Measures of dispersion

Dispersion is all about the amount of the spread or scatter in a distribution. Measures of dispersion fall into two categories:

Measures of absolute dispersion

(i) Range
(ii) Quartile deviation
(iii) Mean deviation
(iv) Standard deviation and variance

Measures of relative dispersion

(i) Coefficient of quartile deviation
(ii) Coefficient of mean deviation
(iii) Coefficient of variation

Range

It is simply the difference between the largest and the smallest values in a distribution.

Quartile Deviation

$$QD = \frac{Q3 - Q1}{2} = \frac{68.07 - 41.2}{2} = 13.435$$

Mean Deviation

$$MD = \frac{\sum |X - \overline{X}|}{n} \; for raw data$$

$$MD = \frac{\sum f\,|X - \overline{X}|}{n} \; for frequency table$$

Example

Scores	x	f	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$	$f\lvert x - \bar{x} \rvert$	fx
40-44	42	2	-15	15	30	84
45-49	47	5	-10	10	50	235
50-54	52	8	-5	5	40	416
55-59	57	12	0	0	0	684
60-64	62	7	5	5	35	434
65-69	67	4	10	10	40	268
70-74	72	2	15	15	30	144
		40			225	2265

$$\bar{x} = 57, MD = \frac{225}{40} = 5.63$$

Variance and Standard Deviation

$$SampleVariance(S^2) = \frac{\sum (x - \bar{x})^2}{n-1} \; forrawdata$$

$$SampleVariance(S^2) = \frac{\sum f(x - \bar{x})^2}{n-1} \; forfrequencydata$$

$$SamplesS\tan dardDeviation(S) = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} \; forrawdata$$

$$SampleS\tan dardDeviation(S) = \sqrt{\frac{\sum f(x - \bar{x})^2}{n-1}} \; forfrequencydata$$

$$PopulationVariance(\sigma^2) = \frac{\sum (x - \mu)^2}{n} \; forrawdata$$

$$PopulationVariance(\sigma^2) = \frac{\sum f(x - \mu)^2}{n} \; forfrequencydata$$

$$StdDev(\sigma) = \sqrt{\frac{\sum (x - \mu)^2}{n}} \; forrawdata$$

$$StdDev(\sigma) = \sqrt{\frac{\sum f(x - \mu)^2}{n}} \; forfrequencydata$$

Short cut method

$$\sigma^2 = \left[\frac{\sum fu^2 - \frac{\left(\sum fu\right)^2}{n}}{n} \right] C^2$$

$$\sigma = \sqrt{\sigma^2}$$

$$S^2 = \left[\frac{\sum fu^2 - \frac{\left(\sum fu\right)^2}{n}}{n-1} \right] C^2$$

$$S = \sqrt{S^2}$$

Note – For large sample sizes ($n \geq 30$) the population standard deviation formula may be used to obtain standard deviation for sample. In such case, we use the sample mean to replace the population mean.

Example

Class	f	x	u	u^2	fu	fu^2
1 – 10	6	5.5	-3	9	-18	54
11 – 20	6	15.5	-2	4	-12	24
21 – 30	12	25.5	-1	1	-12	12
31 – 40	11	35.5	0	0	0	0
41 – 50	10	45.5	1	1	10	10
51 – 60	5	55.5	2	4	10	20
	50				-22	120

$$\sigma^2 = \left[\frac{120 - \frac{(-22)^2}{50}}{50} \right] x10^2 = 220.64$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{220.64} = 14.85$$