Problem Set 9

Statistical Methods In Engineering And Science

Due Date: 10:00 PM, March 11, 2023
Last Update: March 7, 2023

Prof. Alexander Giessing
Winter Quarter, 2023

Study Group: _____ _____ _____

Please upload your solution in a single pdf file on Canvas. Include all calculations, R-code, and figures (if applicable). All data sets are available on Canvas https://canvas.uw.edu/courses/1614615.

Question 1. We continue the analysis of the `olympic1500m.txt` data set from Problem Set 8. But now we analyze the problem as one of constructing a confidence interval for a proportion.

(a) Let X denote the number of races (out of 23 total races) won by a skater starting in the outer lane. If there is no advantage of starting on the outer lane, then it is reasonable to model X as having a $Bin(23, p)$ distribution with $p = 0.5$. Explain why.

(b) Use the normal approximation to the binomial distribution to derive lower 95% and 98% CIs for the unknown proportion p. *(Hint: Lectures of Week 9, Part 1, Slides 16ff. You can use Matlab, Wolfram Alpha, ... to solve the inequality.)*

(c) Based on your 95% CI, are you 95% confident that skaters starting in the outer line are indeed faster? What is your answer based on the 98% CI?

Question 2. Reconsider the data set on 20 measurements of dielectric breakdown voltage for pieces of epoxy resin from this week's lectures:

24.46 25.61 26.25 26.42 26.66 27.15 27.31 27.54 27.74 27.94
27.98 28.04 28.28 28.49 28.50 28.87 29.11 29.13 29.50 30.88.

(a) Suppose that the data are normally distributed. Construct a 95% exact CI for the mean breakdown voltage.

(b) Construct a 95% Empirical Bootstrap CI for the mean breakdown voltage. Compare with the CI in part (a). *(Hint: Use the R code from this week's lab!)*

Question 3. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distr. on the interval $[0, \theta]$.

(a) Show that $U = \max_{1 \leq i \leq n} X_i/\theta$ has pdf

$$f_U(u) = \begin{cases} nu^{n-1} & 0 \leq u \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(Hint: Lectures of Week 5, Part 1.)

(b) Use $f_U(u)$ to verify that

$$P\left(\alpha^{1/n} < \max_{1 \leq i \leq n} X_i/\theta \leq 1\right) = 1 - \alpha,$$

and use this to derive a $100(1-\alpha)\%$ CI for θ.

(Hint: First, recall how to compute probabilities via densities. Second, for the CI adapt the steps of how to construct a CI for the mean, i.e. find suitable random variables L_n and U_n. See Lectures of Week 8, Part 2.)

Question 4. Please fill in the course evaluation. I would really appreciate your input! If at least 75% of you fill the course evaluation, I will award each of you **10 points extra points**. You should have received the link to the evaluation in an email. I've also posted the link on Canvas. Thank you for a great quarter :-)

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