Problem Set 1 **STAT 394/ MATH 394**
Probability I

Due Date: 10:00 PM, March 5, 2022 Prof. Alexander Giessing
Last Update: April 4, 2022 Spring Quarter, 2022

Study Group: _____ _____ _____

Question 1. **[5 + 5 pts]** Consider tossing a coin three times and define the following events:

$$A = \{\text{“1st toss shows heads”}\}, \quad B = \{\text{“2nd toss shows heads”}\}, \quad \text{and} \quad C = \{\text{“3rd toss shows heads”}\}.$$

(a) Describe in words the events $A \cap B$, $A \cap B^c \cap C$, $A \cup B \cup C$.

(b) Which sets of outcomes correspond to the event $D = \{\text{“3rd toss shows heads for the first time”}\}$?

Question 2. **[5 + 5 pts]** Show mathematically and illustrate with a Venn-diagram that the following is true:

(a) $A \cup B = (B \setminus A) \cup (A \setminus B) \cup (A \cap B)$.

(b) $A \setminus B$, $B \setminus A$, $A \cap B$ are disjoint.

Question 3. **[5 + 5 + 5 pts]** You roll a fair die twice.

(a) Describe the probability triplet $(\Omega, \mathcal{A}, \mathbb{P})$ for this experiment.

(b) Let A be the event that the second roll is larger than the first. Compute the probability $\mathbb{P}(A)$.

(c) Let B be the event that the sum of both rolls equals 10, and C be the event that the highest roll is 5. Describe in words the event $B \setminus C$ and compute the probability $\mathbb{P}(B \setminus C)$.

Question 4. **[5 + 5 pts]** σ-algebras.

(a) Let $\mathcal{A}$ be a σ-algebra and $A, B \in \mathcal{A}$. Show that $\mathcal{A}$ contains $A \cap B$, $A \setminus B$, and $A \Delta B$, where $A \Delta B$ is the “symmetric difference” between A and B defined as $A \Delta B := (A \setminus B) \cup (B \setminus A)$.

(b) Let $\mathcal{A}$ be a σ-algebra of subsets of Ω and suppose that $B \in \mathcal{A}$. Show that $\mathcal{G} = \{A \cap B \mid A \in \mathcal{A}\}$ is a σ-algebra of subsets of B. *(This one might be difficult.)*

Question 5. **[5 + 5 pts]** Let A and B be events with probabilities $\mathbb{P}(A) = 3/4$ and $\mathbb{P}(B) = 1/3$.

(a) Show that $1/12 \leq \mathbb{P}(A \cap B) \leq 1/3$.

(b) Find lower and upper bounds on $\mathbb{P}(A \cup B)$.

Question 6. **[0 pts]** **[Optional]** Suppose that you toss a fair coin repeatedly. Show that, with probability one, you will toss a head eventually. *(Hint: A similar approach as with the typewriter example on the slides will help.)*