Documentation to supplement the HPS demo codes

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The HPS demo code Git repo includes 1 and 2-D demo codes. The codes are not meant to be a library but instead to make the use of HPS for research and educational purposes accessible with little overhead for the user. An effort has been made to make the codes consistent so that modifications are easy for users.

1 Subroutines that the user may want to modify

The codes are designed to solve problems of the form

- $\alpha y'' + k^2 b(x) = f(x)$ with Dirichlet boundary conditions in 1D where α and k are constants, and
- $\Delta u + k^2(1 - b(x))u = f(x)$ with a collection of boundary conditions in 2D.

The following names of subroutines are consistent throughout all of the codes.

- `create_source` evaluates the body load function $f(x)$.
- `create_bdy_data` creates boundary data that gets fed into the solver.
- `upward_sweep` builds particular solution operators and **only needs to be computed once per $f(x)$.**
- `downward_sweep` constructs the solution utilizing the precomputed particular solution operators and the boundary data. This needs to be called once per boundary condition.
- `process_leaf` constructs the local spectral operators on each leaf. If you want to change the differential operator approximated, you need to change the matrix $\boldsymbol{A}$ accordingly.

2 1D codes

There are two codes provided for 1D problems: Both codes solve a boundary value problem with a known solution u_{ex}. `main_1D_HPS_converge.m` checks the convergence of the method for a point xp in the geometry Ω. It also numerically illstrates the convergence order of the HPS method as the mesh is uniformly refined.

3 2D codes

For 2D boundary value problems, there are two folders containing codes: Laplace and Helmholtz. The codes in the Laplace folder are for problems that do not have oscillatory solutions; i.e. Laplace like. The codes in the Helmholtz folder are for problems with oscillatory solutions.

3.1 Laplace folder

The following codes are in the Laplace folder

- `Laplace_nobody.m` solves elliptic problems with no body load. This means you only need a downward sweep of the solver and the precomputation does not build any particular solution opertors.
- `Laplace_bodyload.m` solves elliptic problems with body loads. You can use this code for no body load problems but it will be more expensive than the previous code.

The following paper should be referenced if you use any of the codes in the Laplace folder. [4, 5, 2]

3.2 Helmholtz

The following codes are in the Helmholtz folder

- `Helmholtz_nobody.m` solves a homogeneous boundary value problem with Dirichlet boundary data. The downward sweep requires impedance boundary data. This is constructed out of the Dirichlet boundary data utilizing the constructed DtN operator.
- `Helmholtz_free.m` solves the free space Helmholtz problem with local deviations from constant coefficient. The second kind integral equation presented in [3].
- `Helmholtz_DIR.m` solves Helmholtz problems with Dirichlet boundary conditions. You need to replace `udir` with your Dirichlet boundary data.
- Additional codes to come...

For all of these codes the value kh denotes the wavenumber.
The following papers should be cited when using the Helmholtz codes. [3, 5, 3, 1, 2]

References

[1] N. N. Beams, A. Gillman, and R. J. Hewett. A parallel shared-memory implementation of a high-order accurate solution technique for variable coefficient Helmholtz problems. *Computers & Mathematics with Applications*, 79(4):996–1011, 2020.

[2] P. Geldermans and A. Gillman. An adaptive high order direct solution technique for elliptic boundary value problems. *SIAM Journal on Scientific Computing*, 41(1):A292–A315, 2019.

[3] A. Gillman, A. H. Barnett, and P.-G. Martinsson. A spectrally accurate direct solution technique for frequency-domain scattering problems with variable media. *BIT Numerical Mathematics*, 55(1):141–170, 2015.

[4] A. Gillman and P. G. Martinsson. A direct solver with \$o(n)\$ complexity for variable coefficient elliptic PDEs discretized via a high-order composite spectral collocation method. *SIAM Journal on Scientific Computing*, 36(4):A2023–A2046, 2014.

[5] P. Martinsson. A direct solver for variable coefficient elliptic PDEs discretized via a composite spectral collocation method. *Journal of Computational Physics*, 242:460–479, 2013.