AE6102 Course Project - Hyperplane Arrangement Visualization Sandbox

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Terminology

1. Ambient Space - (Here) Any n-dimensional Euclidean Space, i.e $\mathbb{R}^n$
2. Hyperplane - A hyperplane in an n-dimensional Euclidean Space, i.e $\mathbb{R}^n$ is any $n-1$-dimensional subspace of it, which can also be an affine subspace (that is, all the points in the subspace may be translated by a fixed distance in any direction)
3. Hyperplane Arrangement - A set of hyperplanes(may be empty) in a given ambient space is called as an hyperplane arrangement.
4. Central Face - The subset of ambient space obtained by intersection of all the hyperplanes in an hyperplane arrangement is its central face.
5. Rank of an Hyperplane Arrangement - The rank of an hyperplane arrangement is the difference of dimension of central face and dimension of ambient face.
6. Linear Hyperplane Arrangement - A hyperplane arrangement is saif to be linear if all the hyperplanes pass through the origin of ambient space.
7. Reflection Arrangements - A hyperplane arrangement is said to be a reflection arrangement, when if we take reflection in a single hyperplane, we get the same hyperplane arrangement, that is taking reflections doesn't destroy the hyperplane arrangement.
8. Coxeter Cell Complex of a reflection arrangement - A coxeter complex is obtained by first quotienting out the ambient space by the central face, and then taking the intersection of the quotient by a unit sphere of appropriate dimension.

Abstract

Hyperplane Arrangements is one of the branches of mathematics which has a visual element attached to it in the form of its simple geometric constructions but has tremendous use in the fields of combinatorics. Recently, Hyperplane Arrangements (specifically those arrangements which are said to be reflection arrangements) have seen tremendous use in Coxeter Theory and Hopf Algebras, and has contributed to some rich results in these fields. The really interesting and exciting part about these results are the fact that, all these results can be viewed, explained and understood through the visual geometry of hyperplane arrangements. Thus, when it comes to understanding such complicated theory, the visualization really plays a crucial role. Even though hyperplane arrangements can be made in an ambient space which can have any number of dimensions, almost all the theory that has been developed can be explained while dealing with our usual 3-dimensional space. Not only that, even hyperplane arrangements of ranks one and two (with ambient space as the usual flat plane, i.e $\mathbb{R}^2$) serve as rich examples. Thus having an visual environment to play with these hyperplane arrangements can be extremely useful to anyone who wants to study the rich theory it comes with.

Outline and Deliverables

The goal of my project is to create a visualization sandbox for hyperplane arrangements which will be made with the help of Mayavi and TraitsUI, where the ranks of arrangement is less than or equal to 3.

To avoid complexity, we will assume that all hyperplane arrangements are linear.

There will be UI elements where the user can set the dimension of ambient space($1, 2$ or 3), number of hyperplanes, and set the hyperplanes by giving their equations(setting the coefficients of the variables x, y, z of a hyperplane's equation) as well.

Since Coxeter Theory involves reflection arrangements, there will be a special toggle for having reflection arrangements - for rank two, reflection arrangements can be specified by the number of hyperplanes only, for rank three, this is complex, but in general, we will only consider reflection arrangements which are of the following four types (here, we are considering any $\mathbb{R}^n$)

Reflection arrangement	Hyperplanes
Coordinate arrangement	$x_i = 0$, for $1 \leq i \leq n$
Braid arrangement or type A	$x_i = x_j$, for $1 \leq i, j \leq n$
Arrangement of type B	$x_i = \pm x_j,\ x_i = 0$, for $1 \leq i, j \leq n$
Arrangement of type D	$x_i = \pm x_j$, for $1 \leq i, j \leq n$

If time permits -

Aditionally, for reflection arrangements - the option to view its Coxeter Cell Complex will be made available

The final submission will be a github repository which will house the script for this application, along with any necessary support files. There will also be a video explaining how to use the application, and its demo.

Timeline

1. First write the code for the background part, the part where the planes will be drawn based on user-input, based on the dimension of the ambient space, no reflection arrangements yet.
2. Once that is done, a basic UI will be created for this visualisation, using TraitsUI.
3. After this, background work will be done for reflection arrangements, starting with rank two, and then rank three.
4. This will be followed by the development of UI for the above backend work.

References

MA 862 Course Material