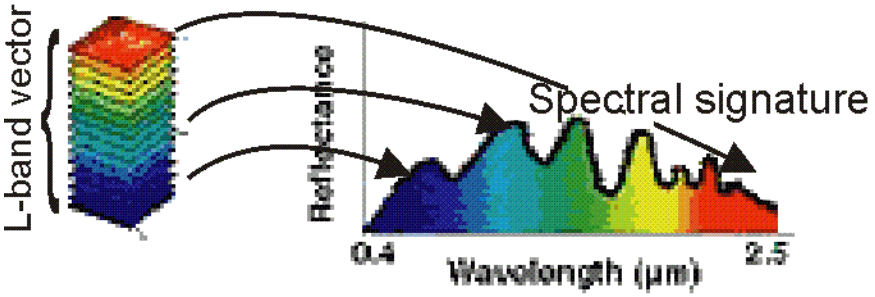
"Machine Learning and Computational Statistics"

Project

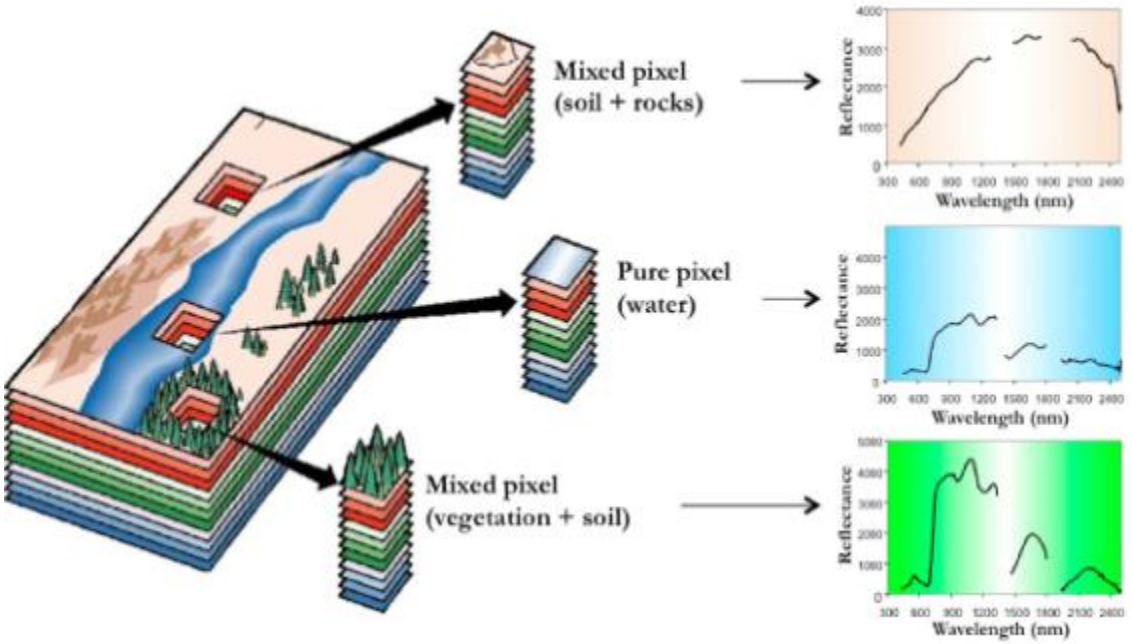
Introduction

This project is devoted to Hyperspectral Images (HSI) processing.

Hyperspectral images (HSIs): An HSI depicts a specific scene at several (L) narrow continuous spectral bands (actually, they visualize the reflectance of the depicted scene in various spectral bands). It can be represented by a MxNxL three-dimensional cube, where the first two dimensions correspond to the spatial information, while the third corresponds to the spectral information. Thus, the (i,j) pixel in such an image, $i=1,\ldots,M$, $j=1,\ldots,N$, is represented by an L-dimensional vector (constituted by the corresponding spectral bands), called the ***spectral signature*** of the pixel.



In several remote sensing applications, the HSIs (taken from satellites) that depict specific scenes of the earth surface at a specific spatial resolution (that is, a single pixel may represent an area from 3x3 m^2, to, e.g., 100x100m^2 or more). That is, each pixel is likely to depict more than one materials depicted in the corresponding area of the scene. Such pixels are called ***mixed pixels*** and they are the vast majority of the pixels in the image. On the other hand, there are (usually) a few pixels that depict a single material. These are called ***pure pixels***.



Processing in HSIs: The usual processing procedures in HSIs follow two main directions, namely, the ***spectral unmixing*** and the ***classification*** (supervised, unsupervised).

(a) **Spectral unmixing (SU):** The problem here is stated as follows: Assume that a set of m spectral signatures corresponding to the pure pixels in the HSI under study is given[1]. For a given pixel in the image, the aim is to determine the percentage (***abundance***) to which each pure material contributes in its formation. It is clear, that SU provides ***sub-pixel information*** for a given pixel. Speaking in mathematical terms, let

(i) y be the (column L-dimensional) spectral signature of the pixel under study,

(ii) $x_1,\ldots x_m$, be the spectral signatures (column L-dimensional vectors) of the pure pixels in the image (each one corresponding to a pure material met in the image) and

(iii) θ, the m-dimensional ***abundance vector*** of the pixel (its q-th coordinate corresponds to the percentage to which the q-th pure pixel contributes to the formation of the pixel under study).

Adopting the ***linear spectral unmixing hypothesis***, the above quantities are related as follows

$$y = X\theta + \eta,$$

[1] Actually, in most real cases, these pure pixels are selected from the image via a procedure called ***Endmember Extraction***.

where η is an L-dimensional i.i.d., zero mean Gaussian noise vector. Note that, physically, the entries of θ should be nonnegative and (ideally) they should sum to one.

(b) **(Supervised) classification:** In this case, the problem is stated as follows: Assume that all the pixels in the HSI under study are known to belong to one out of m ***known classes***. Given a specific pixel, the aim is to determine the most suitable class to assign it.

All questions in this project refer to the so called "Salinas" HSI, which depicts an area of the Salinas valley in California, USA. It is a 220x120 spatial resolution HSI and consists of 204 spectral bands (from 0.2μm – 2.4μm) and its spatial resolution is 3.7m (that is, the HSI is a 220x120x204 cube). The data that will be used are in the files "***Salinas_cube.mat***" (the Salinas hypercube) and "***Salinas_gt.mat***" (the class label for each pixel).

NOTES:

1. You will need first to run the attached python code to retrieve the quantities mentioned below.
2. Only the **pixels** with **nonzero class label** will be **taken into consideration** in this project.

Description of the project

Part 1 (spectral unmixing)

The "**Salinas**" HSI includes **7** endmembers, each one corresponding to a certain material (cultivation in our case), as described in the following table:

Endmember	Material
1	Grapes
2	Broccoli
3	Fallow 1
4	Fallow 2
5	Fallow 3
6	Stubble
7	Celery

The aim here is to perform **unmixing** on **each one** of the **pixels** in the image **with nonzero label**, with respect to the **7** endmembers (obtained after the execution of the attached python code) using the following five different spectral unmixing methods:

(a) Least squares (as it was presented in the class),
(b) Least squares imposing the sum-to-one constraint
(c) Least squares imposing the non-negativity constraint on the entries of $\boldsymbol{\theta}$
(d) Least squares imposing both the non-negativity and the sum-to-one constraint on the entries of $\boldsymbol{\theta}$**.**
(e) LASSO, i.e., impose sparsity on $\boldsymbol{\theta}$ via l_1 norm minimization.

** You may use either sklearn, scipy.optimize or the solvers.qp() function from the CVXOPT package, which should be downloaded.

(A) For **each method**:
  (i) Derive the corresponding **7** abundance maps (one for each endmember/material)
  (ii) Compute the **reconstruction error** (for each (**non-zero class label**) pixel $\boldsymbol{y}_i$ compute the quantity $||\boldsymbol{y}_i - X\boldsymbol{\theta}_i||^2$ and then take the average over those pixels).

(B) Compare the results obtained from the above five methods (focusing on the abundance maps and the reconstruction error) and comment briefly on them (utilize the class information given in "***Salinas_gt.mat***").

Part 2 (classification)

In this case, we **consider** also the image **pixels** with **non-zero class label**. The task is to assign each one of them to the most appropriate class, among the ***7 known classes***. To this end four classifiers will be used: (i) the naïve Bayes classifier, (ii) the minimum Euclidean distance classifier, (iii) the k-nearest neighbor classifier and (iv) the Bayesian classifier. For each one of them the ***same training***, ***test*** and ***operation sets*** will be used (obtained after the execution of the attached python code). Note that, **in practice**, we have at our disposal a set of points, which we split into the training and the test set. The operational set mentioned above corresponds to data presented to the classifiers after their training and evaluation, that is, during their ***operational mode***.

(A) **For each classifier**:
  (i) Train it based on the training set performing 10-fold cross validation (that is, the training procedure will be repeated 10 times). Report the estimated validation error as the mean of the ten resulting error values (compute also the associated standard deviation).
  (ii) After that, use the whole training set to train the classifier and evaluate their performance on the test set as follows: First, **compute** the **confusion matrix** (a 7x7

matrix, whose (*i*,*j*) element is the number of pixels that belong to the *i*-class and are assigned from the classifier to the *j*-th class. Clearly, the "more diagonal" the matrix, the better the performance of the classifier is) and identify the classes that are not well separated by the classifier (if any). Then, **compute** the **success rate** of the classifier (sum of the diagonal elements of the confusion matrix and divide by the sum of all elements of the matrix).

(B) Compare the results of the classifiers and comment on them (Relate the confusion matrices obtained from each classifier to each other and pay attention to non-diagonal entries that are "not nearly zero". For example, a non-diagonal entry equal to 1 does not carry any significant information).

NOTE: You can use any ready-to-use python implementations of the above classifiers, except the minimum distance classifier, which should be implemented by you. For the cross-validation, you can also use relevant products, ready-for-use (e.g. function *cross_val_score* from sklearn.model_selection tool).

Part 3 (combination)

Comment briefly on the possible correlation of the results obtained from the spectral unmixing procedure with those obtained from classification.