

Problem Set 6

Statistical Methods In Engineering And Science

Due Date: 10:00 PM, February 20, 2023

Prof. Alexander Giessing

Last Update: February 14, 2023

Winter Quarter, 2023

Study Group: _____

Please upload your solution in a single pdf file on Canvas. Include all calculations, R-code, and figures (if applicable). All data sets are available on Canvas <https://canvas.uw.edu/courses/1584511>.

Question 1. The “Roman Casino” in Seattle is for sale. You consider buying it, but you want to know how much money you are going to make. The current owner, the Maverick Gaming LLC, is only willing to share the following information with you: The roulette game “Red or Black” is played about 1,000 times a night, 365 days a year. Each time the game is played the owner of the casino wins \$1 with probability $20/38$ and loses \$1 with probability $18/38$.

Explain in detail why the weak law of large numbers can be used to determine the revenue (per year) of the casino, and determine how much it is. (For simplicity, assume that Seattleites only know to play “Red or Black”).

Question 2. Let X_1, \dots, X_{730} be i.i.d. random variables, with pdf f given by

$$f(x) = \begin{cases} 4(1-x)^3 & \text{if } 0 \leq x \leq 1, \\ 0 & \text{o/w.} \end{cases}$$

Use the central limit theorem to approximate the probability $P(X_1 + \dots + X_{730} < 153.3)$.

Question 3. Given a random sample X_1, \dots, X_n from $Exp(\lambda)$ with unknown $\lambda > 0$ you want to estimate the expectation $1/\lambda$. From the lecture you know that the sample average $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ is an unbiased estimator for $1/\lambda$. Now, consider a more generally estimator of the form $T_c = c \times (\sum_{i=1}^n X_i)$ for $c \in \mathbb{R}$. You are interested in the MSE of this estimator and you would like to know whether there are choices for $c \in \mathbb{R}$ such that T_c has smaller MSE than \bar{X}_n .

- (a) Compute the $MSE(T_c)$ for arbitrary $c \in \mathbb{R}$.
- (b) For which $c \in \mathbb{R}$ does the estimator T_c perform best in the MSE sense? Compare the MSE of this “best” estimator with the MSE of the unbiased estimator \bar{X}_n (which is just the estimator T_c with $c = 1/n$).

Question 4. Suppose that you have two unbiased estimators U and V with the same variance $\text{Var}(U) = \text{Var}(V)$. Based on the MSE criterion you cannot choose one estimator over the other. Consider now $W = (U + V)/2$.

(a) Show that W is unbiased.

(b) Show that

$$\frac{\text{Var}((U + V)/2)}{\text{Var}(U)} = \frac{1}{2} + \frac{1}{2}\rho(U, V),$$

where $\rho(U, V)$ is the correlation coefficient between U and V .

(c) Why do the results for part (a) and (b) imply that we should use W instead of U or V ?