Problem Set 5 **Statistical Methods**
In Engineering And Science

Due Date: 10:00 PM, May 5, 2023 Prof. Alexander Giessing
Last Update: April 24, 2023 Spring Quarter, 2023

Study Group: ______________ ______________ ______________

Please upload your solution in a single pdf file on Canvas. Include all calculations, R-code, and figures (if applicable). All data sets are available on Canvas https://canvas.uw.edu/courses/1584511.

Question 1. Suppose that you want to model the lifetime of a product using a cumulative distribution function from the so-called Weibull family, e.g.

$$F(x) = \begin{cases} 1 - e^{-3x^2} & \text{for } x \geq 0, \\ 0 & o/w. \end{cases}$$

Construct a random variable Z with this distribution using only a $Unif(0,1)$ variable.

Question 2. The ports of Los Angeles (A) and Long Beach (B) are two of the busiest ports on the West Coast. Denote the number of cargo ships being cleared at A and B every day by X and Y, respectively. The following table shows the joint pmf of X and Y.

		y		
	$p(x,y)$	1	3	4
	1	0.04	0.16	0.13
x	2	0.2	0.11	0.02
	3	0.09	0.07	0.18

(a) Find the marginal distribution, expected value, and standard deviation of X at any given day.

(b) Find the probabilities $P(X + Y \leq 5)$ and $P(X + Y \leq 5 \mid X \leq 2)$.

(c) Compute $P(X \leq 2, Y \leq 3)$. Using this probability (and two other probabilities) check whether X and Y are independent.

(d) Compute the covariance between X and Y.

(e) The POLA management plans to transition to a 24/7 workshift, which will double the number of cargo that can be cleared any day. How will the results from part (a) and (d) change?

Question 3. Suppose that the distances of javelin throws of professional athletes follow approximately a normal distribution with mean 301 ft and standard deviation 4 ft. (Mean and standard deviation are indeed plausible, see https://en.wikipedia.org/wiki/Javelin_throw. The assumption of normality is a simplification; I believe that it is justifiable.)

(a) What is the minimum distance thrown by someone who belongs to the top 5% of the athletes?

(b) What percent of the athletes throws the javelin farther than 310 ft?

(c) Consider a competition with 20 javelin throwers. Suppose that the javelin throwers are mutually independent of each other. What is the probability that the winner has thrown the javelin 311 to 323 ft far? *(Hint: Express the probability in terms of the distribution function Φ of a standard normal random variable, then use R to compute the values.)*

Question 4. Let X, Y, Z be random variables. Show the following:

(a) $\mathrm{Cov}(X,Y) = \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y]$.

(b) $\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ if X and Y are independent.

(c) $\mathrm{Cov}(aX + bY + c, dZ + e) = ad\mathrm{Cov}(X,Z) + bd\mathrm{Cov}(Y,Z)$ for all $a,b,c,d,e \in \mathbb{R}$.

Question 5. Let X and Y be two independent random variables with the following pdfs:

$$f_X(x) = \begin{cases} \frac{3}{4}x(x-2) & \text{if } 2 \leq x \leq 3, \\ 0 & o/w, \end{cases} \qquad \text{and} \qquad f_Y(y) = \begin{cases} 1 & \text{if } 1 \leq y \leq 2, \\ 0 & o/w. \end{cases}$$

(a) Derive the joint pdf $f(x,y)$ of X and Y.

(b) Let $Z = -2Y + 1$. Derive the cdf and pdf of Z.

(c) Compute the covariance $\mathrm{Cov}(Y + X, -2Y + 1)$.

(d) Are Y and $\left(Y - \frac{3}{2}\right)^2$ uncorrelated and/or independent? Justify your answer mathematically and intuitively.