \cup \text{dom}(S)$ and $\text{range}(R \cap S) \subseteq \text{range}(R) \cap \text{range}(S)$

Sol → (i) $R \cup S = \{(1,2), (2,4), (3,3), (1,3), (4,2)\}$
(ii) $R \cap S = \{(2,4)\}$
(iii) $R - S = \{(1,2), (3,3)\}$
(iv) $S - R = \{(1,3), (4,2)\}$
(v) $R \oplus S = \{(1,2), (3,3), (1,3), (4,2)\}$
$R \oplus S = (R - S) \cup (S - R)$.

$\text{dom}(R \cup S) = \{1, 2, 3, 4\}$
$\text{dom}(R) = \{1, 2, 3\}$ $\text{dom}(S) = \{1, 2, 4\}$
$\text{domain}(R) \cup \text{domain}(S) = \{1, 2, 3, 4\}$

range of $R \cap S = \{4\}$.
Range of $R = \{2, 4, 3\}$
Range of $S = \{3, 4, 2\}$
Range of $(R) \cap$ Range of $(S) = \{2, 4, 3\}$
$\Rightarrow$ range of $(R \cap S) \subseteq$ Range of $R \cap$ Range of S.

Properties of Relation :-

① Reflexive Property :- A relation R on set A is said to be reflexive if $a^R a$ $\forall a \in A$.

e.g. If R is the relation on $A = \{1, 2, 3\}$.
$a^R b$ iff $a \leq b$, $a, b \in A$.
$R = \{\underline{(1,1)}, (1,2), (1,3), \underline{(2,2)}, (2,3), \underline{(3,3)}\}$
R is reflexive

Note $\rightarrow$ Reflexive relation can be identity but identity can't be reflexive.

② Symmetric :- A relation R on set A is said to be symmetric if $a^R b$ then $b^R a$.
Eg. $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$ if $a + b = 3$.
$R = \{(1,2), (2,1)\}$

③ Antisymmetric :- A relation R on set A is said to be antisymmetric whenever (a, b) & (b, a) $\in R$ then $a = b$.
i.e. $a^R b$ & $b^R a \Rightarrow a = b$.

Identity is only example

(4) Transitive :- A relation R on set A is said to be transitive if
$a^R b$ & $b^R c$ then $a^R c$.

Equivalence Relation :- A relation R on set A is an equivalence relation if R is reflexive, symmetric and transitive.

Partial-order Relation :- A relation R on set A is an Partial order relation if R is reflexive, anti symmetric and transitive.

Q Determine whether the relation R on the set of all integer, is reflexive, symmetric, antisymmetric and transitive. where $a^R b$ iff
(i) $a \neq b$ (ii) $ab \geq 0$.

Sol → (i) $a^R b$ iff $a \neq b$.

Reflexive :- $a^R a$
$\Rightarrow a \neq a$
which is not possible
$\Rightarrow$ It is not reflexive

Symmetric : $a^R b$.
$a \neq b$.
$b^R a$
$b \neq a$
which is true
$\Rightarrow$ It is symmetric

Anti-symmetric :- $a^R b$ & $b^R a$
$a \neq b$ & $b \neq a$

$\Rightarrow R$ is not antisymmetric

Transitive :- $a^R b$ & $b^R c$
$a \neq b$ & $b \neq c$.

which is not necessarily true.

$\Rightarrow R$ is not transitive.

$\Rightarrow$ Symmetric only.

(ii) $ab \geq 0$.

Reflexive : $a^R a$

$\Rightarrow a \cdot a \geq 0$
$a^2 \geq 0$

$\Rightarrow$ True
$\Rightarrow$ Reflexive

Symmetric $\Rightarrow a^R b$
$ab \geq 0$
$b^R a$
$ba \geq 0$

$\Rightarrow$ true
$\Rightarrow$ symmetric

Antisymmetric :- $a^R b$ & $b^R a$
$ab \geq 0$ & $ba \geq 0$

$a = b$ is not necessarily true

So, not a antisymmetric relation

Transitive :- $a^R b$ & $b^R c$
$ab \geq 0$ & $bc \geq 0$
$\Rightarrow$ Not transitive

$\Rightarrow$ Reflexive & Symmetric

Q If R is the relation on the set of ordered pairs of +ve integer such that (a, b), (c, d) ∈ R whenever $ad = bc$. Prove that R is an equivalence Relation.

Sol → $(a, b)^R (c, d)$ iff $ad = bc$

Reflexive :- $a^R a$
$(a, b)^R (a, b)$.
$\Rightarrow ab = ab$
which is true
$\Rightarrow$ Reflexive

Symmetric :- $(a, b)^R (c, d)$
$\Rightarrow ad = bc$
$\Rightarrow bc = ad$
$\Rightarrow cb = da$
$\Rightarrow (c, d)^R (a, b)$

Hence Symmetric

Transitive :- $(a,b)^R (c,d)$ & $(c,d)^R (e,f)$

$ad = bc$ & $cf = de$

$\frac{d}{c} = \frac{b}{a}$ & $\frac{d}{c} = \frac{f}{e}$.

$\Rightarrow \frac{b}{a} = \frac{f}{e}$.

$\Rightarrow be = af$

$\Rightarrow af = be$

$= (a,b)^R (e,f)$.

$\Rightarrow$ transitive

Since R is Reflexive, Symmetric & transitive
$\therefore$ It is an equivalence relation.

Matrix Representation of Relations :-

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Q Let $P = \{1,2,3,4\}$, $Q = \{a,b,c,d\}$.
$R = \{(1,a), (2,c), (2,d), (4,b)\}$. Find M_R.

Sol →

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Ans

Q If R & S be relation on a set S, represented by matrices:

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \ \& \ M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find matrices that represent

(a) $R \cup S$ (b) $R \cap S$ (c) $R \cdot S$ (d) $S \cdot R$ (e) $R \oplus S$.

Sol → (a) $R \cup S \Rightarrow M_{R \cup S} = M_R \vee M_S$.

$$M_{R \cup S} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ Ans}$$

(b) $R \cap S \Rightarrow M_{R \cap S} = M_R \wedge M_S$.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ Ans}$$

(c) $M_{RS} = M_R \cdot M_S$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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(d) $M_{SR} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Ans

(e) $M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$ Ans

Q If R is the relation of $A = \{1, 2, 3\}$ such that $(a, b) \in R$ iff $a + b$ = even. find relation matrix M_R. find also relation matrices of R^{-1}, $\bar{R}$ & R^2.

Sol→ $A = \{1, 2, 3\}$

$R = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$M_{R^{-1}} = (M_R)^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Ans

$M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $M_{R^2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Ans

Warshall's Algorithm :-

With this we convert non-transitive relation to transitive relation

Q Using Warshall's algorithm, find all transitive closure of relation $R = \{(4,4), (4,10), (6,6), (6,8), (8,10)\}$ on set $A = \{4, 6, 8, 10\}$.

Sol →

$$W_0 = M_R = \begin{array}{c|cccc} & 4 & 6 & 8 & 10 \\ \hline 4 & 1 & 0 & 0 & 1 \\ 6 & 0 & 1 & 1 & 0 \\ 8 & 0 & 0 & 0 & 1 \\ 10 & 0 & 0 & 0 & 0 \end{array}$$

We will now compute W_1. To compute W_1.
(i) transfer all 1's from W_0 to W_1
(ii) Location of non-zero entries in C_1
(iii) Location of non-zero entries in R_1

Non-zero entry in $C_1 = 4$
Non-zero entry in $R_1 = 4, 10$
Mark $(4,4), (4,10)$ as 1

$$W_1 = \begin{array}{c|cccc} & 4 & 6 & 8 & 10 \\ \hline 4 & 1 & 0 & 0 & 1 \\ 6 & 0 & 1 & 1 & 0 \\ 8 & 0 & 0 & 0 & 1 \\ 10 & 0 & 0 & 0 & 0 \end{array}$$

Now following same steps we will find W_2, W_3, W_4

Non-zero entry in C_2 : 6

Non-zero entry in R_2 : (6, 8)

Mark (6, 6), (6, 8) as 1

$$W_2 = \begin{array}{c} 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-zero entry in C_3 : 6

Non-zero entry in R_3 : 10

Mark (6, 10) as 1

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-zero entry in C_4 : 4, 6, 8

Non-zero entry in R_4 : —

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R^+ = \{(4,4), (4,10), (6,6), (6,8), (6,10), (8,10)\}$

Ans

Q Using Warshall Algorithm find transitive Closure relation of $R = \{(1,2), (2,3), (3,3)\}$ where $A = \{1, 2, 3\}$.

Sol → $W_0 = M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$C_1 :$ —
$R_1 : 2$

$W_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$C_2 : 1$
$R_2 : 3$
mark $(1,3)$ as 1

$W_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$C_3 = 1, 2, 3$
$R_3 = 3.$
mark $(1,3), (2,3), (3,3)$ as 1

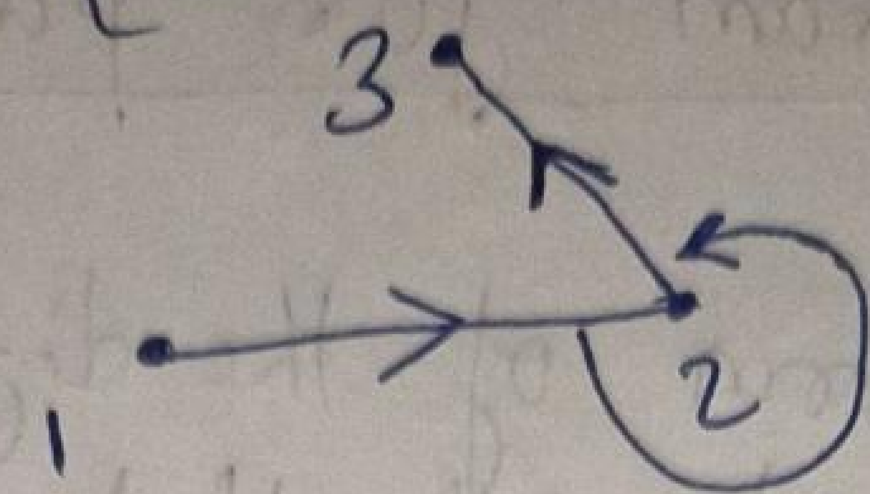
$W_3 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$R^+ = \{(1,2), (1,3), (2,3), (3,3)\}$ Ans

Representation of Relation by graph :-

To represent R on set A graphically, each element of A is represented by a point known as node or vertices. To show a is related to b, an ~~arch~~ arc is drawn from a to b. This arc is called ~~vert~~ edges or arcs. Direction is indicated by an arrow. Resulting diagram is called directed graph or di-graph.

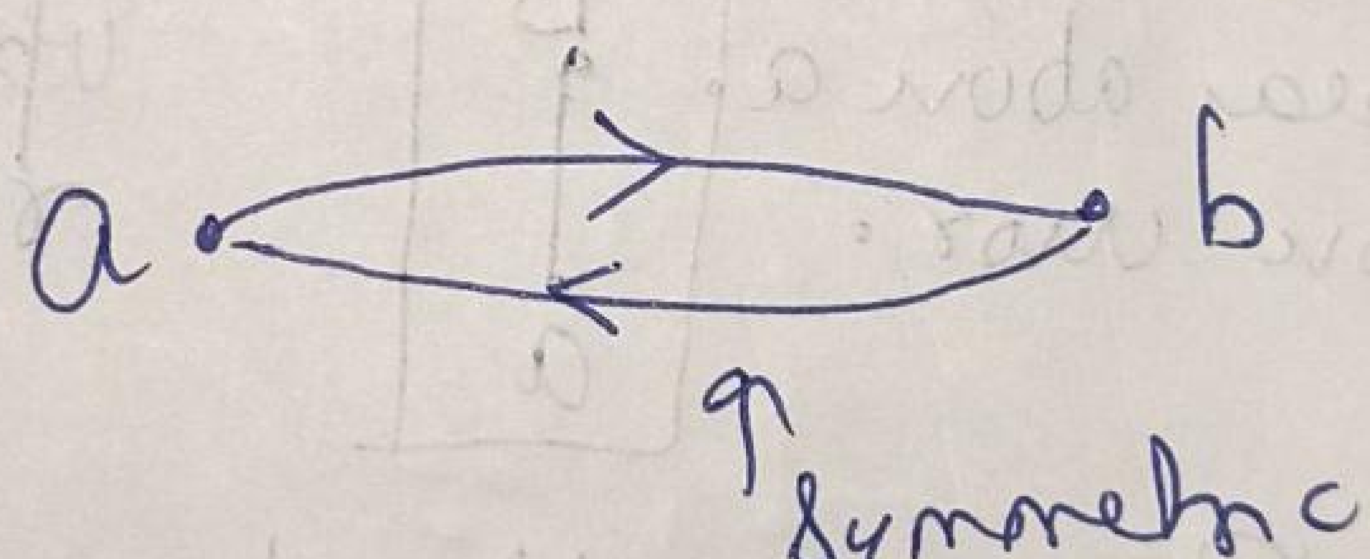
Eg. $R = \{(1,2), (2,3), (2,2)\}$



* How to Identify types of relation by looking at di-graph ?

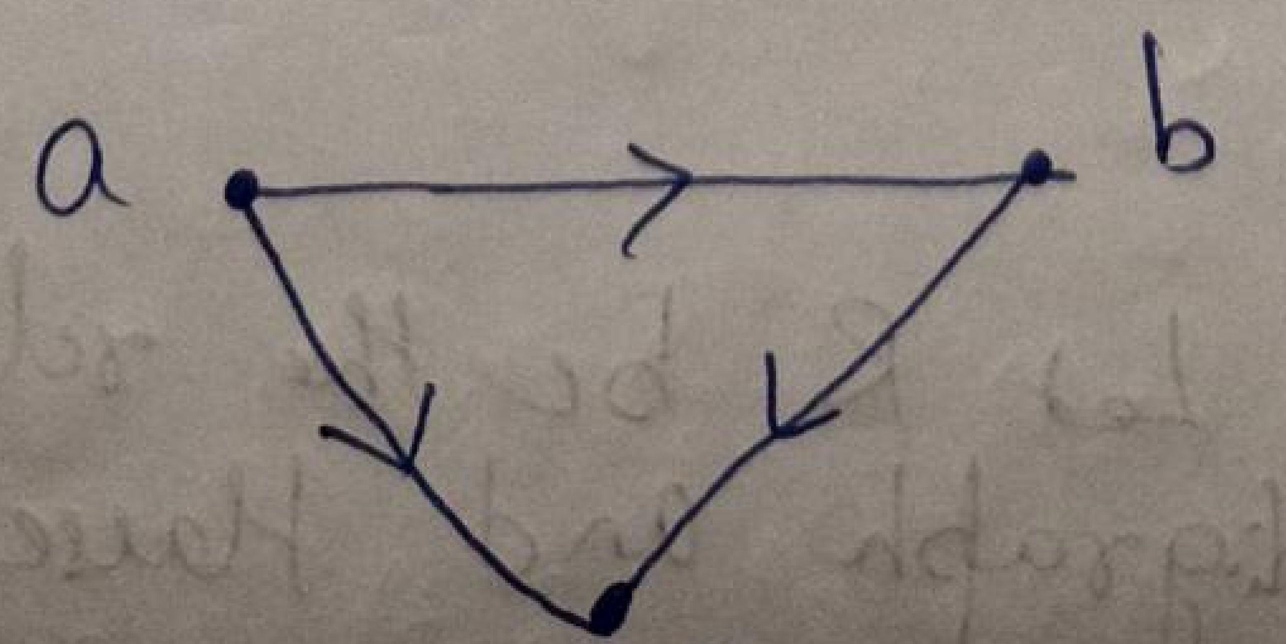
(i) Reflexive :- A relation R is reflexive if and only if there is a self-loop at every vertex.

(ii) Symmetric :- A relation R is symmetric iff there is any parallel opposit edge between a and b.



(iii) Antisymmetric :- iff there is no parallel edge

(iv) Transitive :- if and only if whenever there is an edge between a ~~and~~ to b and from b to c then there is edge between a to c.



Imp

Hasse Diagram for partial ordering

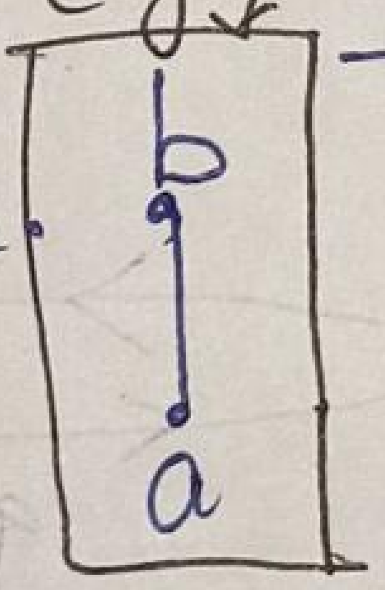
The simplified form of the digraph of a partial ordering on a finite set that contain sufficient information about the partial ordering is called Hasse diagram.

How to simplify ?

Step 1 → Since partial ordering is reflexive, so remove all self loop from all vertex.

Step 2 → If $a^R b$, b appear above a & remove arrow.

e.g. →

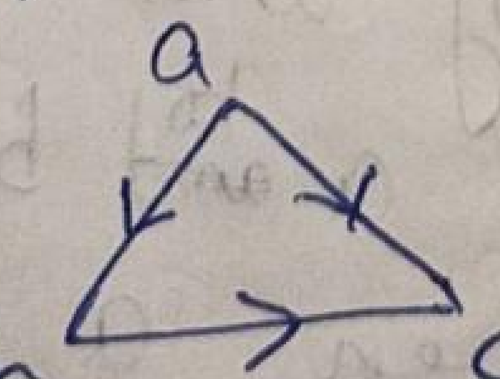


All edge are directed upward. No direction required.

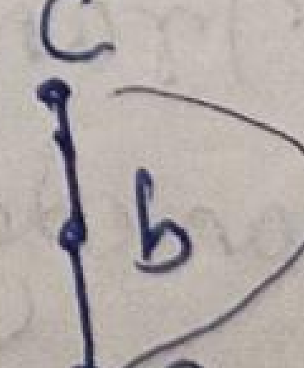
Step 3 :- Remove all edge whose existence is implied by transitive property.

$a^R b$, $b^R c$, $a^R c$.

⇒



⇒



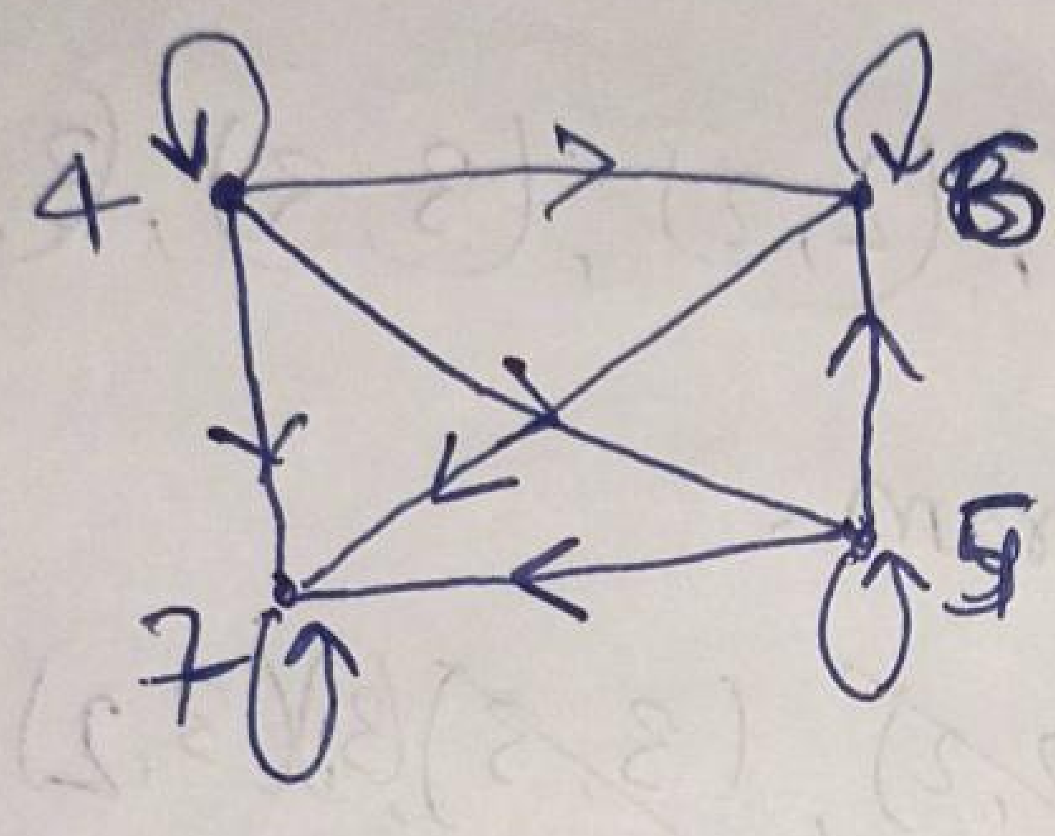
← No need to have edge from a to c.

Q. $A = \{4, 5, 6, 7\}$, Let R be the relation $\leq$ on A. Draw the digraph and Hasse diagram of R.

Sol → $A = \{4, 5, 6, 7\}$

$R = \{(4,4), (4,5), (4,6), (4,7), (5,5), (5,6), (5,7), (6,6), (6,7), (7,7)\}$

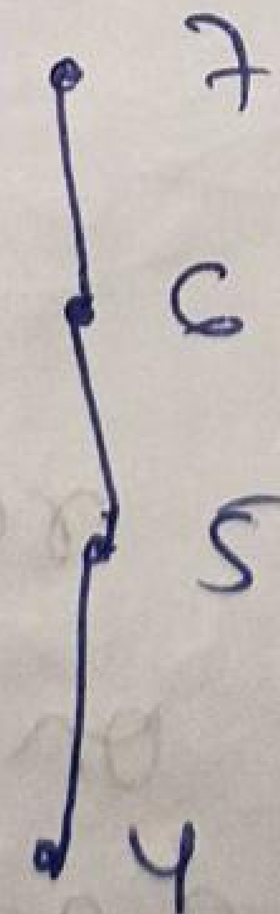
<u>Di-graph :-</u>



Hasse Diagram :-

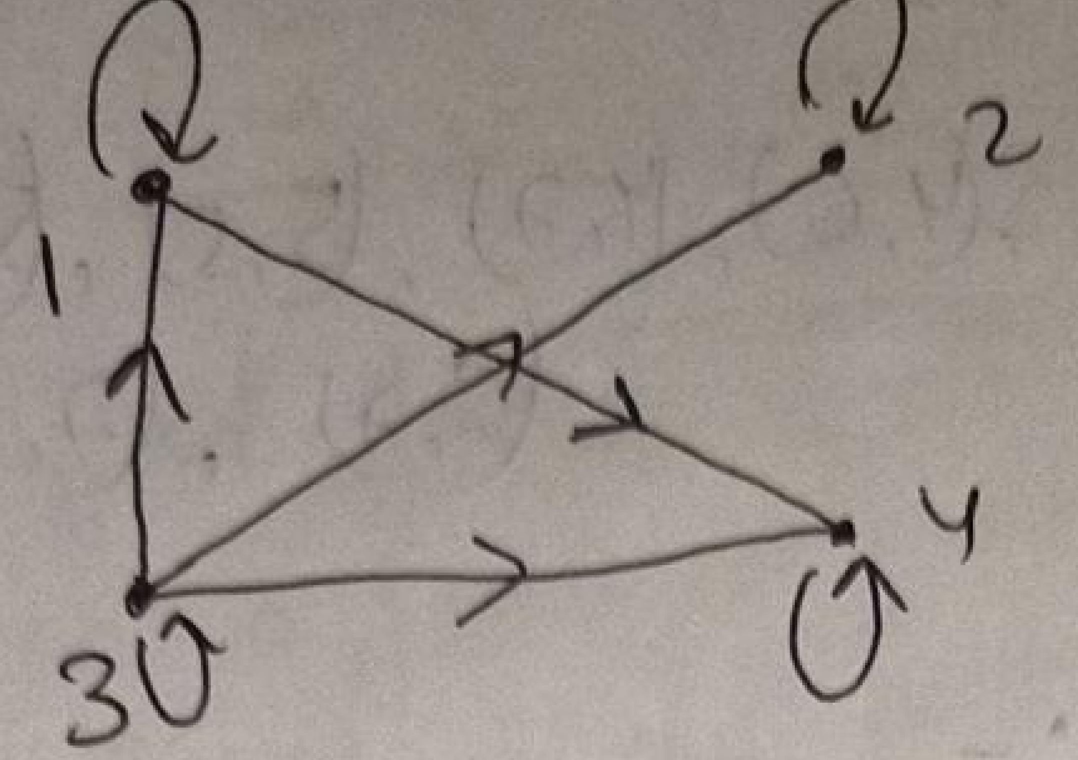
$R = \{\cancel{(4,4)}, (4,5), \cancel{(4,6)}, \cancel{(4,7)}, \cancel{(5,5)}, (5,6), \cancel{(5,7)}, \cancel{(6,6)}, (6,7), \cancel{(7,7)}\}$

$R = \{(4,5), (5,6), (6,7)\}$



Ans

Q ~~A = {1, 2, 3, 4}~~. Draw Hasse Diagram from the given digraph G for partial ordering relation on set $A = \{1, 2, 3, 4\}$



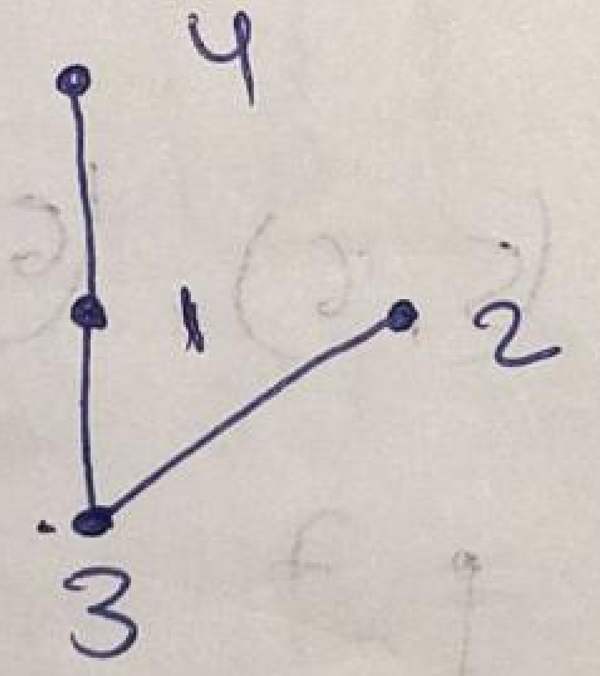
Sol → from graph.

$R = \{(1,1), (1,4), (2,2), (3,3), (3,1), (3,4), (4,4)\}$

For Hasse diagram.

$R = \{\cancel{(1,1)}, (1,4), \cancel{(2,2)}, \cancel{(3,3)}, (3,1), (3,2), \cancel{(3,4)}, \cancel{(4,4)}\}$

$R = \{(1,4), (3,2), (3,1), (3,4)\}$

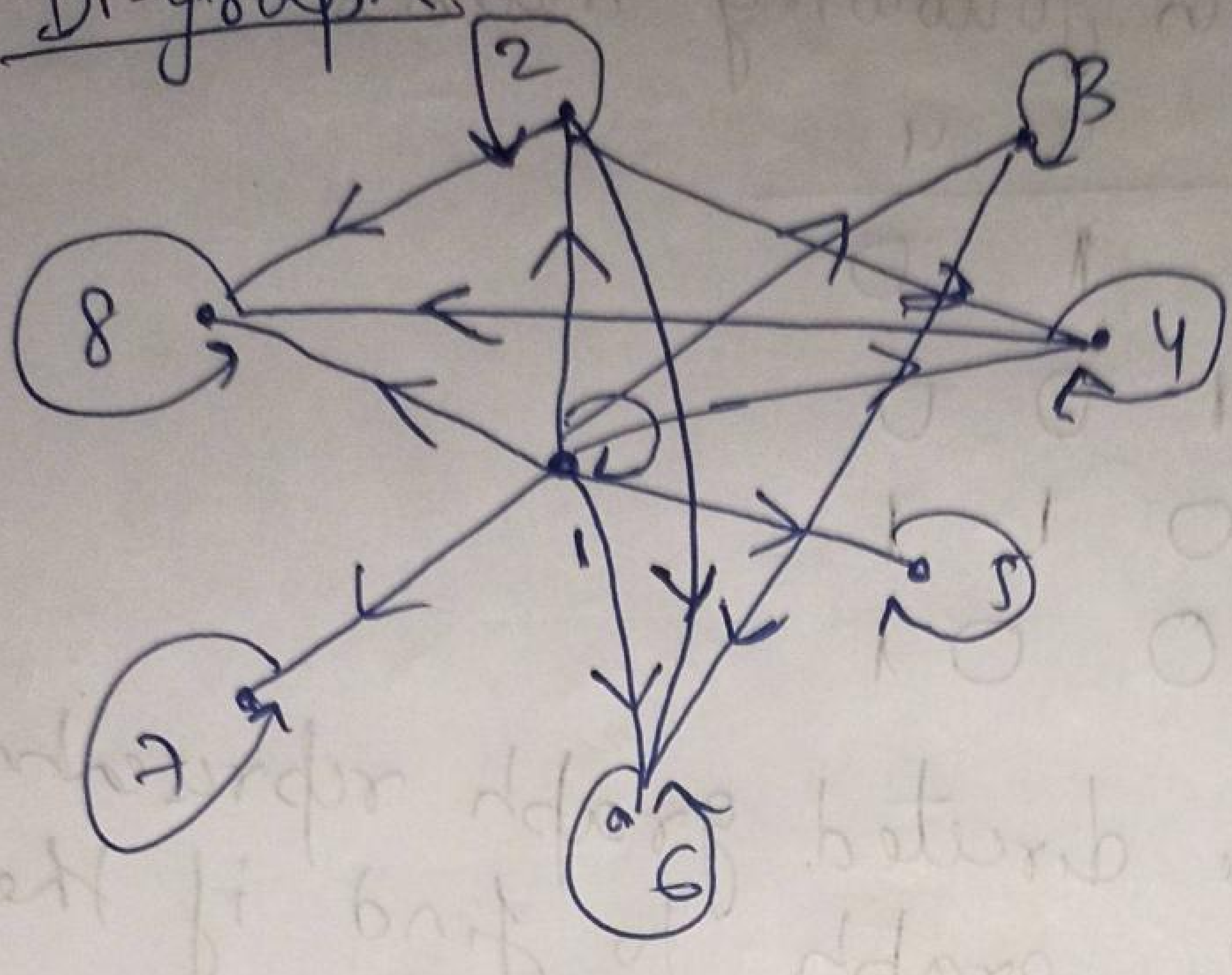


Ans

Q Draw the digraph representing the relation (a, b) a divides b on the set $A = \{1,2,3,4,5,6,7,8\}$. Reduce it to Hasse diagram representing the partial ordering.

Sol → $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (5,5), (4,8), (6,6)\}$

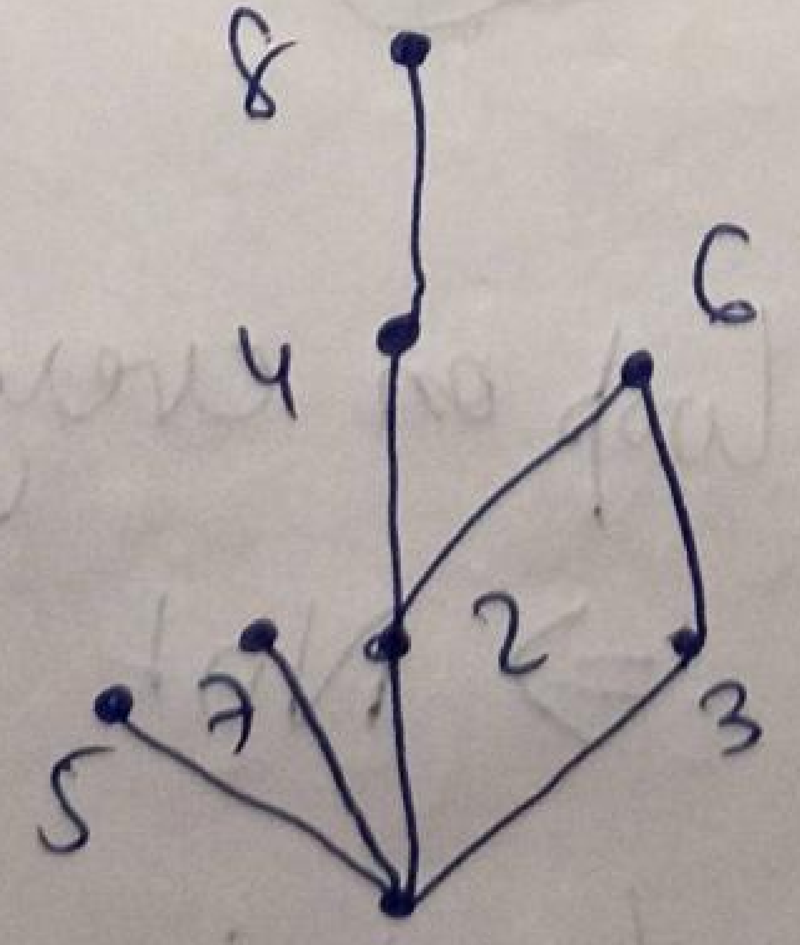
Di-graph :-



Hasse Diagram :-

$R = \{ \cancel{(1,1)}, (1,2), (1,3), \cancel{(1,4)}, (1,5), \cancel{(1,6)}, (1,7), \cancel{(1,8)},$
$\cancel{(2,2)}, (2,4), (2,6), \cancel{(2,8)}, \cancel{(3,3)}, (3,6), \cancel{(4,4)}, (4,8), \cancel{(5,5)},$
$\cancel{(6,6)}, \cancel{(7,7)}, \cancel{(8,8)} \}$

$R = \{ (1,2), (1,3), (1,5), (1,7), (2,4), (2,6), (3,6), (4,8) \}$

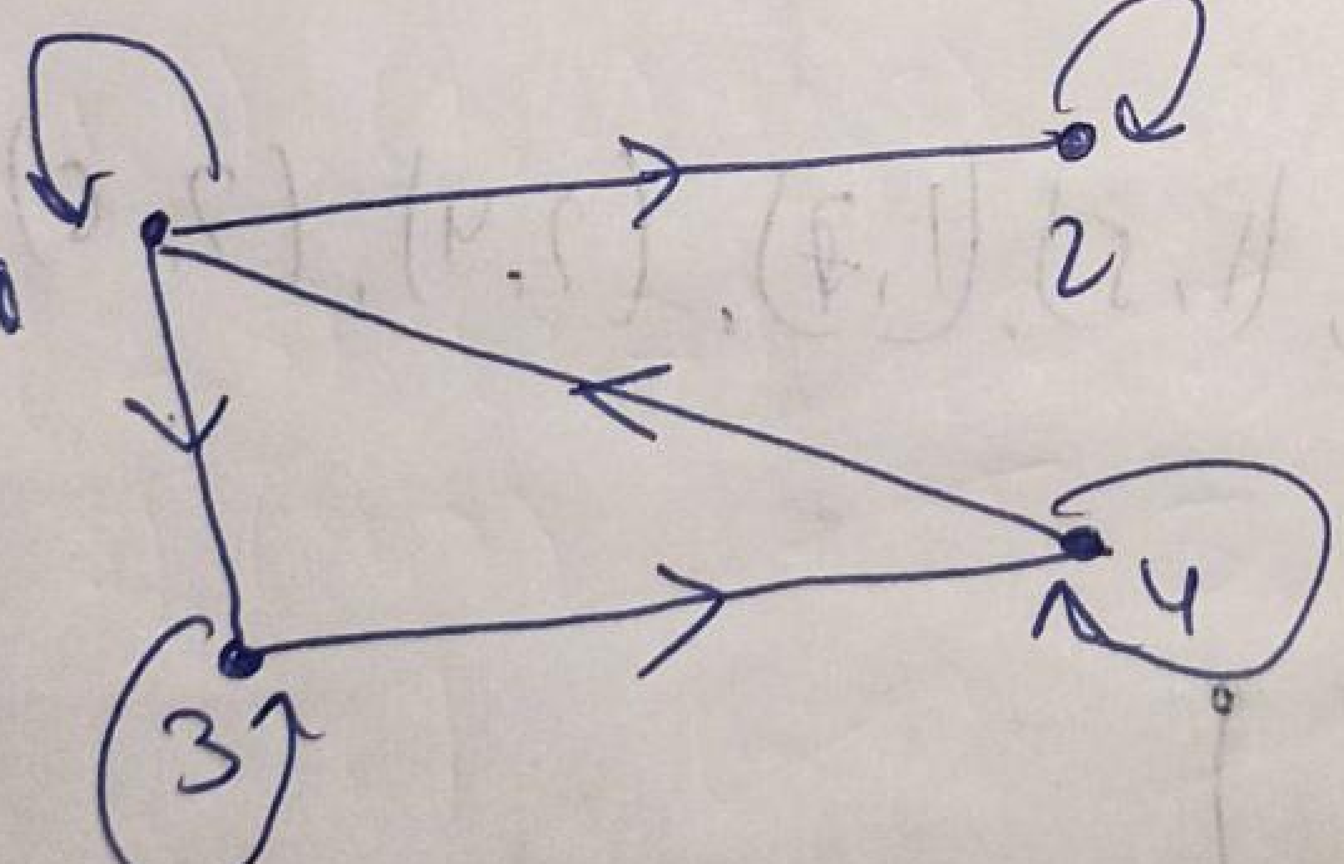


Q List the ordered pair in relation on {1, 2, 3, 4} Corresponding in following matrix

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 1 & 0 & 0 & 1 \end{array}$$

Also draw the directed graph representing this relation. Use the graph to find if the relation is reflexive, symmetric or transitive.

Sol → $R = \{(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)\}$

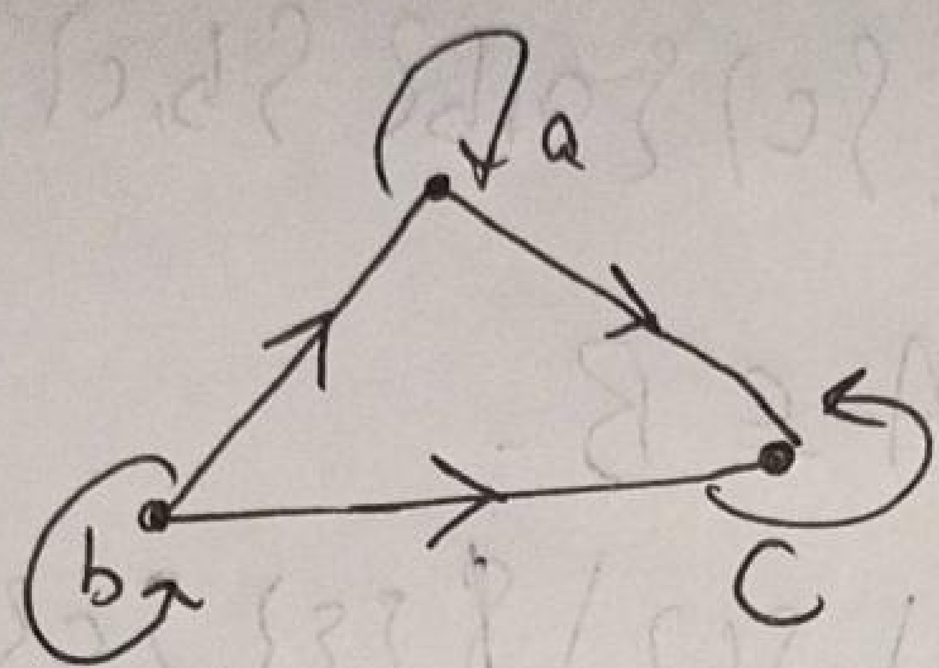


Since there is self loop on every vertex ⇒ Reflexive

No parallel edge ⇒ Not Symmetric

(1,3) . (3,4) but (1,4) is not there. So, Not transitive.

Q. List the ordered pair in the relation represented by di-graph given in the figure. Also use the graph to prove that relation is partial ordering. Also draw the directed graph representing R^{-1} and $\bar{R}$.



Sol → $R = \{(a,a), (a,c), (b,a), (b,b), (c,c), (b,c)\}$

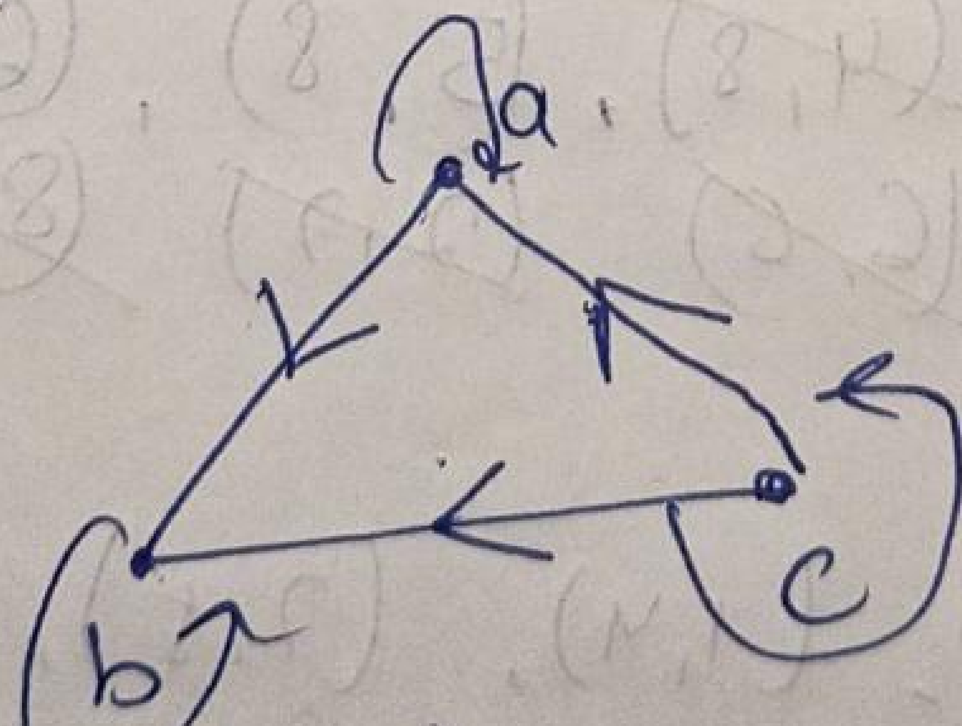
Since there is self loop on every node ⇒ Reflexive

Since there is no parallel edge ⇒ Anti-Symmetric

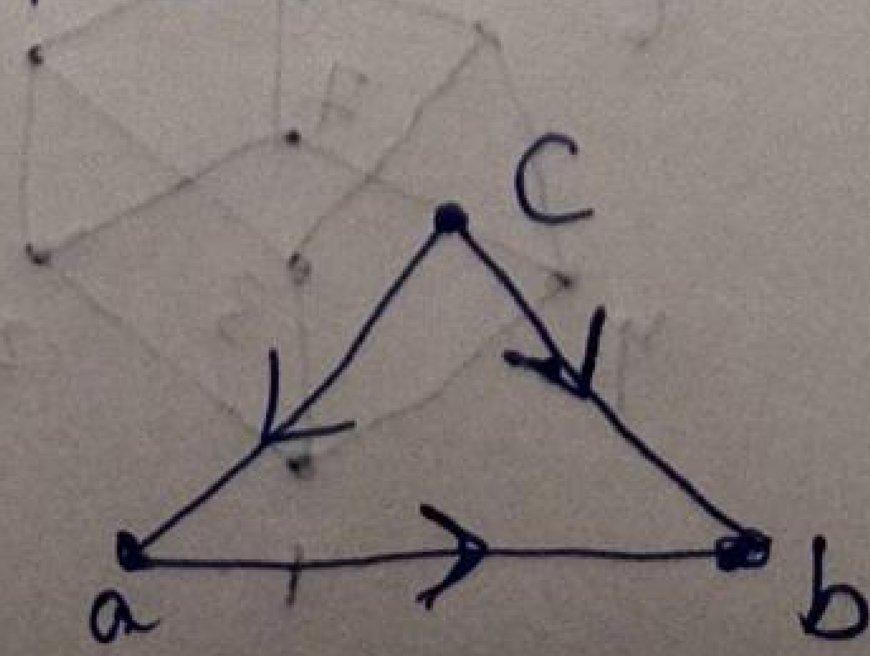
$(b,a), (a,c)$ & (b,c) so transitive.

So, graph relation is partial ordering.

R^{-1} :



$\bar{R}$: $\{(a,b), (c,a), (c,b)\}$



Q Draw the Hasse diagram representing the partial ordering {(A, B) : A ⊆ B} on power set P(S) where S = {a, b, c, d}.

Sol → Power set of S P(S).

= {{}, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c}}

Relation (a, b) : A ⊆ B.

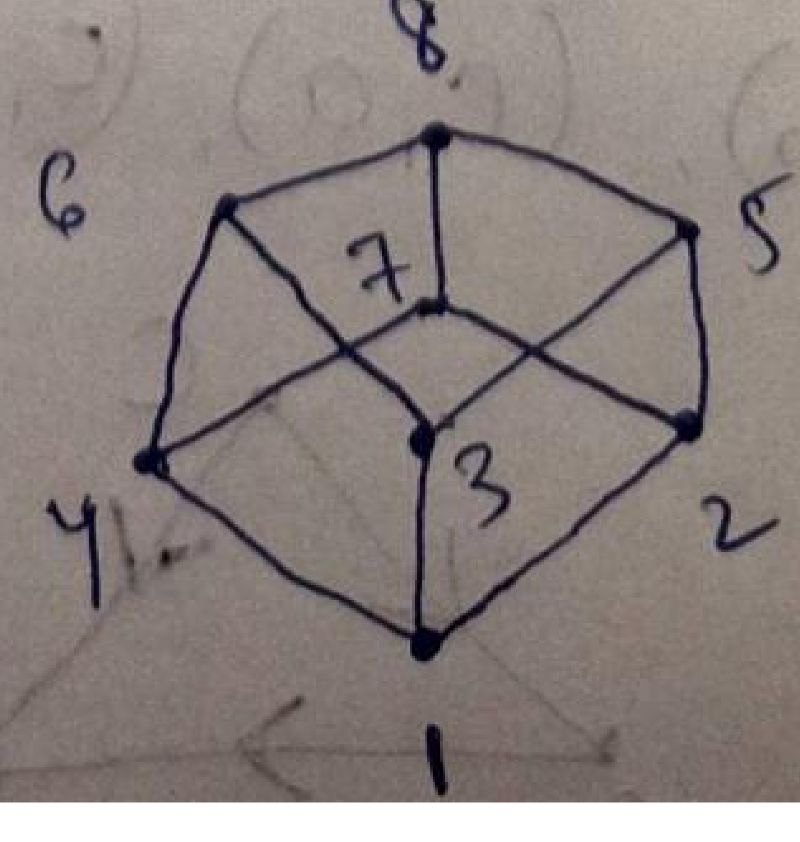
({}, {}) ({}, {a}) ({}, {b}) ({}, {c}), ({}, {a,b}) — so on

or

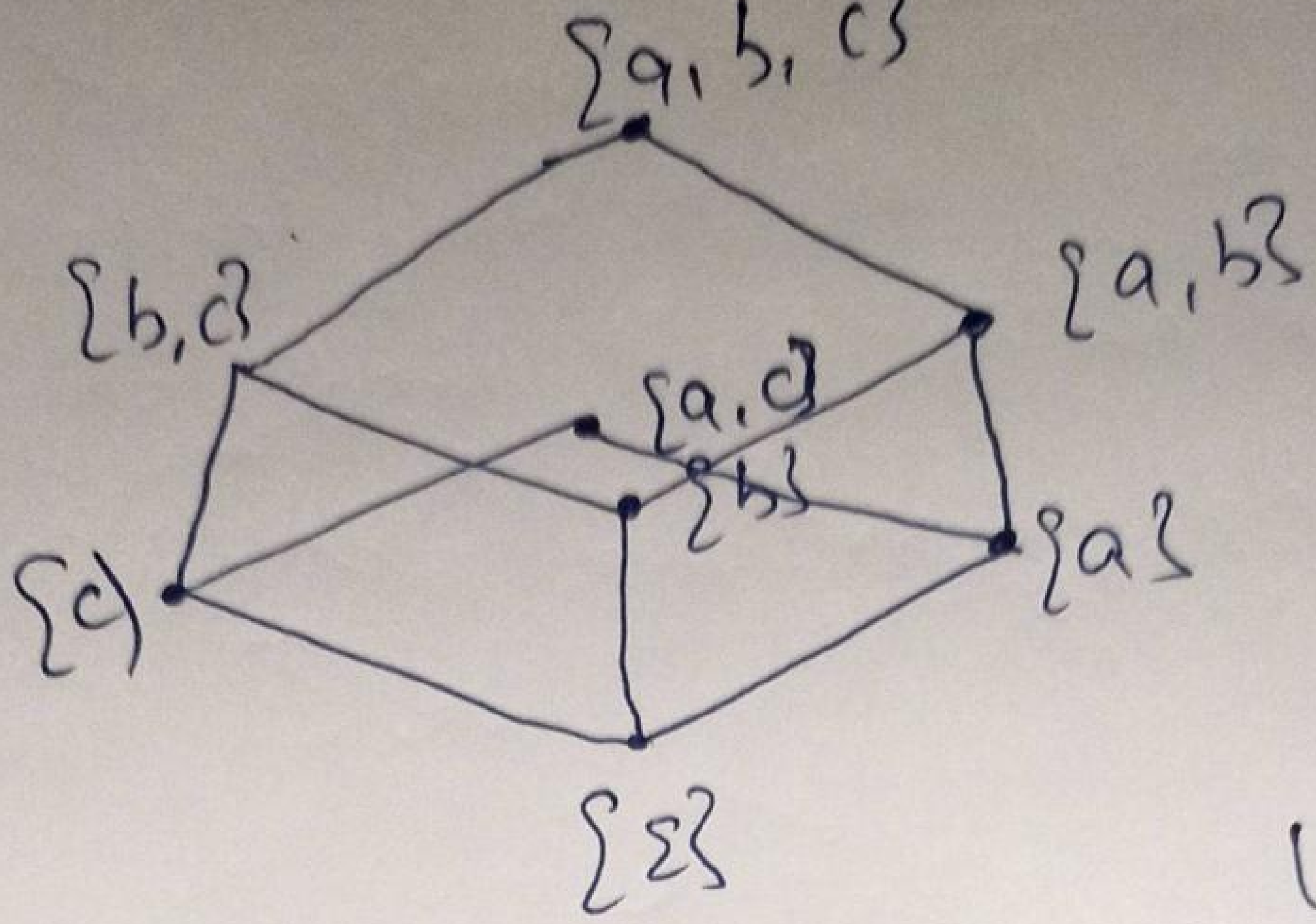
A = (1, 2, 3, 4, 5, 6, 7, 8)

R = {~~(1,1)~~, (1,2), (1,3), (1,4), ~~(1,5)~~, ~~(1,6)~~, ~~(1,7)~~, ~~(1,8)~~
(2,5), (2,7), ~~(2,8)~~, (3,5), (3,6), (4,6),
~~(3,8)~~, (4,7), ~~(4,8)~~, (5,8), (6,8), (7,8)}
~~(2,2)~~ ~~(3,3)~~ ~~(4,4)~~, ~~(5,5)~~, ~~(6,6)~~ ~~(7,7)~~ ~~(8,8)~~}

R = {(1,2), (1,3), (1,4), (2,5), (2,7), (3,5), (3,6),
(4,6), (4,7), (5,8), (6,8), (7,8)}



⇒



Ans

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