n-Cells Identification Model

A. Giuliano Mirabella Galvín

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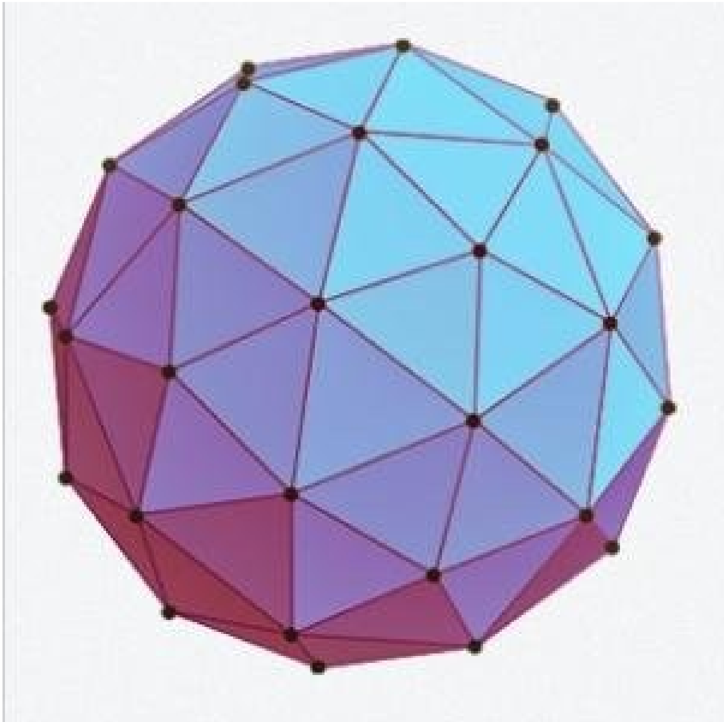

Contents

1 Introduction

This document contains the theory underneath the TANI project [1], a Topological Analysis of nD Images toolkit for Python.

2 Hypothesis

Let V be a discrete n-dimensional space, within every point can be identified by its coordinates:

$$(u_0, u_1, \dots, u_{n-1}, u_n)$$
$$u_i \in \mathbb{N}$$

And J an extension of V such that "point density" is doubled and therefore coordinates can be multiple of $\frac{1}{2}$:

$$(\frac{1}{2}u'_0, \frac{1}{2}u'_1, \dots, \frac{1}{2}u'_{n-1}, \frac{1}{2}u'_n)$$
$$u'_i \in \mathbb{N}$$

Let $P \in J$ be ay point whose coordinates are such:

$$(x_0 + \lambda_0\frac{1}{2}, x_1 + \lambda_1\frac{1}{2}, \dots, x_n + \lambda_n\frac{1}{2})$$
$$x_i \in \mathbb{N}$$
$$\lambda_i \in 0, 1$$

Such coordinates will not identify P anymore; they will instead refer to the r-dimensional entity ("*cell*" from now on) E with:

$$r = n - \sum_{i=0}^{n} \lambda_i$$

This way, P happen to be "the mass center" of E.

Note that:

- when $\lambda_i = 0 \forall i$ then $dim(E) = n$, so E is not a point.
- if $\lambda_i = 1 \forall i$ then $dim(E) = 0$so E is a common 0-dimensional point.

3 Model Outcomes

An $r$dimensional cell (r-cell) E has some special relations with its r-or-less-dimensional surrounding cells.

3.1 Subcells

A cell S is called a *subcell of* E if its coordinates are obtainable from the ones of E by the only modification of λ_i values.

The number of w-dimensional subcells (w-subcells) of an r-cell is given by:

$$s(r, w) = 2^{r-w}\binom{r}{w};$$

If E *exists* so do all its subcells.

The subcells set $S(E)$ is the set of cells that are subcells to E:

$$S(E) =< s : sisasubcellofE$$

Let $\mathbf{E}$ be a vector of m r-cells such $E_0, E_1, \dots, E_m$, then:

$$S(\mathbf{E}) =< s : s \in S(E_i) \forall i \in 0, 1, \dots, m$$

The intersection dimension $i(\mathbb{E})$ is the maximum dimension of $S(\mathbf{E})$ elements.

3.2 Connectivity

A vector of r-cells $\mathbf{E}$ is called α-connected if $i(\mathbf{E}) \geq \alpha$

The potentially connected set $\mathbf{P}(E, \alpha)$ is the set of r-cells that, if had existed would have been α-connected to E. The size of $\mathbf{P}(E)$ is the maximum number of cells E can be α-connected.

$$|\mathbf{P}(E, \alpha)| = \sum_{i=\alpha}^{r-1} s(r, i)$$

4 Further Definitions

TODO :

5 On the Distance

TODO :

6 Non-Squared Mesh-grid Ideas

TODO :

7 An intuitive example in 3D

TODO :

References

[1] GitHub Repository. *Topological Analysis of nD Images*. URL: `https://github.com/agiulianomirabella/tani`.