Homework 2

Due: Friday Feb. 6th, 2026 at 11:59pm ET via Dropbox and Crowdmark.

Crowdmark Submission: answer sheet (with codes as an appendix).

Problem 1:
Consider the following IP:

$$\begin{array}{ll} \max & 3x_1 + 2x_2 \\ \text{s.t.} & 4x_1 + 2x_2 \leq 15 \\ & x_1 + 2x_2 \leq 8 \\ & x_1 + x_2 \leq 5 \\ & x_1, x_2 \geq 0; \text{integer} \end{array}$$

a) Solve the LP relaxation graphically.

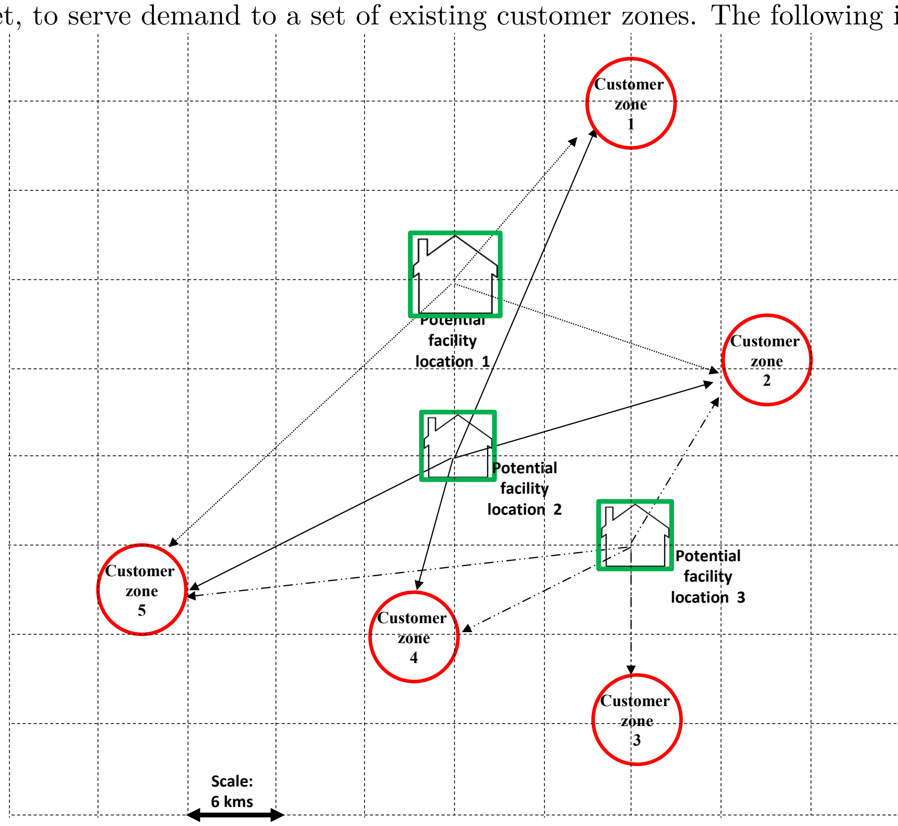
b) Is $2x_1 + x_2 \leq 7$ a valid cut? justify your answer. If not modify it to lead to a valid cut. If yes, add it and resolve.

c) Build the optimal simplex tableau using the information in a) and the revised simplex method. Generate a Gomory cut; add it and resolve.

d) Use branch and bound to solve the IP (without the added cuts). Branch first on x_1.

e) Is $x_2 = 3$ vs $x_2 = 4$ vs $x_2 \leq 2$ a valid branching rule? if so use it to solve the IP. If not modify it and the use it to solve the IP.

Problem 2:
The **Uncapacitated Facility Location problem** aims to locate a set of facilities, among a potential set, to serve demand to a set of existing customer zones. The following is an example:



If the demand is [200 400 150 300 250] and the costs are proportional to the Euclidean distance between the centre of the customer zones and the centre of the potential facilities as in the above figure, then

1. Provide an assignment type formulations of the problem
2. Provide a set covering type formulations of the problem
3. Solve each in Matlab/Python and find the optimal solution.

Note: There is no capacity for the facilities, but there is a fixed cost for establishing the facilities. Come up with the costs yourself, and possibly compare alternatives.

Problem 3:
Consider the following formulations for the **Capacitated Facility Location problem**:

$$
\begin{aligned}
[P1]: \ \min \ & \sum_j f_j z_j + \sum_i \sum_j c_{ij} x_{ij} \\
\text{s.t.} \ & \sum_j x_{ij} = 1, \ \forall i \\
& \sum_i D_i x_{ij} \le V z_j, \ \forall j \\
& x_{ij}, z_j \in \{0,1\}.
\end{aligned}
$$

$$
\begin{aligned}
[P2]: \ \min \ & \sum_j f_j z_j + \sum_i \sum_j c_{ij} x_{ij} \\
\text{s.t.} \ & \sum_j x_{ij} = 1, \ \forall i \\
& \sum_i D_i x_{ij} \le V, \ \forall j \\
& x_{ij} \le z_j, \forall i, \ \forall j \\
& x_{ij}, z_j \in \{0,1\}.
\end{aligned}
$$

$$
\begin{aligned}
[P3]: \ \min \ & \sum_j f_j z_j + \sum_i \sum_j c_{ij} x_{ij} \\
\text{s.t.} \ & \sum_j x_{ij} = 1, \ \forall i \\
& \sum_i D_i x_{ij} \le V z_j, \ \forall j \\
& x_{ij} \le z_j, \forall i, \ \forall j \\
& x_{ij}, z_j \in \{0,1\}.
\end{aligned}
$$

Denote the feasible region of the LP relaxation of each formulation by F1, F2 and F3 respectively. Prove or disprove the following:

1. F3 $\subseteq$ F1
2. F3 $\subseteq$ F2