Computational Combinatorial Computations

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1 q-Zeta Composition Series Computation

Composition U	Sage Code
(a, b)	$\zeta_{q,(a,b)}$ =1/(q^(a*b) + 1)
(a, b, c)	$\zeta_{q,(a,b,c)}$ =(q^(a*b + b*c) - 1)/((q^(a*b + a*c + b*c) - 1)*(q^(a*b) + 1)*(q^(b*c) + 1))
(a, b, c, d)	$\zeta_{q,(a,b,c,d)}$ =(q^(2*a*b + a*c + 3*b*c + b*d + 2*c*d) - q^(2*a*b + a*c + 2*b*c + b*d + c*d) - q^(a*b + a*c + 2*b*c + b*d + 2*c*d) - q^(a*b + a*c + 2*b*c + b*d + c*d) + q^(a*b + a*c + 2*b*c + c*d) + q^(a*b + a*c + b*c + c*d) + q^(a*b + 2*b*c + b*d + c*d) + q^(a*b + b*c + b*d + c*d) - q^(a*b + b*c + c*d) - q^(a*b + b*c) - q^(b*c + c*d) + 1)/((q^(a*b + a*c + b*c + a*d + b*d + c*d) + 1)*(q^(a*b + a*c + b*c) - 1)*(q^(a*b) + 1)*(q^(b*c + b*d + c*d) - 1)*(q^(b*c) + 1)*(q^(c*d) + 1))

The number of terms in the rational function increases very quickly, hence for the sake of the document, I have mentioned the q-Zeta terms for compositions of degrees 2, 3, and 4. For degree 1 composition, the q-Zeta becomes the constant function with value 1.

The q-Zeta terms above were obtained using the **Zaslavsky Forumla** for q-Zeta Series and the calculation for degree 2 and degree 3 compositions has been independently verified by me and Prof. Swapneel Mahajan.

The computational approach relies on using a recursive function, which terminates with q-Zeta for degree 1 compositions, which is identically 1. I have created a function that computes the distance between a composition U and its opposite composition $\bar{U}$. Rest of the elements in this algorithm rely on SAGE Math's built-in support for compositions, under the 'Combinatorics' library.

2 q-Möbius Composition Series Computation

Computation for q-Möbius was done using a similar approach, but using the corresponding Zaslavsky Formula.

Composition U	Sage Code
(a, b, c, d)	$\mu_{q,(a,b,c,d)}$ =-(q^(3*a*b + 5*a*c + 3*b*c + 6*a*d + 5*b*d + 3*c*d) - q^(3*a*b + 4*a*c + 2*b*c + 4*a*d + 3*b*d + 2*c*d) - q^(2*a*b + 4*a*c + 3*b*c + 4*a*d + 4*b*d + 2*c*d) + q^(2*a*b + 4*a*c + 2*b*c + 4*a*d + 3*b*d + 2*c*d) - q^(2*a*b + 3*a*c + 2*b*c + 4*a*d + 4*b*d + 3*c*d) + q^(2*a*b + 3*a*c + 2*b*c + 4*a*d + 4*b*d + 2*c*d) + q^(2*a*b + 3*a*c + 2*b*c + 4*a*d + 3*b*d + 2*c*d) - q^(2*a*b + 3*a*c + 2*b*c + 4*a*d + 3*b*d + c*d) - 2*q^(2*a*b + 3*a*c + 2*b*c + 3*a*d + 3*b*d + 2*c*d) + q^(2*a*b + 3*a*c + 2*b*c + 3*a*d + 3*b*d + c*d) + q^(2*a*b + 3*a*c + 2*b*c + 3*a*d + 2*b*d + c*d) - q^(2*a*b + 3*a*c + b*c + 4*a*d + 3*b*d + 2*c*d) + q^(2*a*b + 3*a*c + b*c + 3*a*d + 2*b*d + 2*c*d) - q^(2*a*b + 3*a*c + b*c + 3*a*d + 2*b*d + c*d) + q^(2*a*b + 2*a*c + b*c + 3*a*d + 3*b*d + 2*c*d) - q^(2*a*b + 2*a*c + b*c + 3*a*d + 2*b*d + c*d) + q^(2*a*b + 2*a*c + b*c + 2*a*d + 2*b*d + c*d) - q^(a*b + 3*a*c + 2*b*c + 4*a*d + 3*b*d + 2*c*d) + q^(a*b + 3*a*c + 2*b*c + 3*a*d + 3*b*d + 2*c*d) - q^(a*b + 3*a*c + 2*b*c + 3*a*d + 2*b*d + c*d) + q^(a*b + 2*a*c + 2*b*c + 3*a*d + 3*b*d + 2*c*d) - q^(a*b + 2*a*c + 2*b*c + 3*a*d + 3*b*d + c*d) + q^(a*b + 2*a*c + 2*b*c + 2*a*d + 2*b*d + c*d) - q^(a*b + 2*a*c + b*c + 3*a*d + 3*b*d + 2*c*d) - q^(a*b + 2*a*c + b*c + 3*a*d + 2*b*d + 2*c*d) + 2*q^(a*b + 2*a*c + b*c + 3*a*d + 2*b*d + c*d) + q^(a*b + 2*a*c + b*c + 2*a*d + 2*b*d + 2*c*d) - q^(a*b + 2*a*c + b*c + 2*a*d + 2*b*d + c*d) - q^(a*b + 2*a*c + b*c + 2*a*d + b*d + c*d) + q^(a*b + 2*a*c + b*c + 2*a*d + b*d) - q^(a*b + a*c + b*c + 2*a*d + 2*b*d + c*d) + q^(a*b + a*c + 2*a*d + b*d + c*d) + q^(a*c + b*c + 2*a*d + 2*b*d + c*d) - 1)/((q^(a*b + a*c + b*c + a*d + b*d + c*d) + 1)*(q^(a*b + a*c + b*c + a*d + b*d) - 1)*(q^(a*b + a*c + a*d + b*d + c*d) - 1)*(q^(a*b + a*c + a*d) + 1)*(q^(a*c + b*c + a*d + b*d + c*d) - 1)*(q^(a*c + b*c + a*d + b*d) + 1)*(q^(a*d + b*d + c*d) + 1))
(a, b)	$\mu_{q,(a,b)}$ =-1/(q^(a*b) + 1)
(a, b, c)	$\mu_{q,(a,b,c)}$ =(q^(a*b + 2*a*c + b*c) - 1)/((q^(a*b + a*c + b*c) - 1)*(q^(a*b + a*c) + 1)*(q^(a*c + b*c) + 1))