Notation: $Y=(Y_1,Y_2,\ldots,Y_n)^T\in\mathbb{R}^N$ and $U=(U_1,U_2,\ldots,U_q)^T\in\mathbb{R}^q$ are random vectors (response and random/mixed effects); $\beta \in \mathbb{R}^p$ is the vector of fixed effects; $X \in \mathbb{R}^{n \times p}$ and $Z \in \mathbb{R}^{n \times q}$ are model matrices, where x_i^T or z_i^T denote the *i*th row of X or Z respectively. Generalized linear mixed model (GLMM):

 $(Y_i|U=u) \sim \text{indep.} f_{Y_i|U}(y_i|u) \text{ for } i \in \{1, 2, ..., n\}, U \sim f_U(u).$ $f_{Y_i|U}(y_i|u) = h(y_i) \exp\left(\langle T(y_i), \eta \rangle - A(\eta)\right),$ $\mu_i = \mathrm{E}(Y_i|U=u) = f(\eta),$ $g(\mu_i) = x_i^T \beta + z_i^T u.$ Z = 0Normality Generalized linear model (GLM): Linear mixed model (LMM):

 $Y_i \sim \text{indep.} f_{Y_i}(y_i) \text{ for } i \in \{1, 2, \dots, n\},\$ $(Y|U=u) \sim \mathcal{N}(X\beta + Zu, \sigma^2 I),$ $f_{Y_i}(y_i) = h(y_i) \exp(\langle T(y_i), \eta \rangle - A(\eta)),$ $U \sim \mathcal{N}(0, V)$. $\mu_i = \mathrm{E}(Y_i) = f(\eta),$ $g(\mu_i) = x_i^T \beta.$ Z =Normality Linear model (LM): $Y \sim \mathcal{N}\left(X\beta, \sigma^2 I\right).$ Normality, $\lambda = 0$ Normality, $\lambda = 0$

Linear model (LM):
$$Y \sim \mathcal{N}\left(X\beta, \sigma^2 I\right).$$
 Normality, $\lambda = 0$

 $E(Y_i) = X\beta,$ $E(Y_i) = X\beta$, $\hat{\beta} = \operatorname{argmin}_{b \in \mathbb{R}^p} \left\{ \frac{1}{2n} \|y - Xb\|_2^2 + \lambda \|b\|_1 \right\}.$ $\hat{\beta} = \operatorname{argmin}_{b \in \mathbb{R}^p} \left\{ \frac{1}{2n} \|y - Xb\|_2^2 + \lambda \|b\|_2^2 \right\}.$

Ridge regression:

LASSO: