## Harmonic Mean

The harmonic mean (HM) of a set of observations is defined as the reciprocal of the arithmetic mean of the reciprocals of the observations.

***Definition for a raw data***

If $x_1, x_2, x_3, ..., x_n$ are 'n' observations

$$HM = \frac{1}{\left(\frac{1}{x_1} + \frac{1}{x_2} + .. + \frac{1}{x_n}\right)/n} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + .. + \frac{1}{x_n}} = \frac{n}{\sum\left(\frac{1}{x}\right)}$$

***Definition for a frequency data***

If $x_1, x_2, x_3, ..., x_n$ are 'n' observations with the corresponding frequencies $f_1, f_2, f_3, ..., f_n$

$$\text{then HM} = \frac{1}{f_1 \times \frac{1}{x_1} + f_2 \times \frac{N}{x_2} + .. + f_n \times \frac{1}{x_n}} = \frac{N}{\sum\left(\frac{1}{x}\right)}$$

where $N = \sum f$

**Note 1** HM can be calculated only for non zero and non negative values.

**Note 2** HM is appropriate for finding average speed when distance travelled at different speeds are equal. Weighted HM is appropriate when the distances are unequal. HM is suitable to study rates also.

**Note 3** Weighted HM = $\frac{N}{\sum\left(\frac{w}{x}\right)}$ where w's are the weighted assigned.

**Example 11**

Calculate the HM of 2, 3, 4, 5 and 7



**Solution**

$$HM = \frac{n}{\sum\frac{1}{x}} = \frac{5}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{7}}$$

$$= \frac{5}{\frac{210+140+105+84+60}{420}} = \frac{5 \times 420}{599} = \mathbf{3.50}$$

**Example 12**

Calculate HM of 5, 11, 12,16, 7, 9, 15, 13, 10 and 8

**Solution**

| X | 1/x | X | 1/x |
|---|---|---|---|
| 5 | 0.2000 | 9 | 0.1111 |
| 11 | 0.0909 | 15 | 0.0667 |
| 12 | 0.0833 | 13 | 0.0769 |
| 16 | 0.0625 | 10 | 0.1000 |
| 7 | 0.1429 | 8 | 0.1250 |
| | | Total | 1.0593 |

$$HM = \frac{n}{\sum\left(\frac{1}{x}\right)} = (10/1.0593) = 9.44$$

## Merits and Demerits

### Merits

It is rigidly defined. It has clear cut mathematical formula.

It is based on all the items. The magnitude of every item is considered for its computation.

It is affected less by extreme items than A.M. or even G.M.

It gives lesser weight to larger items and greater weight to lesser items.

It can be algebraically manipulated. The H.M. of the combined set can be calculated from the H.M.s and sizes of the sets. For example,

$$HM_{12} = \frac{N_1 + N_2}{\frac{N_1}{HM_1} + \frac{N_2}{HM_2}}$$

It is suitable to find the average speed.

**Demerits**

It is neither simple to understand nor easy to calculate.

It has less sampling stability than the A.M.

Theoretically, it cannot be calculated for open-end data.

It cannot be found graphically.

It is not defined for qualities. It is not calculated when atleast one item or one mid value is zero or negative.

It gives undue weightage to small items and least weightage to largest items. It is not used for analysing business or economic data.

## Median

Median is defined as the middle most observation when the observations are arranged in ascending or descending order of magnitude. That means the number of observations preceding median will be equal to the number of observations succeeding it. Median is denoted by M.

***Definition for a raw data***

For a raw data if there are odd number of observations, there will be only one middle value and it will be the median. That means, if there are n observations arranged in order of their magnitude, the size of (n+1)/2 th observation will be the median. If there are even number of observations the average of two middle values will be *th*the median. That means, median will be the average of $n/2^{th}$ and $\left(\frac{n}{2}+1\right)$ observations.

***Definition for a frequency data***

For a frequency distribution median is defined as the value of the variable



which divides the distribution into two equal parts. The median can be calculated using the following formula.

$$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$$

where, $l$ - lower limit of median class

Median class - the class in which $N/2^{lh}$ observation falls

N - total frequency

m - cumulative frequency up to median class

c - class interval of the median class

f - frequency of median class

found to lie with in that interval.

**Example 13**

Find the median height from the following heights (in cms.) of 9 soldiers.
160, 180, 175, 179, 164, 178, 171, 164, 176

**Solution**

Step 1. Heights are arranged in ascending order:
160, 164, 164, 171, 175, 176, 178, 179, 180.

Step 2. Position of median = $\frac{n+1}{2}$ is calculated. It is $\frac{9+1}{2} = 5$.

Step 3. Median is identified ($5^{th}$ value) M = **175cms.**

It is to be noted that $\frac{+1}{2}$ may be a fraction, in which case, median is found as follows.

**Example 14**

Find the median weight from the following weights (in Kgs) of 10 soldiers. 75, 71, 73, 70, 74, 80, 85, 81, 86, 79

**Solution**

Step 1. Weights are arranged in ascending order:

70, 71, 73, 74, 75, 79, 80, 81, 85, 86

Step 2. Position of median $\frac{n+1}{2} = \frac{10+1}{2} = 5\frac{1}{2}$ is calculated

Step 3. Median is found. It is the mean of the values at $5^{th}$ and $6^{th}$ positions and so M = $\frac{75+79}{2}$ = **77Kgs.**

**Example 15**

Find the median for the following data.

| Height in cms | : | 160 | 164 | 170 | 173 | 178 | 180 | 182 |
|---|---|---|---|---|---|---|---|---|
| No. of soldiers | : | 1 | 2 | 10 | 22 | 19 | 14 | 2 |

**Solution**

Step 1. Heights are arranged in ascending order. Cumulative frequencies (c.f) are found. (They help to know the values at different positions)

| Height in cms. | No. of Soldiers | C.f. |
|---|---|---|
| 160 | 1 | 1 |
| 164 | 2 | 3 |
| 170 | 10 | 13 |
| 173 | 22 | 35 |
| 178 | 19 | 54 |
| 180 | 14 | 68 |
| 182 | 2 | 70 |
| Total | 70 | |



Step 2. Position of median, $\frac{N+1}{2} = \frac{70+1}{2} = 35\frac{1}{2}$ is calculated.

Step 3. Median is identified as the average of the values at the positions 35 and 36. The values are 173 and 178 respectively.

$$\therefore M = \frac{173+178}{2} = \mathbf{175.5cm}$$

**Example 16**

Calculate median for the following data

| Class | : | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 |
|---|---|---|---|---|---|---|
| f | : | 5 | 10 | 15 | 12 | 8 |

**Solution**

| Class | f | CF |
|---|---|---|
| 0-5 | 5 | 5 |
| 5-10 | 10 | 15 |
| 10-15 | 15 | 30 |
| 15-20 | 12 | 42 |
| 20-25 | 8 | 50 |
| Total | 50 | |

$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c$ Median class is 10-15

Here $l = 10, N/2 = 50/2 = 25, c = 5, m = 15, f = 15$

$$\therefore M = 10 + \frac{(25-15)5}{15}$$

$$= 10 + \frac{10 \times 5}{15} = 10 + \frac{10}{3} = 10+3.33 = \mathbf{13.33}$$

**Example 17**

Calculate median for the data given below.

| Classes | : | 0-6 | 7-13 | 14-20 | 21-27 | 28-34 | 35-41 |
|---|---|---|---|---|---|---|---|
| f | : | 8 | 17 | 28 | 15 | 9 | 3 |

**Solution:**

| Class | f | Actual class | CF |
|---|---|---|---|
| 0-6 | 8 | 0.5-6.5 | 8 |
| 7-13 | 17 | 6.5-13.5 | 25 |
| 14-20 | 28 | 13.5-20.5 | 53 |
| 21-27 | 15 | 20.5-27.5 | 68 |
| 28-34 | 9 | 27.5-34.5 | 77 |
| 35-41 | 3 | 34.5-41.5 | 80 |
| Total | 80 | | |

Median class is 13.5-20.5, $l$ = 13.5, N/2 = 80/2 = 40

c = 7, m = 25, f = 28

$$M = l + \frac{\left(\frac{N}{2} - m\right)}{f} \times c = 13.5 + \frac{(40 - 25)}{28} \times 7$$

$$13.5 + \frac{15 \times 7}{28} = 13.5 + \frac{15}{4}$$

13.5+3.75

**17.25**