Problem Set 3

Statistical Methods In Engineering And Science

Due Date: 10:00 PM, January 27, 2023
Last Update: January 20, 2023

Prof. Alexander Giessing
Winter Quarter, 2023

Study Group: _____ _____ _____

Please upload your solution in a single pdf file on Canvas. Include all calculations, R-code, and figures (if applicable). All data sets are available on Canvas https://canvas.uw.edu/courses/1614615.

Question 1. You independently and uniformly at random draw three times from the numbers $3, 9, 0$. (That is, each number has equal probability of being drawn and each draw is independent of the other draws.) Let X_i denote the ith draw, $i = 1, 2, 3$, and $\bar{X} = (X_1 + X_2 + X_3)/3$.

(a) Find the pmf of X_i for $i = 1, 2, 3$.

(b) Find the pmf of $\bar{X}$.

(c) Find the cdf of $\bar{X}$.

(d) Compute the expected value and standard deviation of X_1 and $\bar{X}$. What do you observe?

(e) Draw pmf and cdf of $\bar{X}$ by hand. (Do not use R.)

Question 2. A store orders a batch of 2000 flashlights. The odds that a flashlight is defective are 0.15%. Let X be the number of defective flashlights in the batch.

(a) What is the distribution of X? What is/ are the value(s) of the paramter(s) of this distribution?

(b) What are expected value and standard deviation of X?

(c) What is the probability that the batch contains no defective flashlights? One defective flashlight? More than two defective ones?

Question 3. Airlines often oversell tickets. Suppose that 104 passengers have tickets for a plane with only 100 seats. Let X be the number of ticketed passengers who show up for the flight. Assume that the probability mass function of X is given by

x	96	97	98	99	100	101	102	103	104
$p(x)$	.15	.12	.14	.25	.17	.06	.05	.04	.02

(a) Use R to plot the pmf and cdf of X.

(b) What are the probabilities of $P(X \leq 100)$ and $P(98 < X \leq 102)$?

(c) Find the pmf of the random variable $Y = |X - 100|$. What is $\mathrm{E}[Y]$?

(d) Suppose that tickets were sold for \$200 and passengers who do not get a seat are entitled to \$300 compensation. Let Z be the profit of the airline generated by overselling 4 tickets, i.e.

$$Z = \begin{cases} 4 \times \$200 - (X - 100) \times \$300 & \text{if } X > 100, \\ 4 \times \$200 & \text{o/w.} \end{cases}$$

What are the expected profit $\mathrm{E}[Z]$ and the standard deviation $\mathrm{S.D.}(Z)$?

Question 4. Recall the setting of Problem 3. Suppose that the airline implements the strategy of overselling 4 tickets for a plane with 100 seats for 1000 consecutive days.

(a) Use below R code to simulate the number of ticketed passengers who show up (X) and the profit (Z) for 1000 consecutive days. Report the first 10 simulation results for the numbers of passengers X and the profits Z.

```
> x <- c(96, 97, 98, 99, 100, 101, 102, 103, 104)
> p <- c(.15, .12, .14, .25, .17, .06, .05, .04, .02)
> X <- sample(x, 1000, replace=T, prob=p) # sample 1000 times from the distribution
> Z <- 800-300*(X-100)*(X > 100) # compute the profit
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(b) Plot the histogram for X based on the simulated data in part (a). Compare the histogram in this problem with the pmf of X in Problem 3(a).

(c) What percentage of the simulated X satisfies $X \leq 100$ and $98 < X \leq 102$? Compare these percentages with the corresponding probabilities in Problem 3(b).[1]

(d) What are the average and standard deviation of the simulated profits Z? How do they compare with the theoretical ones of Problem 3(d).

[1] You may find the subsetting commands for R in Section 3.6 of "STAT 390 R Intro 2" useful. Check the document on Canvas under "Files" $\rightarrow$"Lab Notes".