

## Additional Formulas

### Summation Formulas:

$$\begin{aligned}\sum_{i=1}^n c &= cn \\ \sum_{i=1}^n i &= \frac{n(n+1)}{2} \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum_{i=1}^n i^3 &= \frac{n^2(n+1)^2}{4}\end{aligned}$$

### Trapezoidal Rule (Area):

$$\int_a^b f(x)dx \approx \frac{(b-a)}{2n}[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

with

$$\text{Max Error} \leq \frac{(b-a)^3}{12n^2}[\max|f''(x)|]$$

### Simpson's Rule (Area):

$$\int_a^b f(x)dx \approx \frac{(b-a)}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)]$$

with

$$\text{Max Error} \leq \frac{(b-a)^5}{180n^4}[\max|f^{(4)}(x)|]$$

## Additional Formulas

**Arc Length:**

$$s = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$$

$$s = \int_c^d \sqrt{1 + (g'(y))^2} \, dy$$

**Surface of Revolution:**

$$S = 2\pi \int_a^b r(x) \sqrt{1 + (f'(x))^2} \, dx$$

$$S = 2\pi \int_c^d r(y) \sqrt{1 + (g'(y))^2} \, dy$$

**Work Done by a Variable Force:**

$$W = \int_a^b F(x) \, dx$$

**Force Exerted by a Fluid:**

$$F = w \int_c^d h(y)L(y) \, dy$$

**Taylor Series Expansion for  $f(x)$ :**

$$p_n(x) = f(c) + \frac{f^{(1)}(c)(x-c)}{1!} + \frac{f^{(2)}(c)(x-c)^2}{2!} + \dots + \frac{f^{(n)}(c)(x-c)^n}{n!} + \dots$$

**Maclaurin Series Expansion for  $f(x)$ , where  $c = 0$ :**

$$p_n(x) = f(0) + \frac{f^{(1)}(0)x}{1!} + \frac{f^{(2)}(0)x^2}{2!} + \dots + \frac{f^{(n)}(0)x^n}{n!} + \dots$$

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### Summary of Tests for Series:

Test	Series	Condition(s) of Convergence	Condition(s) of Divergence	Comment
$n$ th-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	This test cannot be used to show convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$ r  < 1$	$ r  \geq 1$	Sums: $S = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n \rightarrow \infty} b_n = L$		Sums: $S = b_1 - L$
$p$ -Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	