Recursividade

Recursividade

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International

Service

Association


ZERO
DE UTILIDADE
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caso **recursivo**

caso **base**

caso **recursivo**

(sub-caso ou caso anterior)

---

caso **base**

caso **recursivo**

(sub-caso ou caso anterior)

caso **base**

(caso atômico ou inicial)

caso **recursivo**

(o que fazer a cada nível da recursão)

---

caso **base**

caso **recursivo**

(o que fazer a cada nível da recursão)

caso **base**

(o que fazer ao atingir o caso base)

cálculo fatorial

```
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n--; }
   return fat;
}
```

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1
- 2! = 2 * 1
- 1! = 1
- 0! = 1

cálculo fatorial

- 5! = 5 * 4 * 3 * 2 * 1
- 4! = 4 * 3 * 2 * 1

- 5! = 5 * 4!

- 4! = 4 * 3 * 2 * 1
- 3! = 3 * 2 * 1

- 4! = 4 * 3!

cálculo **fatorial**

caso **recursivo**

caso **base**

cálculo fatorial

caso recursivo

n! = n * (n-1)!

---

caso base

cálculo **fatorial**

caso **recursivo**

$n! = n * (n-1)!$

---

caso **base**

$0! = 1! = 1$

cálculo fatorial

```c
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n--; }
   return fat;
}
```

```c
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```


LEVEL 1
REALITY
No one...
We think
Cobb, Arthur, Ariadne, Eames, Saito, Yusuf and Robert Fischer Jr.
To drug Fischer Jr. and bring his subconscious into a dream.
There isn't one. The timer counts down and the machine shuts off.


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LEVEL 2
VAN CHASE
Yusuf
"The Chemist"
Cobb, Arthur, Ariadne, Eames, Saito, Yusuf and Robert Fischer Jr.
Fisher Jr. is kidnapped. They force him to give them random numbers which are used later, and begin planting the idea in his head that his father wants him to break up the company.
Yusef drives the van off a bridge. That fails. A second Kick occurs when the van hits the water.


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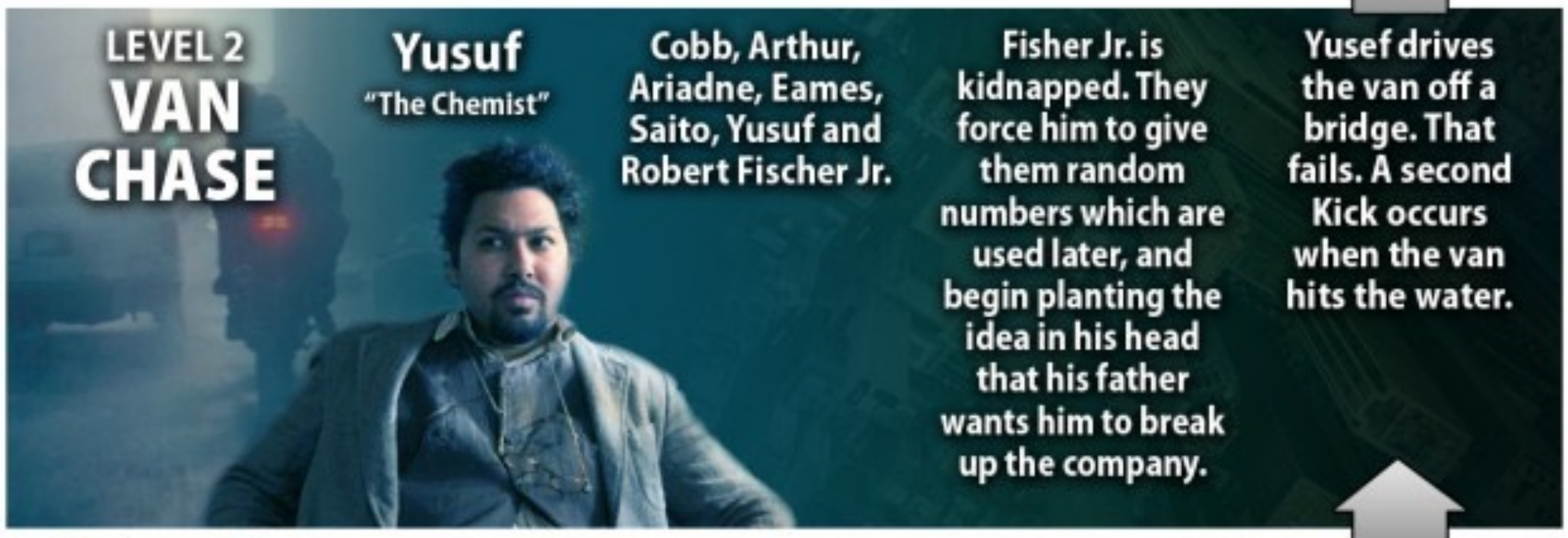
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Arthur blows up an elevator, simulating freefall.


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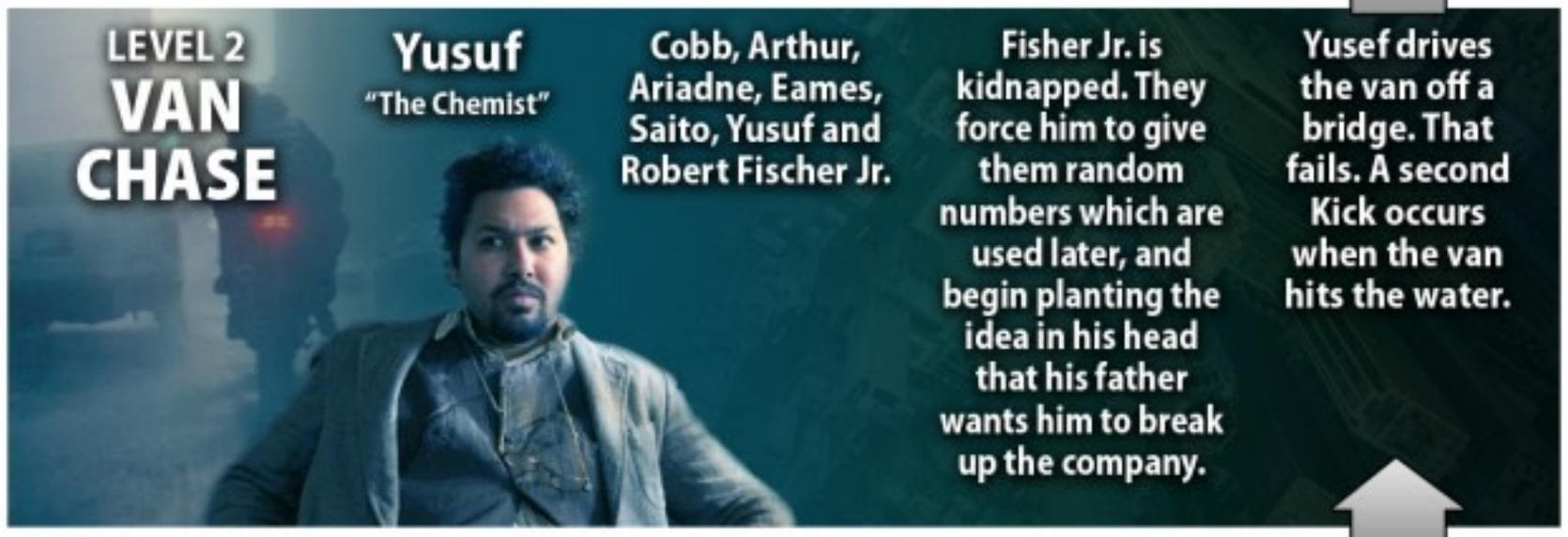
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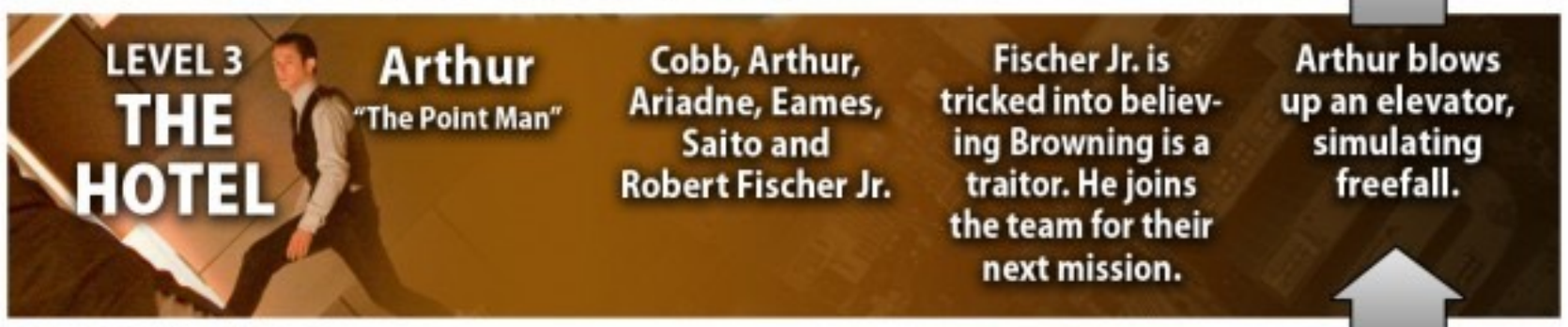
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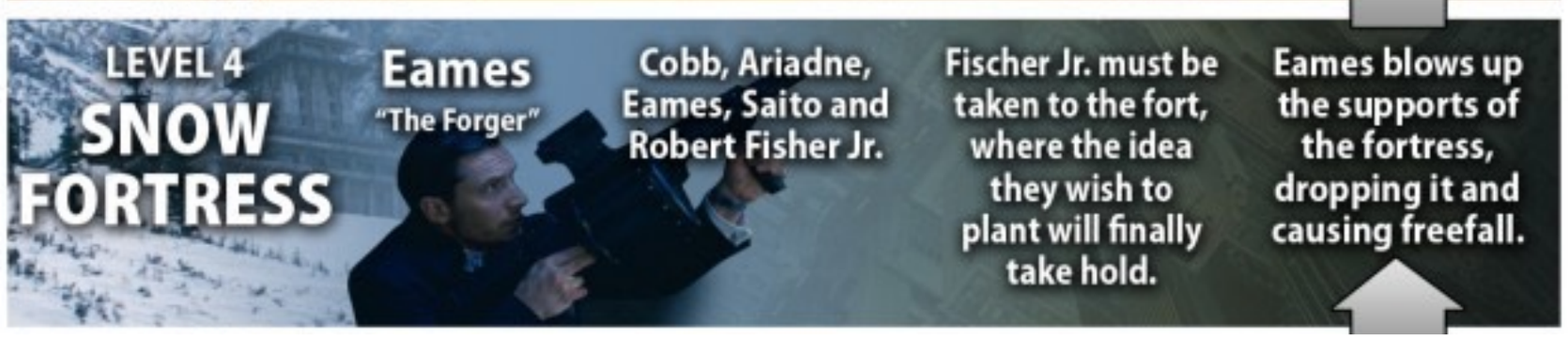
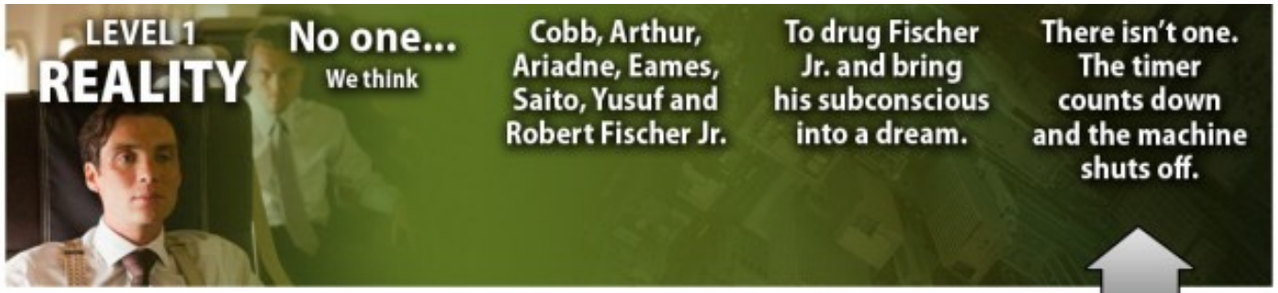

LEVEL 4
SNOW FORTRESS
Eames
"The Forger"
Cobb, Ariadne, Eames, Saito and Robert Fisher Jr.
Fischer Jr. must be taken to the fort, where the idea they wish to plant will finally take hold.
Eames blows up the supports of the fortress, dropping it and causing freefall.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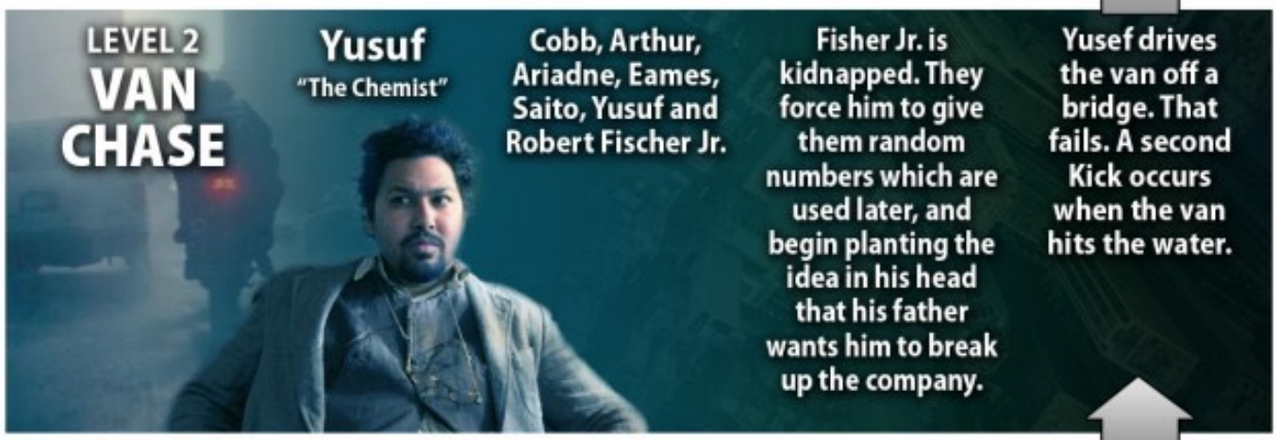
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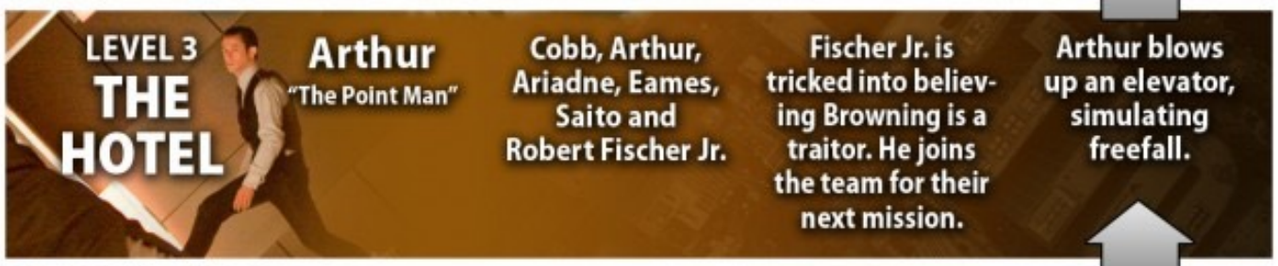
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LEVEL 5
LIMBO
No one
It's a shared state
Cobb, Ariadne, Saito, Robert Fischer Jr. and Mal's projection
To get Fischer Jr. and Saito out.
Ariadne and Fischer fall off a building. Cobb and Saito shoot themselves.
Artwork by Matt Sinopoli


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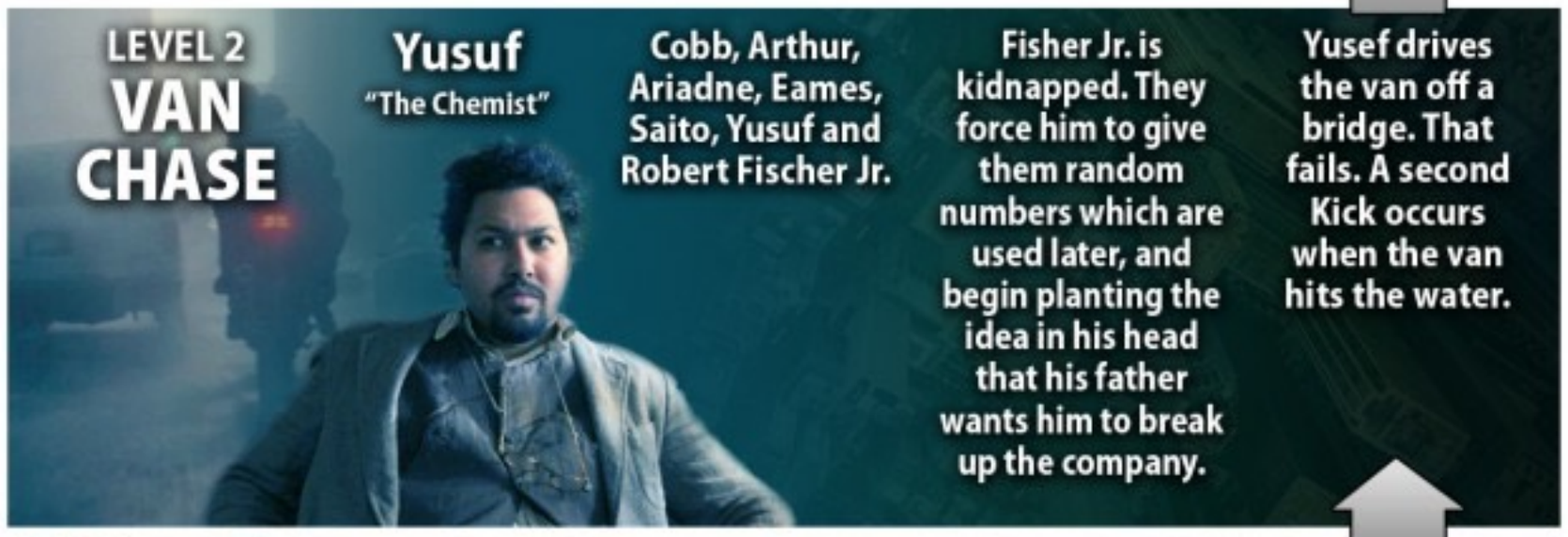
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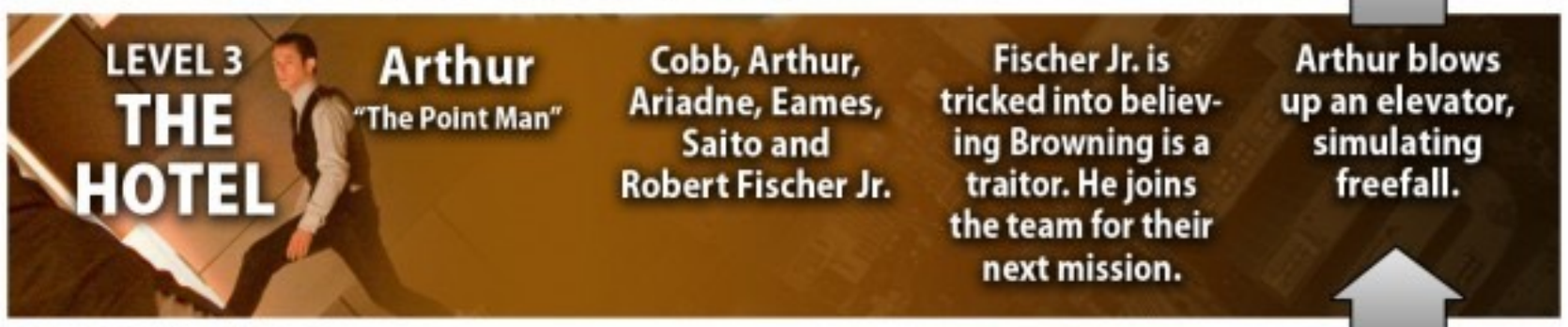
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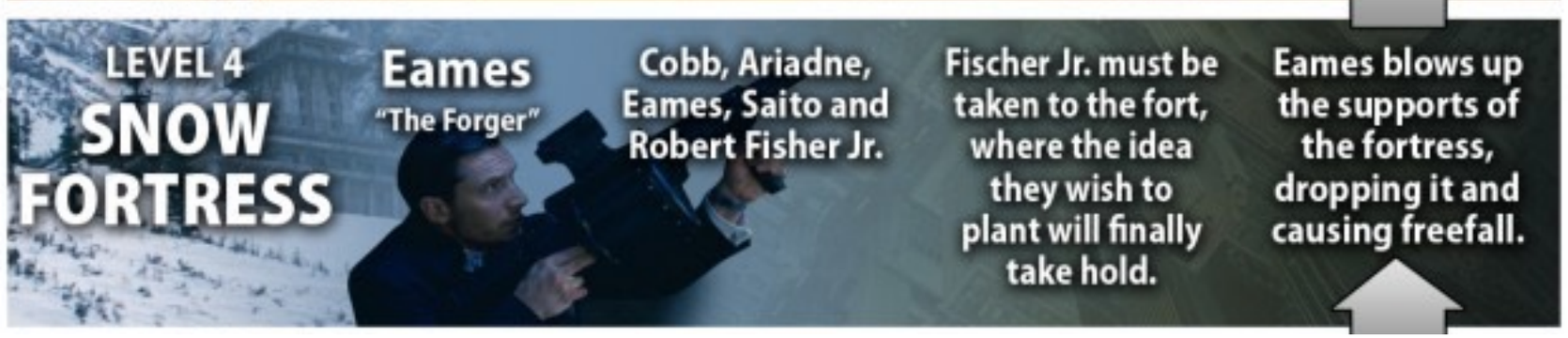
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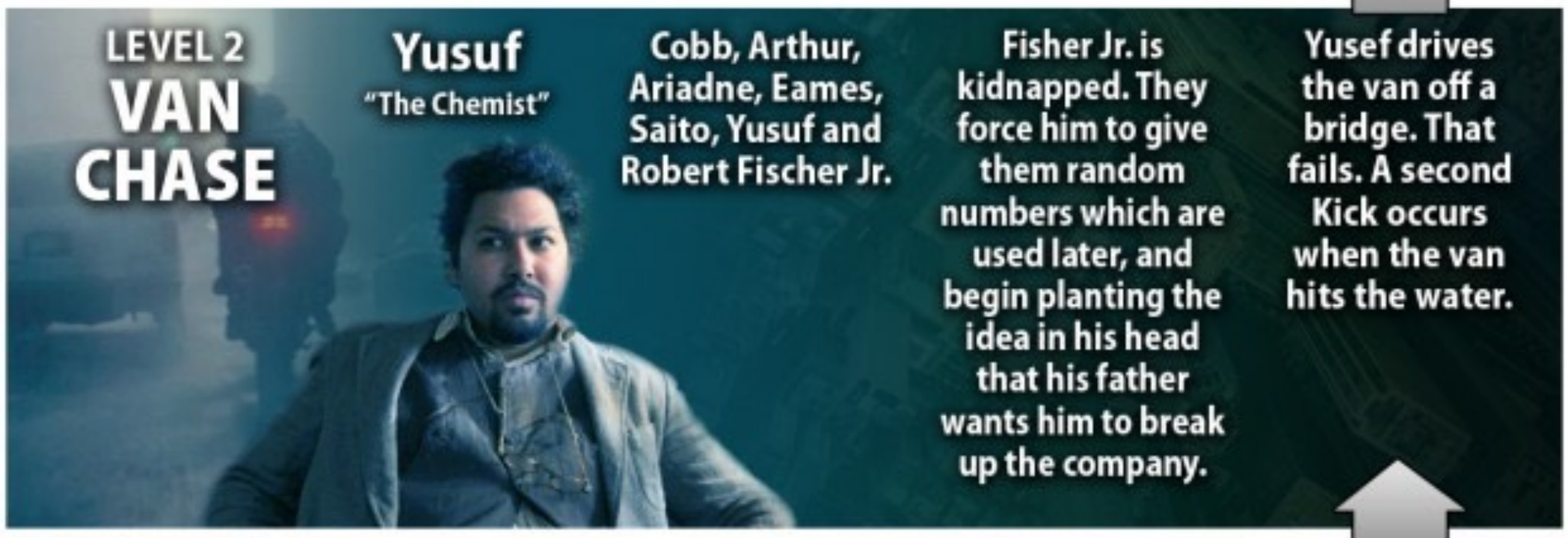
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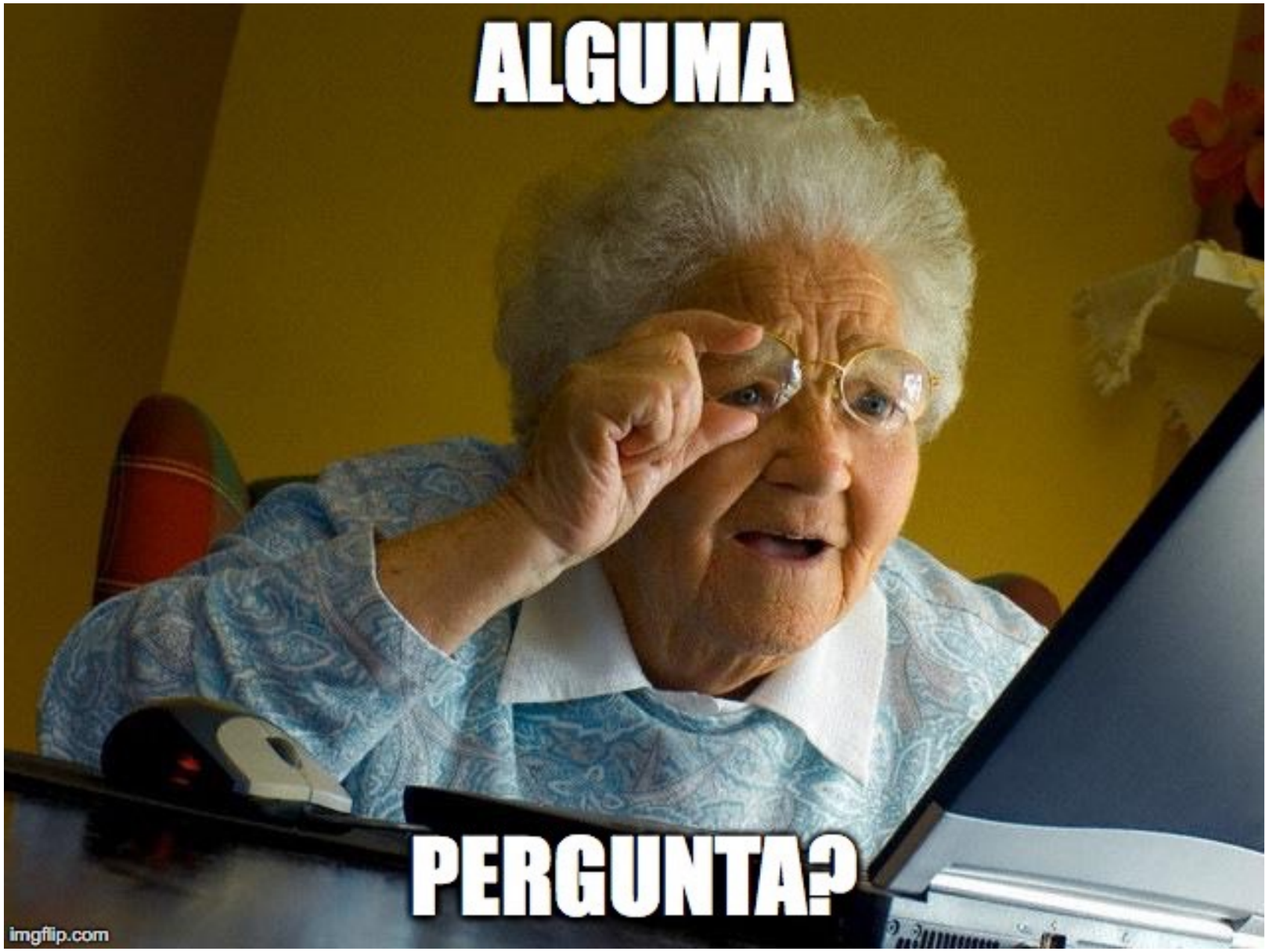

ALGUMA
PERGUNTA?
imgflip.com

cálculo fatorial

```c
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n--; }
   return fat;
}
```

```c
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

```
int fat_iter(int n)
{
   int fat = 1;
   while (n > 1) { fat *= n; n--; }
   return fat;
}
```

Melhor caso

- O(n)

Pior caso

- O(n)

cálculo fatorial

Caso **base**

- ?

Caso **recursivo**

- ?

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

Caso **base**

- 2

Caso **recursivo**

- 2 + T(n-1)

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo **fatorial**

$$T(n) = T(n-1) + 2$$

$$T(n) = (T(n-2) + 2) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = (T(n-3) + 2) + 4$$

$$T(n) = T(n-3) + 6$$

cálculo fatorial

$$T(n) = T(n-1) + 2$$

$$T(n) = T(n-2) + 4$$

$$T(n) = T(n-3) + 6$$

...

$$T(n) = T(n-k) + 2k$$

cálculo **fatorial**

$$T(n) = T(n-k) + 2k$$

Fazendo $k = n$:

$$T(n) = T(n-n) + 2n$$

$$T(n) = T(0) + 2n$$

$$T(n) = 2 + 2n$$

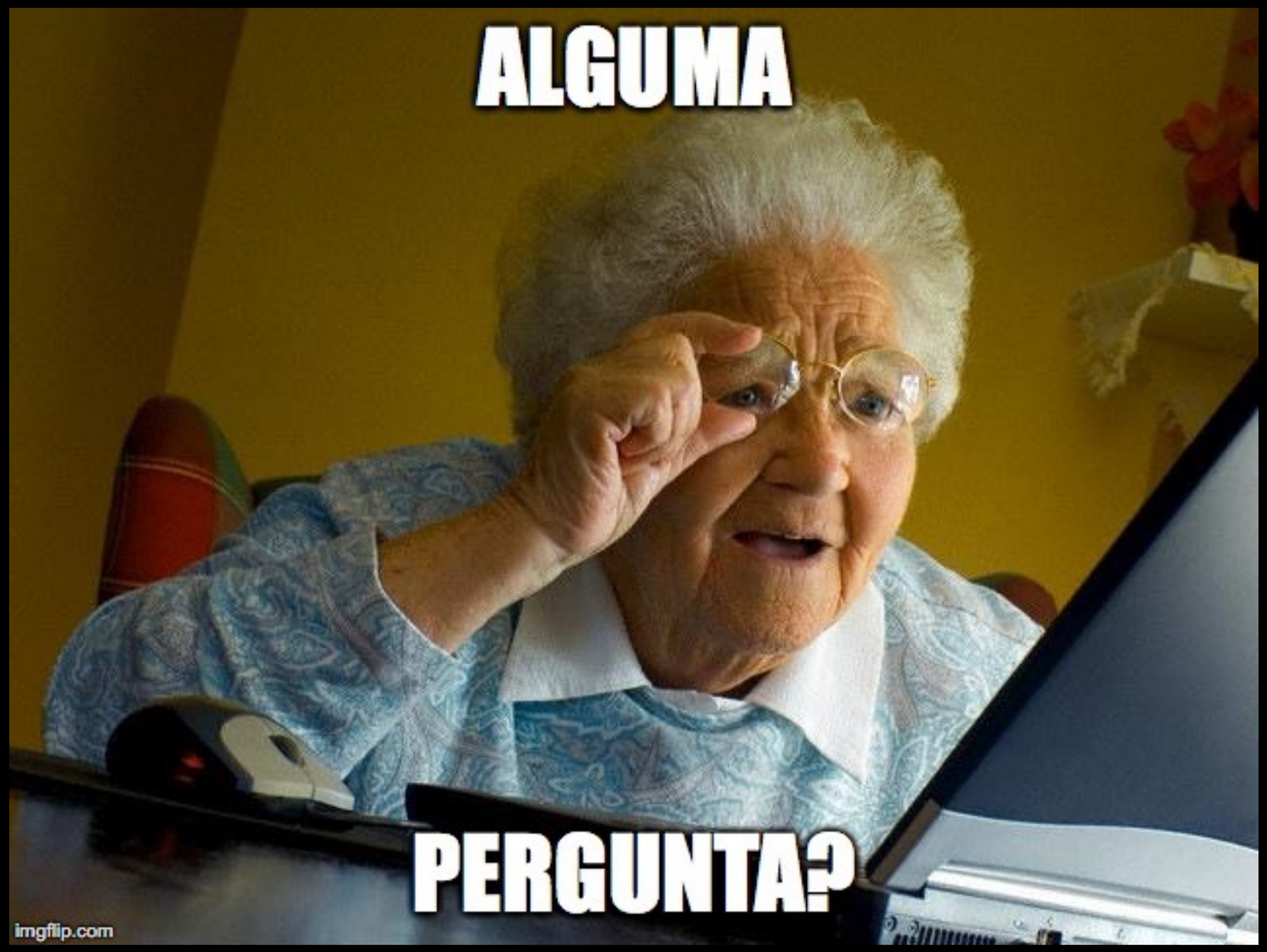
cálculo fatorial

- T(n) = 2 + 2n
- O(n)

```
int fat_rec(int n)
{
   if (n <= 1) return 1;
   return n * fat_rec(n-1);
}
```

cálculo fatorial

iterativo	recursivo
O(n)	O(n)


ALGUMA
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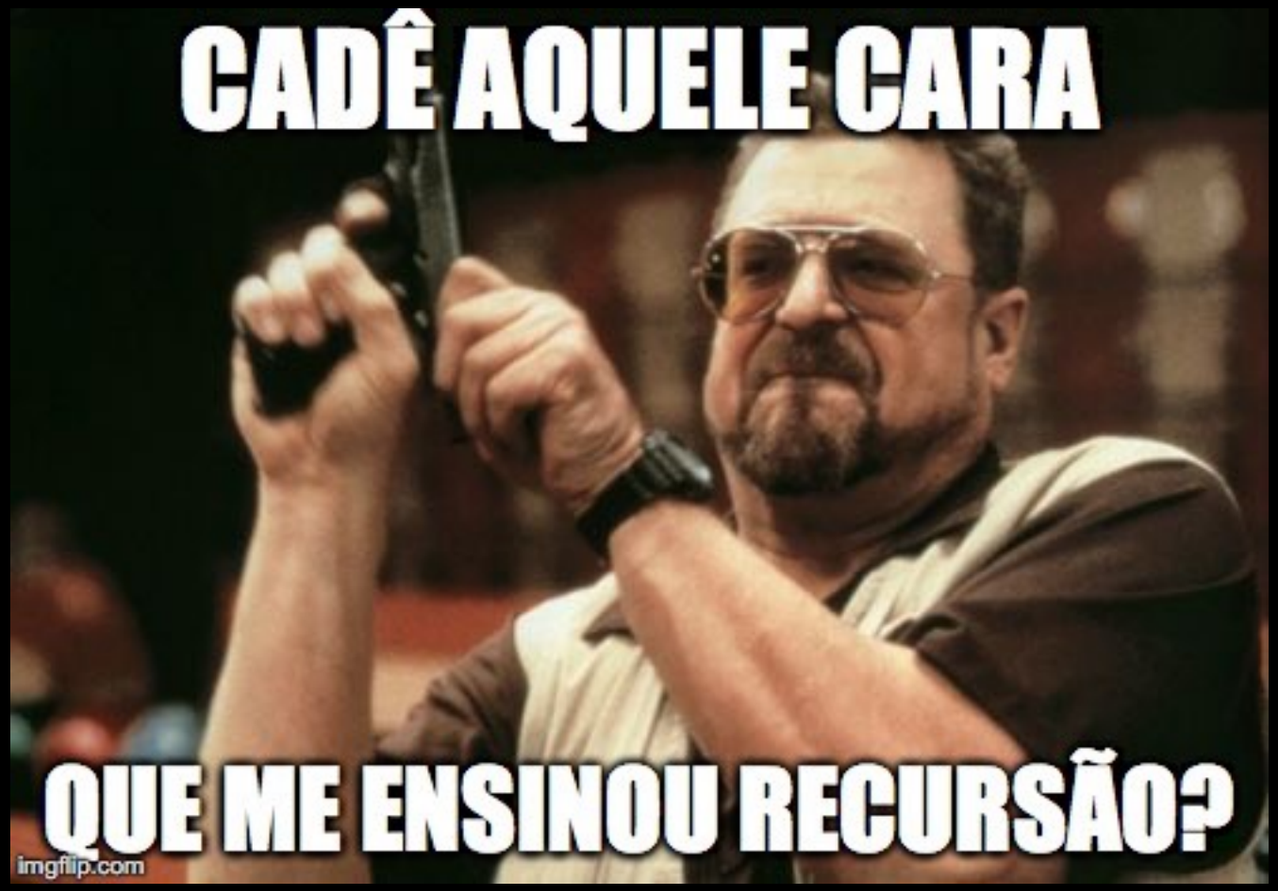

CADÊ AQUELE CARA
QUE ME ENSINOU RECURSÃO?
imgflip.com

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

dividir para conquistar

dividir para conquistar

1	4	7	10	15	16	18	21

1	4	7	10

15	16	18	21

1	4

7	10

15	16

18	21

dividir para conquistar

busca(3)

O(log n)

1	4	7	10	15	16	18	21

1	4	7	10

1	4

dividir para conquistar

busca(3)

O(n)

1	7	4	15	10	21	18	16

1	7	4	15

10	21	18	16

1	7

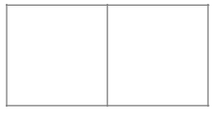
4	15

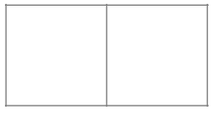
10	21

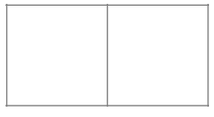
18	16

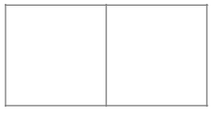
dividir para conquistar

aT(n/b)









dividir para conquistar

$aT(n/b)$

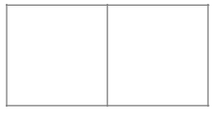
a	fator de arborescência
b	fator de divisão

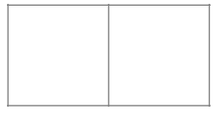
dividir para conquistar

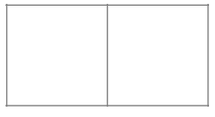
T(n/2)

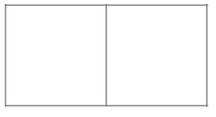
dividir para conquistar

2T(n/2)



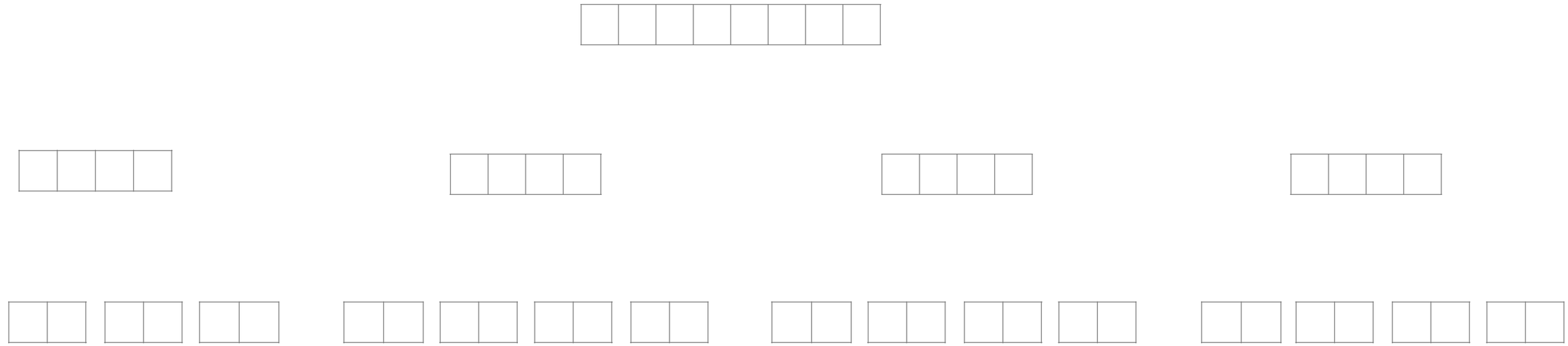






dividir para conquistar

$4T(n/2)$



dividir para conquistar

4T(n/4)

dividir para conquistar

conquistar

$$T(n) = \underbrace{aT(n/b)}_{\text{dividir}} + f(n)$$

busca binária

```
int bin_rec(int v[], int chave, int inicio, int fim)
{
   int tamanho = fim - inicio;
   if (tamanho == 0) { return -1; }
   int meio = inicio + floor(tamanho / 2);
   if (chave == v[meio]) { return meio; }
   else if (chave < v[meio]) {
      return bin_rec(v, chave, inicio, meio);
   }
   return bin_rec(v, chave, meio + 1, fim);
}
```

dividir para conquistar

conquistar

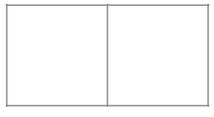
$$T(n) = \underbrace{aT(n/b)}_{\text{dividir}} + f(n)$$

dividir para conquistar

$T(n/2) + 1$

1

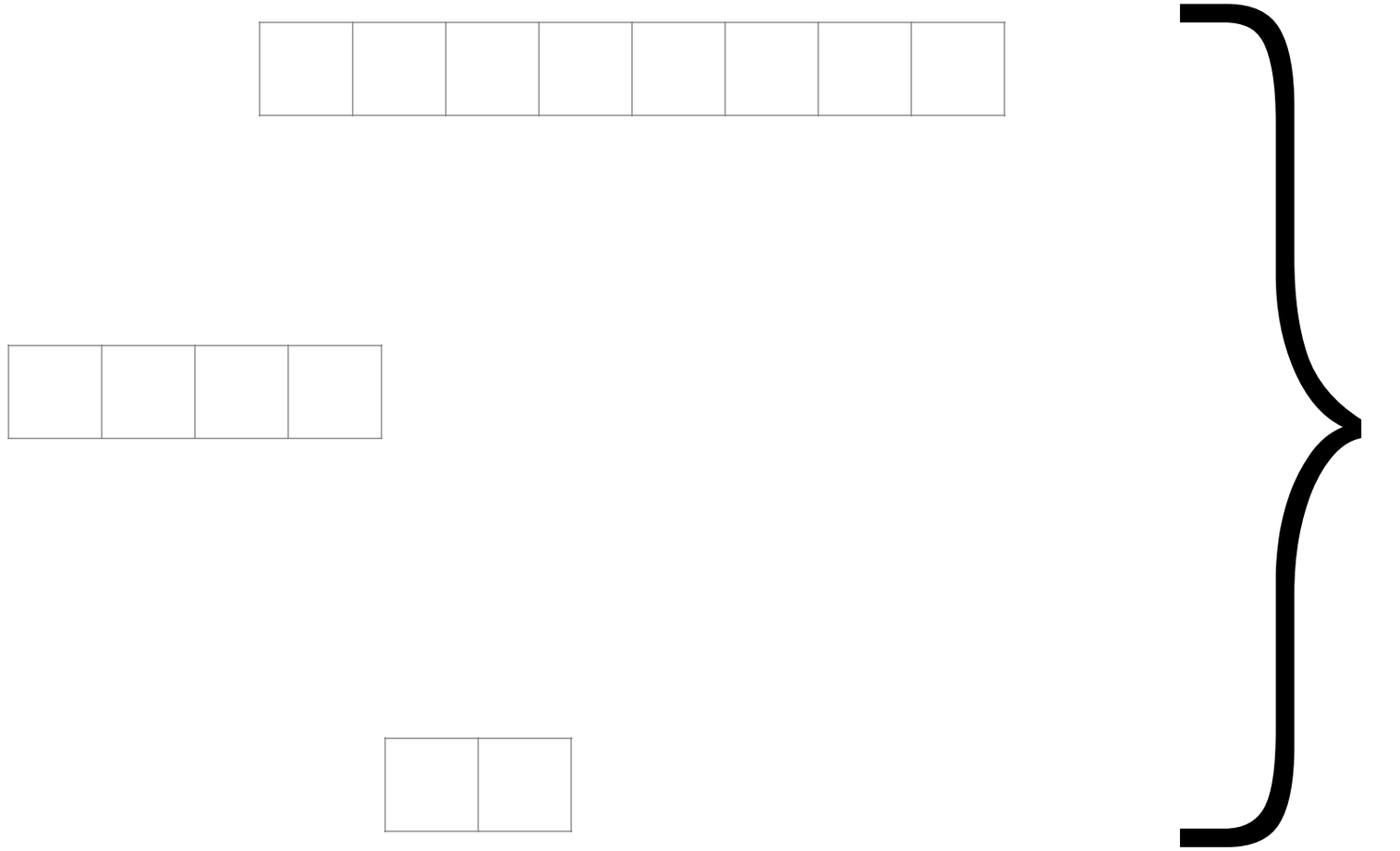
1



1

dividir para conquistar

T(n/2) + 1



log n

dividir para conquistar

$2T(n/2) + 1$

1

2

4

dividir para conquistar

$2T(n/2) + n$

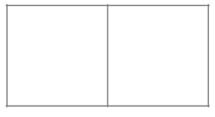
n

n

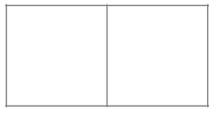
n/2

n/2

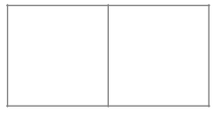
n



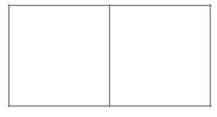
n/4



n/4



n/4



n/4

n

dividir para conquistar

$2T(n/2) + n^2$

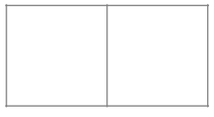
n^2

n^2

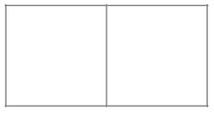
$(n/2)^2$

$(n/2)^2$

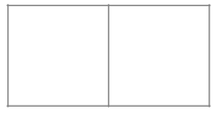
$n^2/2$



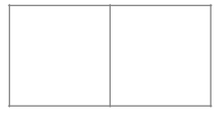
$(n/4)^2$



$(n/4)^2$



$(n/4)^2$



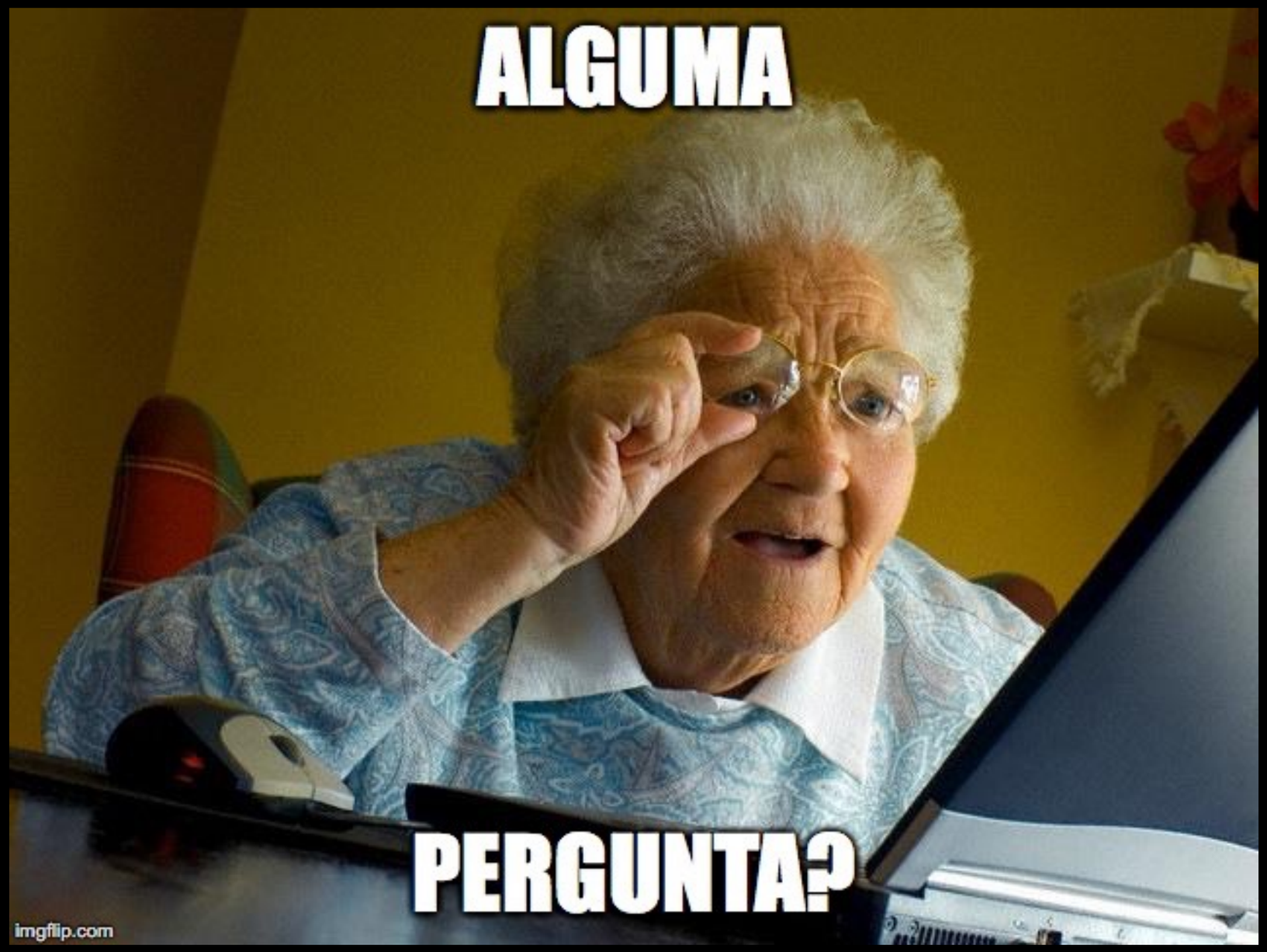
$(n/4)^2$

$n^2/4$

dividir para conquistar

conquistar

$$T(n) = \underbrace{aT(n/b)}_{\text{dividir}} + f(n)$$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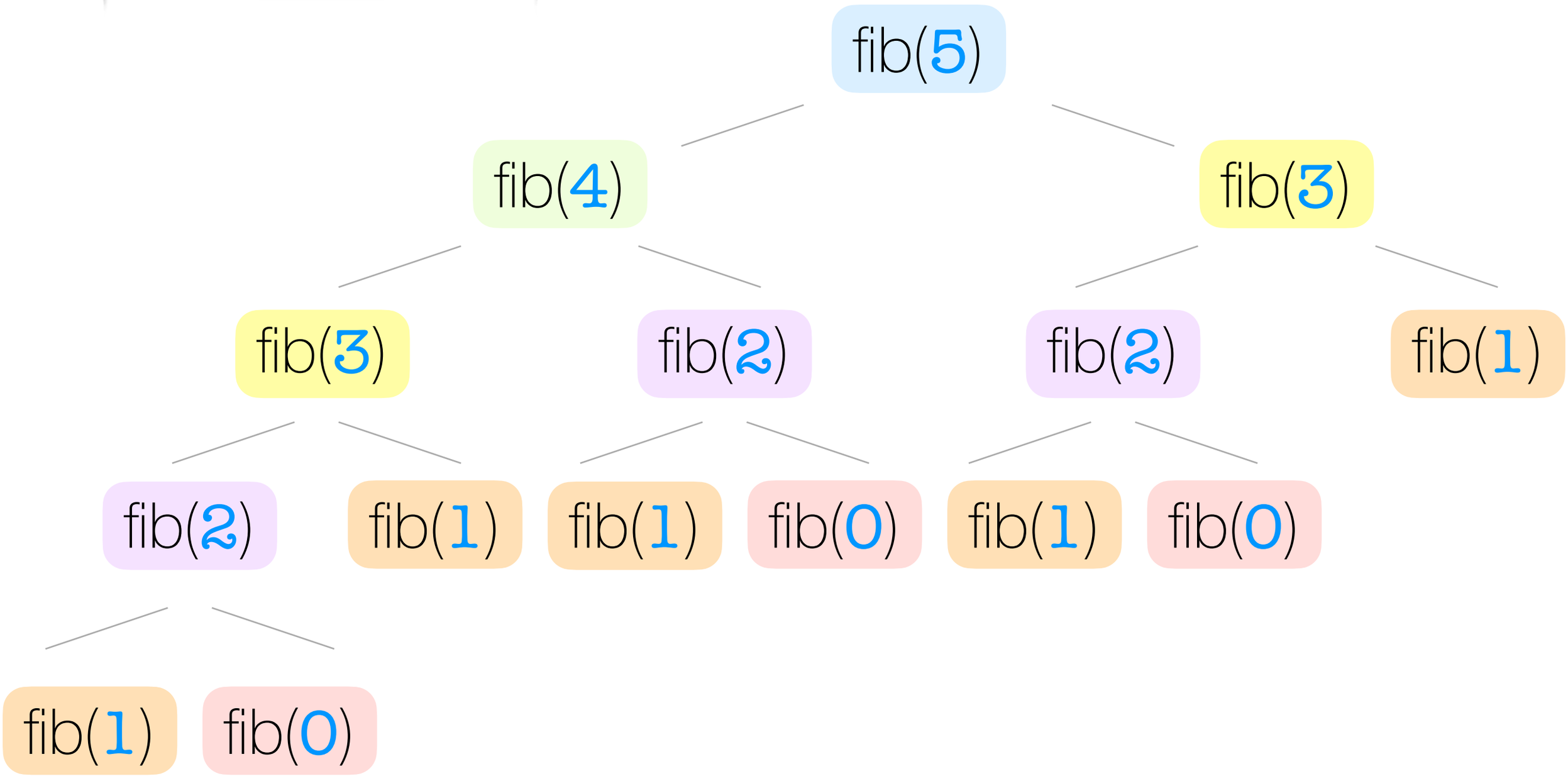
dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

sequência de fibonacci

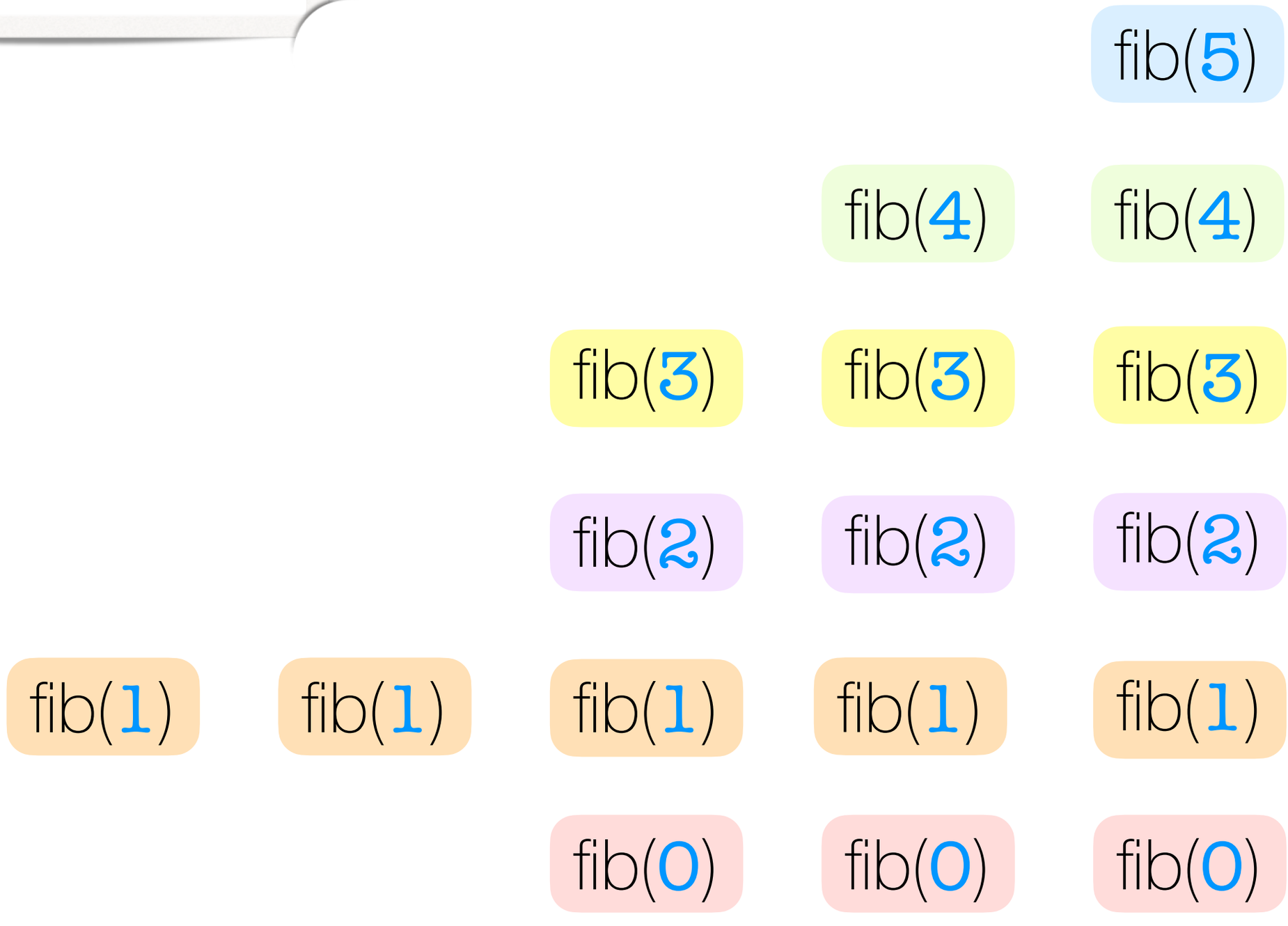
```
int fib_rec(int n)
{
   if (n <= 1) return n;
   return fib(n-1) + fib(n-2);
}
```

- 1 1 2 3 5 8 13 21 34 ...
- fib(n) = fib(n-1) + fib(n-2)
- fib(0) = 0, fib(1) = 1

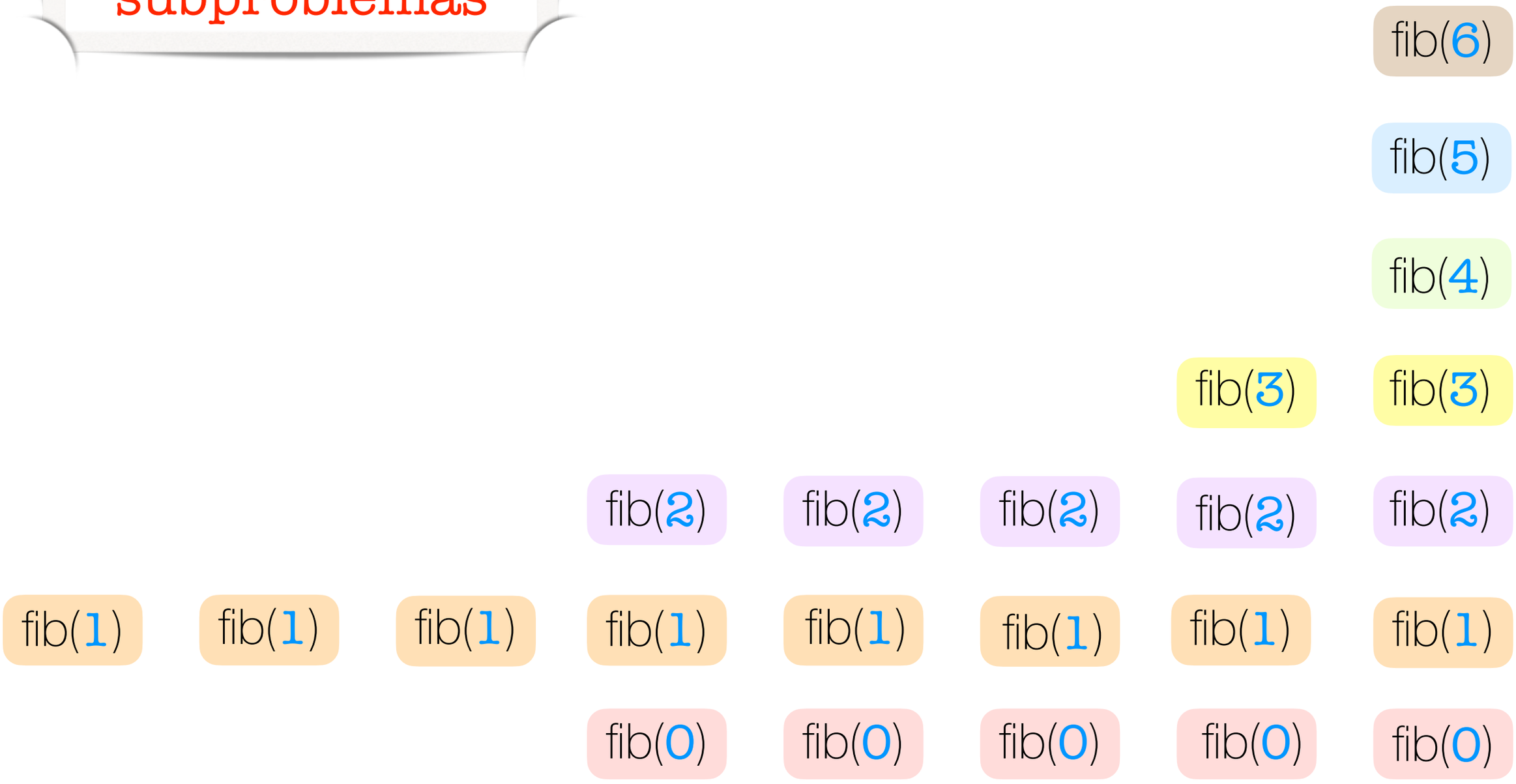
intersecções entre subproblemas



intersecções entre subproblemas



intersecções entre subproblemas



intersecções entre subproblemas

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5) fib(6)

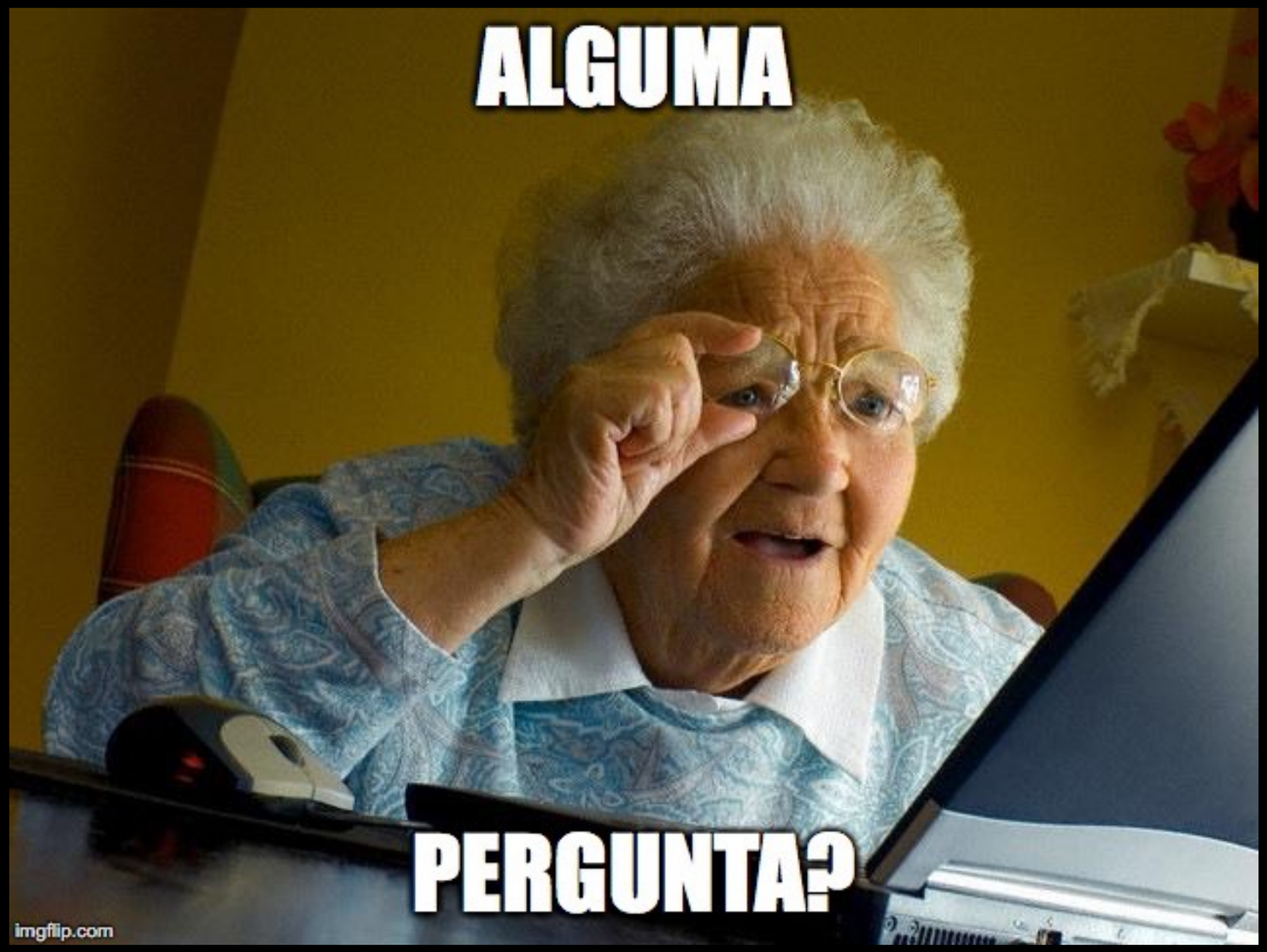
(programação dinâmica)

```c
int fib_iter(int n)
{
  int fib, f1 = 1, f2 = 0;
  for (int i = 2; i <= n; i++) {
     fib = f1 + f2;
     f2 = f1;
     f1 = fib;
  }
  return fib;
}
```

```c
int fib_rec(int n)
{
   if (n <= 1) return n;
   return fib(n-1) + fib(n-2);
}
```

fib(0) fib(1) fib(2) fib(3) fib(4) fib(5) fib(6)

(programação **dinâmica**)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dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos

recursão de cauda

```
int fat_rec(int n)
{
  if (n <= 1) return 1;
  return n * fat_rec(n-1);
}
```

recursão de cauda

```
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n* f);
}
```

```
int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

recursão de cauda

```c
int fat_cauda(int n, int f)
{
  if (n <= 1) return f;
  return fat_cauda(n-1, n* f);
}

int fat_rec(int n)
{
    return fat_cauda(n, 1);
}
```

```c
int fat(int n)
{
    int f = 1;
    while (n > 1) {
        f = n * f;
        n = n - 1;
    }
    return f;
}
```

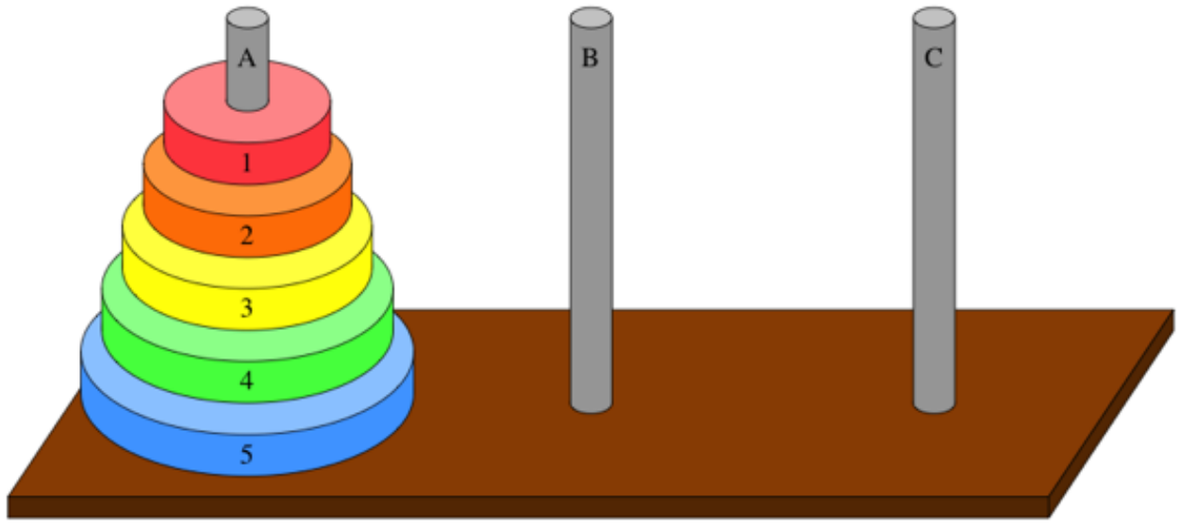
recursão de cauda

recursão de cauda

compilador

iterativo

dividir para conquistar	intersecções entre subproblemas
recursão de cauda	demais casos




```
hanoi (altura, origem, destino, aux)
    if (altura == 1)
        “Movendo disco 1 da torre origem pra torre destino.”
        return
    }
    hanoi (altura - 1, origem, aux, destino);
    “Movendo disco altura da torre origem pra torre destino.”
    hanoi (altura - 1, aux, destino, origem);
}
```

dividir para conquistar	
intersecções entre subproblemas	
recursão de cauda	
demais casos