

# Cascading Failures

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Below, you will find some recommendations for topics to discuss for your presentation on Cascading Failures. This is not an exhaustive list of topics and you are free to alter or depart from this outline depending on how you see fit. After the outline, you will find some arguments that may help in editing your .qmd file.

For the PSC-290 students, please proceed through the headings as if you are instructing a course. You'll be expected to discuss the mathematical formalisms of these different techniques for generating and simulating failure cascade in networks and generating considerations for what types of variable systems follow the different cascade types.

and comment on their estimation. Also, please adjust your .qmd file to be week6\_2.qmd when you submit to me.

## Failures in Graphs?

Well established in this class is the fact that graphs are made up of nodes (components) and edges (links). We have many real-world examples of graphs to take from: the internet, power grids, ecology, physiology, and of course psychological systems.

In these networks often times we see the interplay of processes internal and external to the system. Think of a transmission line malfunctioning in an electrical power grid. Or possibly a perturbation to a system that disrupts communication among nodes. These two examples hint at an important dynamical process termed cascading failure or avalanche (if you're feeling fancy). Although rare, they can have severe consequences. In general, a cascading failure refers to some trigger mechanism that produces further failures in the system which ultimately may lead to a complete collapse of the system (Valdez et al., 2020). Further failures occur because components vary in their ability to handle a certain load that is redistributed to them after an initial failure.

Similarly, in social systems we might think of a cascading failure being dependent on the information other people may have and subsequently their decisions. This is quite common in fads or things that are "hyped-up". We might choose to wear Levi jeans because many of our peers are wearing Levi jeans. We might choose to see one movie over another based on reviews or recommendations by friends. Economists define these scenarios as *binary decisions with externalities*<sup>1</sup>. Important to note is that a personal threshold must be reached before a transition occurs in conjunction with what others have decided. Furthermore, rather than components failing it is a component transitioning from one state to another.

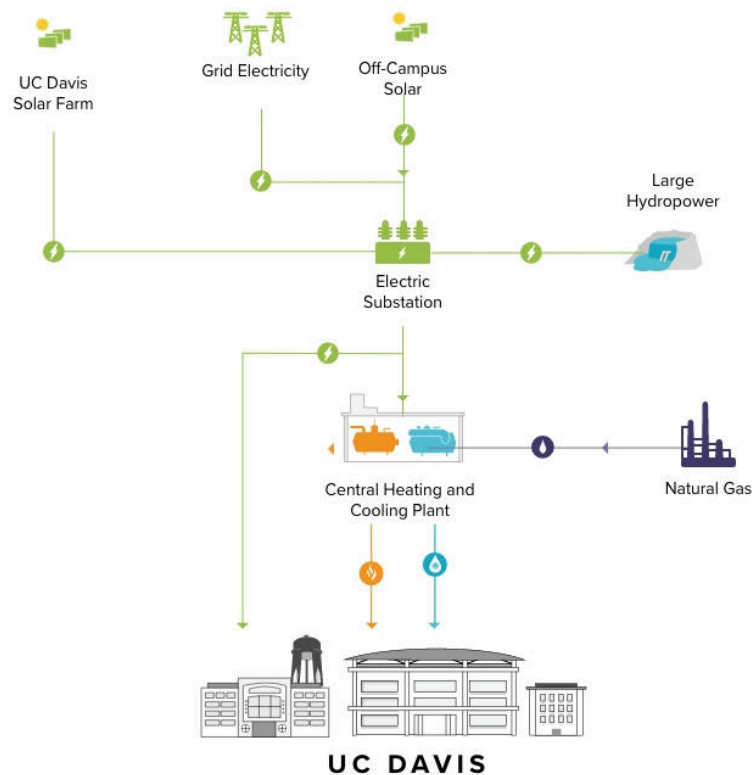
Cascading failures leads us to questions such as:

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<sup>1</sup>A simple model of global cascades on random networks (Watts, 2002)

- How does the structure of a network impact its resilience to failures?
- How can we enhance a system to mitigate further cascading failures?
- Where are the vulnerable components in a system. Are they the high-degree or low-degree nodes?
- When do components transition states (i.e., at a well-defined threshold)?

As an illustrative example:



A group of students are running experiments and simulations in their respective labs. Suddenly a blackout occurs! Welp, there goes their precious research. Let's explain to these poor students that a cascading failure may have occurred.

1. Overloading or zeroing of a component in the electrical power grid. *A group of Davis bikers crash into a substation.*
2. Power is redistributed from the failed component. There is an increase in the loadings of other components.

3. The system is stressed out and loadings are higher than the capacity of components. Leading to a cascading failure.
4. Blackout all around Davis.

## Recap on Graph Structures

- Focus on re-describing the Erdős–Rényi and Barabási–Albert random graphs
  - With Barabási–Albert, talk more in-depth about preferential attachment and the “growth” of graphs
  - E.g., graphs in the real-world add and incorporate vertices over time and the addition of vertices follows some known procedures
  - Define the terms: Preferential attachment and Assortative mixing and how they relate to the evolution of a network’s structure

## Compare and Contrast to these idea of “controlling” a network

## Define Watt’s general form of a Cascading Failure by Thresholds

Define the simple model not the proof. When we meet, you’ll receive code for simulating a cascading failure using Watts’ approach

## The CASCADE model for Cascading Failures

The CASCADE is a probabilistic model for understanding the propagation of cascading failures (Dobson et al., 2007). It was developed to capture the weakening of a system as a cascade proceeds, particularly in the context of blackout of power transmitting systems, but made sufficiently simple and general that it could be applied to cascading failure in various large, interconnected infrastructures. CASCADE focuses on components failing when their load exceeds a threshold, how an initial disturbance loads the system, and how the failure of some components transfers additional load to others.

### Formal definition of the CASCADE model:

The model considers a system with  $N$  components, each with a random initial load.

#### 1. Initial load ( $L_j$ ):

- Each component  $j$  (for  $j = 1, 2, \dots, N$ ) has an initial load  $L_j$ .
- The minimum value of a given load is  $L^{\min}$  and the maximum load is  $L^{\max}$ .
- $L_j$  is distributed uniformly between  $[L^{\min}, L^{\max}]$ .
- The initial loads  $L_1, L_2, \dots, L_N$  are independent of each other.

#### 2. Failure threshold ( $L^{\text{fail}}$ ):

- Components fail if their total load exceeds a certain failure threshold,  $L^{\text{fail}}$

### 3. Load transfer ( $P$ ):

- When a component fails, a fixed amount of additional load,  $P$ , is transferred to each of the other remaining components in the system.

### 4. Initial disturbance ( $D$ ):

- The cascade is initiated by an initial disturbance that adds an amount of load  $D$  to each component.

### 5. Normalized initial load ( $\ell_j$ ):

- The normalized initial load for the  $j^{th}$  vertex is given by  $\ell_j$  where:

$$\ell_j = \frac{L_j - L^{\min}}{L^{\max} - L^{\min}}$$

- This normalization maps the load  $L_j$  to a value  $\ell_j$  that is uniformly distributed on the interval  $[0, 1]$ .

### 6. Normalized parameters:

- The load transfer  $P$  and initial disturbance  $D$  are also normalized. Let  $p$  be the normalized load increment and  $d$  be the normalized initial disturbance:

$$p = \frac{P}{L^{\max} - L^{\min}}$$
$$d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}}$$

### 7. Cascading process:

- After the initial disturbance  $D$ , components whose total load ( $L_j + D$ ) exceeds  $L^{\text{fail}}$  will fail.
- The failure of these components then distributes an additional load  $P$  (or  $p$  in normalized terms) to other components, potentially causing further failures. This process continues iteratively, defining the cascade.

The Watt's description is more simplistic, vertices have no thresholds internal to themselves but are influenced entirely by neighbors. In the CASCADE model, vertices have resilience and an initial load such that they can exhibit more nuanced dynamics in how they fail

## Watt's Cascades vs. CASCADE Model

A brief distinction between the two models is that the Watt's description is more simplistic, vertices have no thresholds internal to themselves but are influenced entirely by neighbors. In the CASCADE model, vertices have resilience and an initial load such that they can exhibit more nuanced dynamics in how they fail. Let's break down their differences.

### Thresholds and Failure

- **Watt’s model:** Failure is determined by the fraction of a node’s *neighbors* that have already failed. A node  $i$  with degree  $k_i$  and threshold  $\phi_i$  fails if at least a threshold fraction  $\phi$  out of its  $k$  neighbors are in state 1 (fail state), otherwise it adopts state 0. The threshold  $\phi_i$  represents the node’s susceptibility to peer influence or stress from neighbors. Vertices do not have an internal “load” capacity in the same way as the CASCADE model; their failure is purely a response to the state of their local neighborhood.
- **CASCADE:** Failure is determined by a node’s internal load exceeding its predefined capacity ( $L^{\text{fail}}$ ). Nodes have an initial load  $L_j$ , and this load increases due to an initial system-wide disturbance  $D$  and subsequent load transfers  $P$  from other failed components. The threshold is an absolute load limit, not directly dependent on the number or state of its neighbors in the topological sense.

### Influence mechanism

- **Watt’s model:** Influence is local and topological. A node is influenced directly and only by the state (failed or not) of its immediate neighbors in the graph. What matters is the structure of the network (who is connected to whom).
- **CASCADE:** Influence is through load redistribution. When a node fails, it transfers a fixed load  $P$  to other components. While the underlying network structure can be implicitly present in how loads might realistically be distributed, the primary mechanism is the global or semi-global impact of load addition.

### Vertex properties

- **Watt’s model:** Vertices are primarily characterized by their degree  $k$  and their individual threshold  $\phi$ . There isn’t an explicit concept of “resilience” beyond the threshold itself.
- **CASCADE:** Vertices are characterized by their initial load  $L_j$  and a common failure load  $L^{\text{fail}}$ . The “resilience” of a node is the difference between its current load and  $L^{\text{fail}}$ . Nodes can have different initial loads, making some inherently closer to failure than others even before the cascade begins. This allows for more nuanced dynamics in how they fail, as a node with a high initial load is more vulnerable.

### Complexity of Dynamics

- **Watt’s model:** The dynamics are driven by changes in neighbor states. The “vulnerable” nodes (low threshold relative to degree) play a crucial role in initiating widespread cascades.
  - In the *low connectivity* regime, the network is sparse, and cascade propagation is mainly limited by connectivity. At the critical point, cascade sizes follow a power law.
  - In the *high connectivity* regime the network is dense. The propagation of cascades is limited by the stability of individual nodes. Most nodes have many neighbors, making them less susceptible to single-node influence. This results in a bimodal distribution of cascade sizes: many small cascades that die out quickly and, rarely, extremely large global cascades.
- **CASCADE:** The system dynamics are driven by accumulating load, with overall stress being a key factor. This stress depends on how close the initial loads are to the failure threshold ( $L^{\text{fail}}$ ) and the magnitudes of  $D$  and  $P$ . It examines the probability distribution of failed components

and how this changes as system parameters, such as overall loading or load transfer ( $P$ ), approach a critical point.

In summary, Watts' model is more focused on the contagion of states through local, fractional influence within a network, where nodes don't have intrinsic load capacities but rather susceptibility thresholds. The CASCADE model, conversely, emphasizes the accumulation of load on components that have defined capacities, where failure is a consequence of exceeding this capacity, leading to further load redistribution.

Given these differences, which cascading failure model would be more appropriate to your research area?

### Can We Know the Thresholds for Vertices?

In the CASCADE model, the threshold is essentially defined by a node's capacity and its load. In Watts' model, the threshold ( $\phi$ ) for a node determines the fraction of its neighbors that must be "on" for it to switch "on."

In **engineered/physical systems**, the thresholds can often be estimated or are known from design specifications. For example, a power line has a maximum current capacity, a router has a maximum data throughput, and a bridge has a maximum weight limit. These thresholds are based on physical properties and engineering design tolerances. However, even in these systems, thresholds can be dynamic, affected by environmental conditions (e.g., temperature affecting power line sag and capacity), aging, maintenance levels, or even "hidden failures" where a component's actual capacity is less than assumed.

In **social/economical/biological systems** a threshold could relate to the point at which an individual or entity changes their behavior, opinion, or state due to influence from others or accumulated stress. In Watts' model, this is the fraction of neighbors adopting a behavior that triggers an individual to also adopt it. For example, in decision making people could assume others have better information and follow their actions. It could also be the desire to align with the perceived majority or norm, or to be influenced by trusted sources. A "failure" can be illustrated by a PhD student who becomes overwhelmed and unable to progress with their research. In this scenario, the threshold could be influenced by factors such as the volume of TA responsibilities, the complexity of the research, and the level of support received from their advisor.

- What is a "threshold" for psychological vertices?

An example of insomnia development. Let's consider insomnia as a complex system where various psychological and behavioral factors interact, much like nodes in a network. We can conceptualize the transition into a state of chronic insomnia, or the worsening of its symptoms, as a form of cascading failure within an individual's "sleep regulation network." Let's build our example using the popular **3P model** (Spielman et al., 1987):

- **Predisposing Factors:** These are like the initial conditions or inherent vulnerabilities of certain nodes in our network. Think of genetic propensity, a tendency towards ruminative thoughts, or a generally anxious disposition. These factors don't cause insomnia on their own but lower the

“failure threshold” of certain psychological states. For example, a “ruminative thought” node might have a naturally lower threshold to become “active” (i.e., start interfering with sleep).

- **Precipitating Factors:** These are the initial “shocks” or “failures” in the system. A stressful life event, a temporary illness, or a change in routine could be a precipitating factor that “activates” or “causes the failure” of an initial set of nodes – perhaps leading to a few nights of poor sleep. In our cascade model, this is the initial  $\phi_0$  of “failed” nodes.
- **Perpetuating Factors:** These are crucial for the “cascade.” These are behaviors and beliefs adopted to cope with initial sleep disturbance but end up reinforcing and exacerbating it.
  - Take, for example, the node “Excessive Time in Bed”: An individual experiences a few nights of poor sleep (precipitating factor). To compensate, they start spending excessive time in bed (a perpetuating behavior). This “Excessive Time in Bed” node now becomes “active.”
  - **Neighboring Nodes & Thresholds:**
    - This “Excessive Time in Bed” node influences other nodes, like “Sleep Fragmentation” and “Weakened Sleep-Wake Cycle.”
    - The “Sleep Fragmentation” node might have a threshold: if “Excessive Time in Bed” is active AND, say, “Anxiety about Sleep” (another node, perhaps activated by the precipitating factor) is also active, its threshold is met, and it “fails” (sleep becomes more fragmented).
    - The “Weakened Sleep-Wake Cycle” node might fail if “Excessive Time in Bed” stays active for a certain duration or intensity.

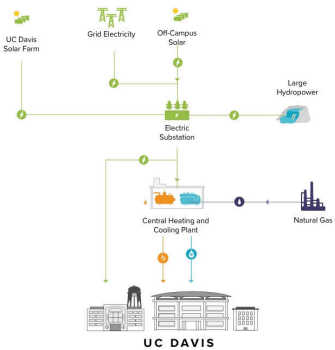
## Weave in Graph Structures

- Do you expect Erdős–Rényi and Barabási–Albert random graphs to exhibit different properties in their susceptibility to cascading failures? Why?
- For Barabási–Albert random graphs, do you expect the property of assortativity to come into play?
  - The fact is, when real-world networks develop, they build in structural stability to be resilient to failure. Assortativity can be a tool for this. Explain.

## Discussion

### Header

Adding a figure can be done via:



When running this locally, change the pathing but when you submit to me, send any images and change paths to “images/[figurename].jpeg”

For hyperlinks to websites, you can use text goes here

## Subheading

To subset an equation on a separate line you can use:

$$y = mx + b$$

To reference equation components in-line, you simply use  $y$  as a single value without breaking the lines.

If you want to create a list, you can simply write:

- List element 1
- List element 2
  - Sublist element 1
- List element 3

Some common symbols for graphs in  $\LaTeX$  format can be found at detextify’s website online. Get a matrix given by:

$$A_{p \times p} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

You might want to italicize or emphasize text by *adding asterisks*. You can also **bold and emphasize text**. R chunks may be added by:

Hidden Message. You can add hidden messages that only appear once clicked on with this surrounding code.

You can also indent text by using the greater than symbol to begin a sentence



## Disclaimer

MC used AI to help with grammar and to generate ideas.

## References

## Bibliography

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