A spatio-temporal model for events on road networks: an application to ambulance interventions in Milan

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Introduction

- In the last years, we observed a surge of interest in the statistical analysis of spatial data lying on or alongside road networks.

- Car crashes, domestic burglaries, and ambulance interventions represent just a few typical examples.

- The most important characteristic of these events is that they are constrained to lie on a restricted spatial domain which cannot be ignored for proper statistical modelling.

Introduction (cont)

- Several authors illustrated the methodological errors that may occur when applying classical planar-space spatial methods to events on road networks.
- Besides, a road network is not a homogeneous space, which implies that readaptations of classical techniques must take into account the new spatial domain.
- In this presentation, we will introduce a non-separable spatio-temporal model for events on road networks. The approach is exemplified using ambulance intervention data.

Emergency intervention data

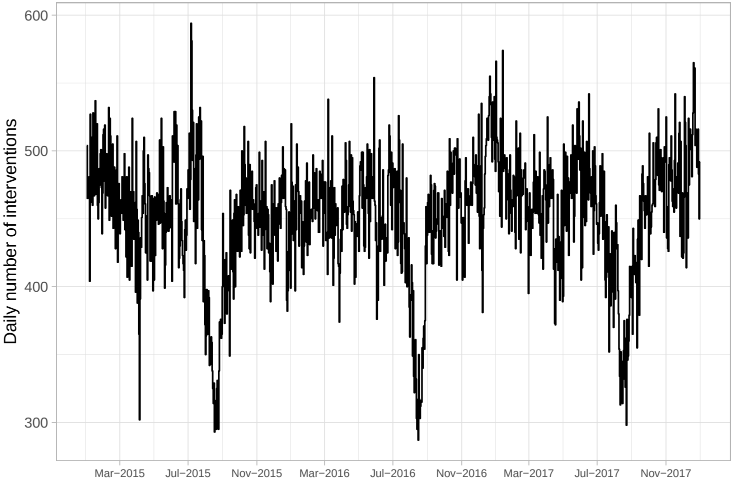
- More precisely, we analysed all emergency events that occurred in the road network of Milan from 2015-01-01 to 2017-12-31. The sample included approximately 500,000 interventions.
- The road network was built using OpenStreetMap. It represents the most important streets in the city and covers approximately 1850km.

Emergency intervention data (cont)

The spatial distribution:

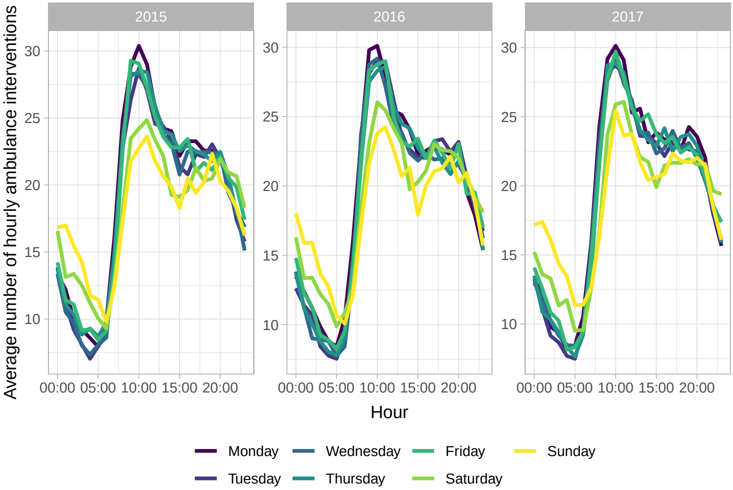
Emergency intervention data (cont)

The temporal distribution:



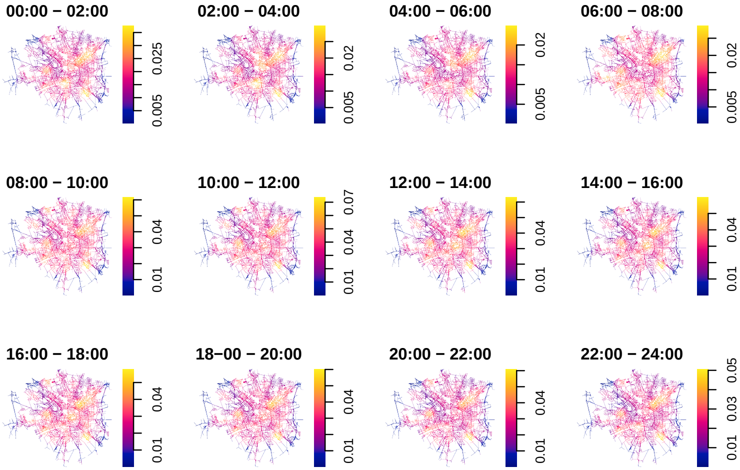
Emergency intervention data (cont)

The intra-day seasonalities:



Emergency intervention data (cont)

The space-time patterns:



The modelling framework

- We modelled the emergency intervention data considering a continuous one-dimensional linear network, say L, and a discrete temporal dimension $\mathcal{T}$ divided into intervals of one hour.
- We assumed that, independently for each $t \in \mathcal{T}$, the point process of ambulance interventions can be modelled as an Inhomogeneous Poisson Process (IHPP) on a linear network with intensity function $\lambda_t(\boldsymbol{s})$.

The modelling framework (cont)

- Furthermore, we assumed that the spatio-temporal intensity function $\lambda_t(\boldsymbol{s})$ can be decomposed as

$$\lambda_t(\boldsymbol{s}) = \mu_t g_t(\boldsymbol{s}), \ t \in \mathcal{T}, \ \boldsymbol{s} \in L.$$

- The term μ_t represents a spatial aggregation. It can be interpreted as the temporal dimension at time t.
- The term $g_t(\boldsymbol{s})$ represents the spatial component. The space-time interactions are modelled using a set of weights that introduce non-separability into the intensity function.

The temporal model

- The temporal dimension was modelled using a semi-parametric Poisson regression with smoothed deterministic calendar covariates, such as the hour of the day or the day of the week.
- In particular, the following log-linear Poisson model was fitted:

$$\log \mu_t = \beta_0 + \mathsf{dow}_t + \mathsf{dow}_t \times s_1(\mathsf{hour}_t) + s_2(\mathsf{week}_t).$$

  The term dow_t represents a factor variable for the day of the week while the terms $s_1(\cdot)$ and $s_2(\cdot)$ represent spline transformations for the hour of the day and the week of the year, respectively.

The spatial model

- The spatial component was estimated non-parametrically using a network-readaptation of a weighted kernel function.
- More precisely, given a set of past events (previously denoted by $\mathcal{T}$), a future hour u, and a location $\boldsymbol{s} \in L$, the weighted kernel estimator can be written as

$$\hat{g}_u(\boldsymbol{s}) = \frac{\sum_{t\in\mathcal{T}} \sum_{i=1}^{y_t} w(t,u) K_N(\boldsymbol{s}, \boldsymbol{s}_{i,t})}{\sum_{t\in\mathcal{T}} y_t w(t,u)}.$$

  where y_t denote the number of ambulance interventions at time t.

The spatial model (cont)

- The term $K_N(\boldsymbol{s}, \boldsymbol{s}_{i,t})$ denotes a network kernel function defined as:
$$K_N(\boldsymbol{s}, \boldsymbol{s_{i,t}}) = \frac{K(\boldsymbol{s} - \boldsymbol{s}_{i,t})}{c_L(\boldsymbol{s}_{i,t})}.$$
- $K(\cdot)$ represents the planar Gaussian kernel while $c_L(\boldsymbol{s}_{i,t}) = \int_L K(\boldsymbol{s} - \boldsymbol{s}_{i,t})\, \mathrm{d}\boldsymbol{s}$ indicates the convolution of kernel K with arc-length measure.
- This estimator can be computed rapidly using Fast Fourier Transform, which is an essential requirement given the volume of emergency data and the length of Milan's road network.

The spatial model (cont)

- The term $w(t,u)$ denotes the weight function that is used to incorporate space-time interactions. It was estimated assuming the following functional form:

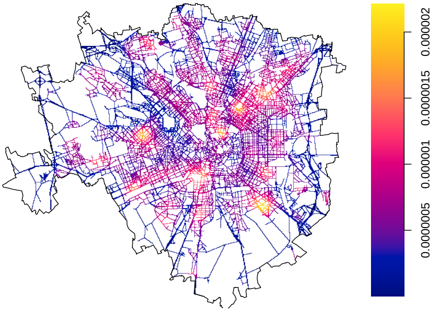
$$w(t,u) = w(u-t) = \rho_1^{u-t} + \rho_2^{u-t} \rho_3^{\sin^2\left(\frac{\pi(u-t)}{24}\right)} \rho_4^{\sin^2\left(\frac{\pi(u-t)}{168}\right)}.$$

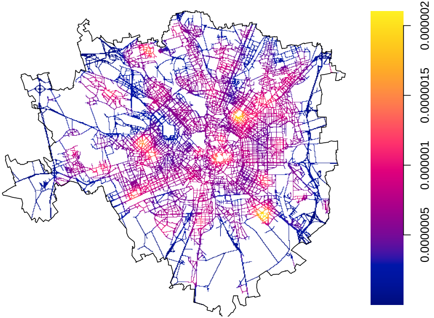
- The parameter ρ_1 is used to capture short-term dependence, while ρ_3 and ρ_4 measure daily and weekly seasonalities, respectively.

- We assume that the weights capture the temporal dependency of past interventions.

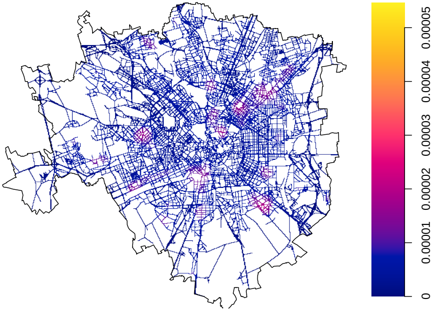
Results

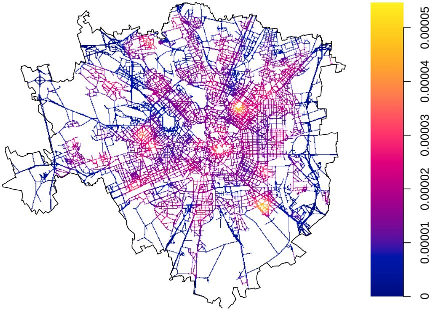
Spatial density function $\hat{g}_u(\boldsymbol{s})$ considering two future time periods: 2018-01-03 at 03:00 (left) and 15:00 (right).





Spatio-temporal intensity function $\hat{\lambda}_u(\boldsymbol{s})$ considering the same time periods as before.





Conclusion and future works

- We investigated the spatio-temporal distribution of the emergency interventions in Milan. Key to the approach was assuming that the events are constrained to lay on the city's road network and that they can be modelled as an IHPP with a temporal and a spatio-temporal dimension.
- The temporal component was estimated via semi-parametric Poisson regression, while the spatial density was estimated using a weighted non-parametric kernel function readapted for network data.
- In the future, we plan to move towards models that account for markers or clustering.

References

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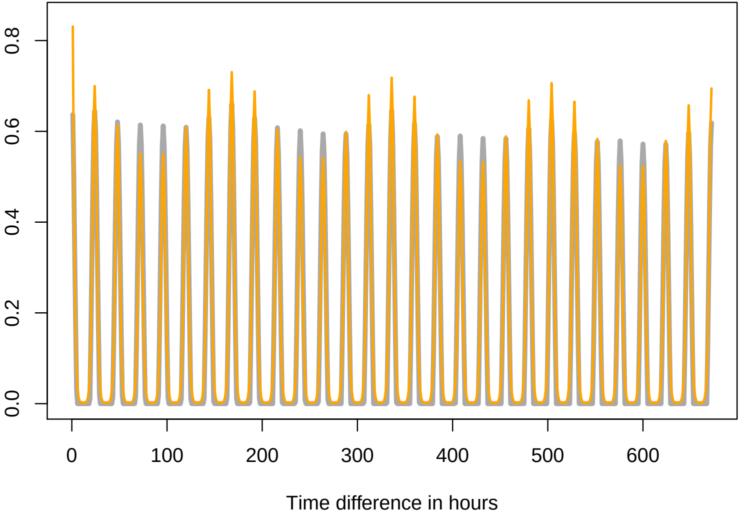
More details

The procedure to estimate $\rho_1, \ldots, \rho_4$

The parameters $\rho_1, \ldots, \rho_4$ were estimated starting from the autocorrelation function of observed counts.



The spatial validation

- The predictive accuracy of $\hat{g}_t(\boldsymbol{s})$ was tested via a network readaptation of the planar relative risk function.

- More precisely, given two network point patterns $\boldsymbol{A}$ and $\boldsymbol{B}$ and a location $\boldsymbol{s} \in L$, the (normalised) relative-risk function is defined as
$$\rho(\boldsymbol{s}) = \frac{g_{\boldsymbol{A}}(\boldsymbol{s})}{g_{\boldsymbol{A}}(\boldsymbol{s}) + g_{\boldsymbol{B}}(\boldsymbol{s})},$$
where $g_{\boldsymbol{A}}(\boldsymbol{s})$ and $g_{\boldsymbol{B}}(\boldsymbol{s})$ denote the spatial densities of $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. The plug-in estimator is given by
$$\hat{\rho}(\boldsymbol{s}) = \frac{\hat{g}_A(\boldsymbol{s})}{\hat{g}_A(\boldsymbol{s}) + \hat{g}_B(\boldsymbol{s})},$$
where $\hat{g}_{\boldsymbol{A}}(\boldsymbol{s})$ and $\hat{g}_{\boldsymbol{B}}(\boldsymbol{s})$ are kernel estimates of $g_{\boldsymbol{A}}(\boldsymbol{s})$ and $g_{\boldsymbol{B}}(\boldsymbol{s})$.

The spatial validation (cont)

- We estimated the relative-risk function $\rho(\boldsymbol{s})$ comparing observed and (out-of-sample) predicted interventions (i.e. the two point patterns mentioned before) for four different temporal aggregations.
- The bandwidths for the two kernel estimates were calculated using a readaptation of Scott's rule of thumb.
- We say that the proposed model successfully predicts the future events when the value of $\rho(\boldsymbol{s})$ lay around 0.5 (since, in that case, the relative risk function cannot distinguish between observed and predicted points).

The spatial validation (cont)