'Financial Contagion'
Application of a Structural Model of Credit Risk to the Network of Interbank Loans

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Conference on Systemic Risk and Financial Stability, 2019

The importance of pricing counterparty risk

Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.

Basel Committee on Banking Supervision, 2011

The problem of pricing risk in a network

The Bank's *solvency contagion model* *examines how deteriorating capital positions lead to revaluation of interbank debt claims, which in turn can affect banks' capital positions further . . . Bank staff's judgement is that . . . the overall impact on the system via this channel remains immaterial . . . See Bank of England, Staff Working Paper No. 662, 'The decline of solvency contagion risk', June 2017.*

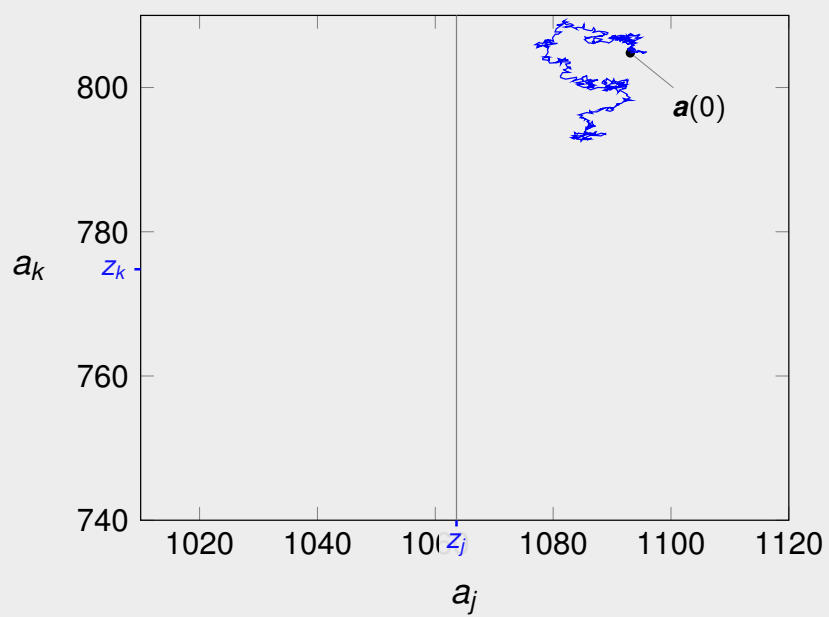Bank of England stress testing results, 2017

Outline

Goings-on in the real economy

Risk-adjusted value as a fraction of face value

Bank j debt valuation u_j

Risk-adjusted value as a fraction of face value

Bank k debt valuation u_k
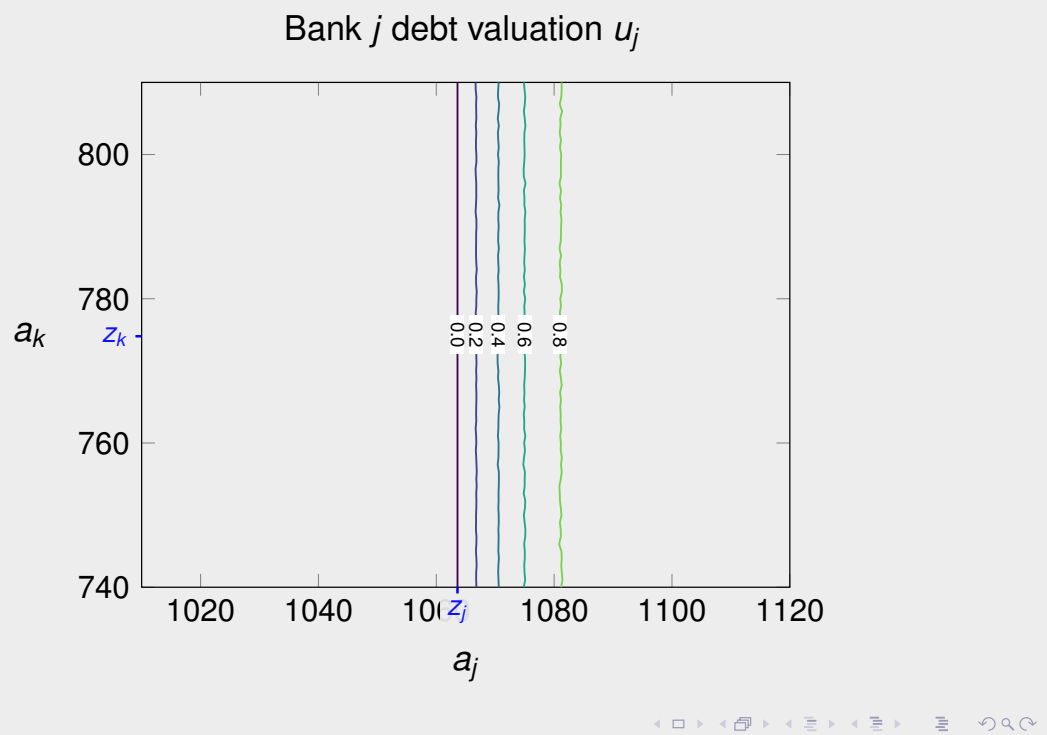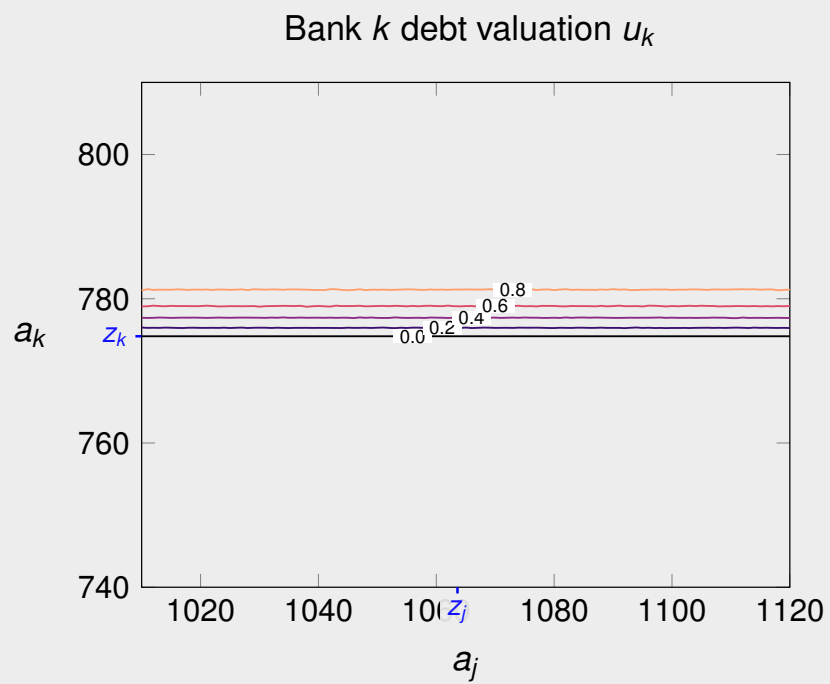
Balance sheets in the unconnected case

Bank j

Assets	Liabilities
a_j	z_j
	E_j

Bank k

Assets	Liabilities
a_k	z_k
	E_k
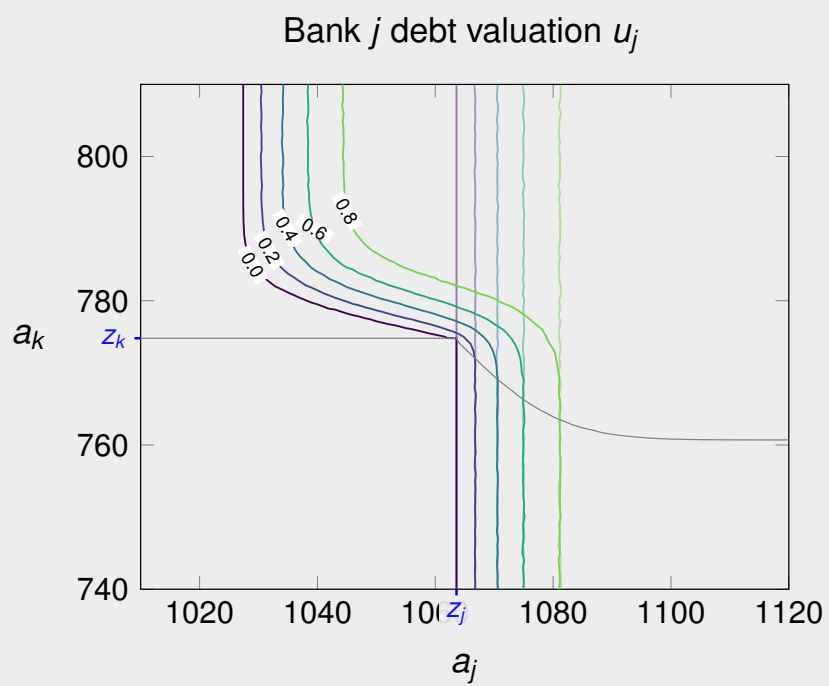
Balance sheets with interbank exposures

Pricing in the effect of interbank exposures
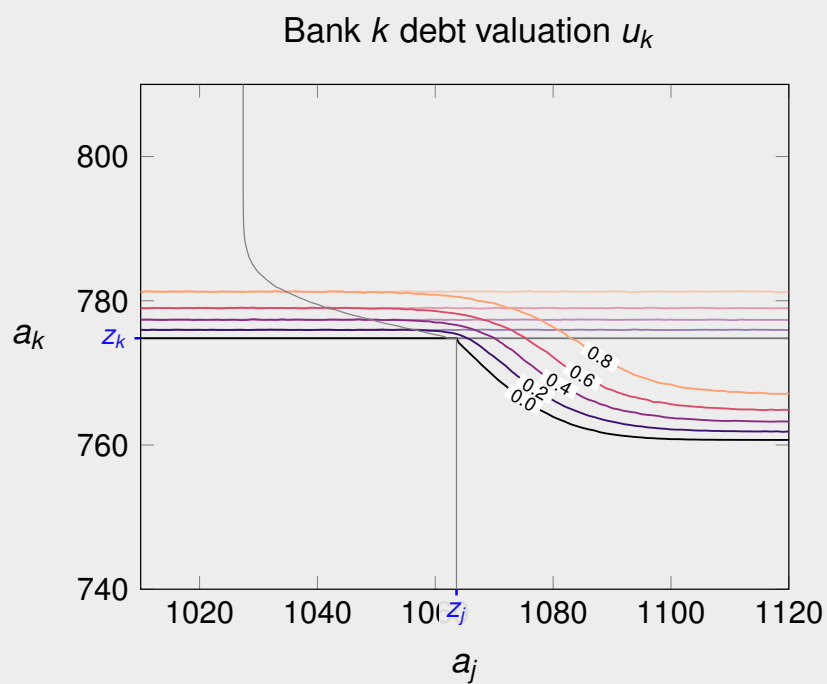
Bank j debt valuation u_j

Pricing in the effect of interbank exposures

Bank k debt valuation u_k

Comparison with the Bank of England model

Bank j debt valuation u_j

Comparison with the Bank of England model

Bank k debt valuation u_k
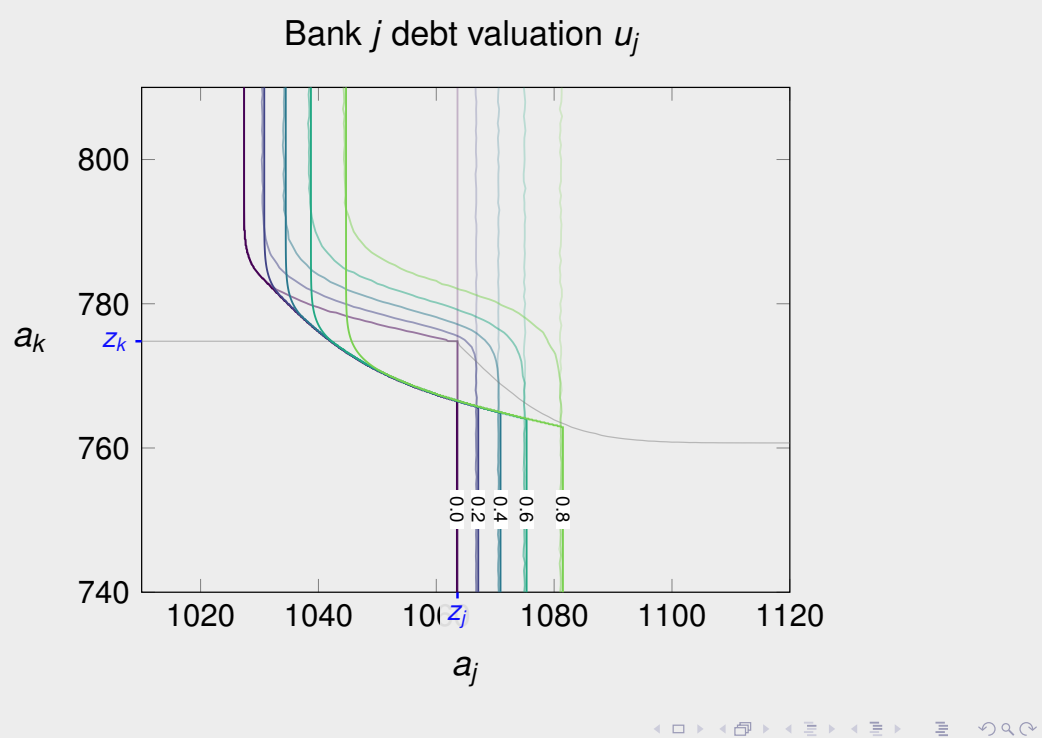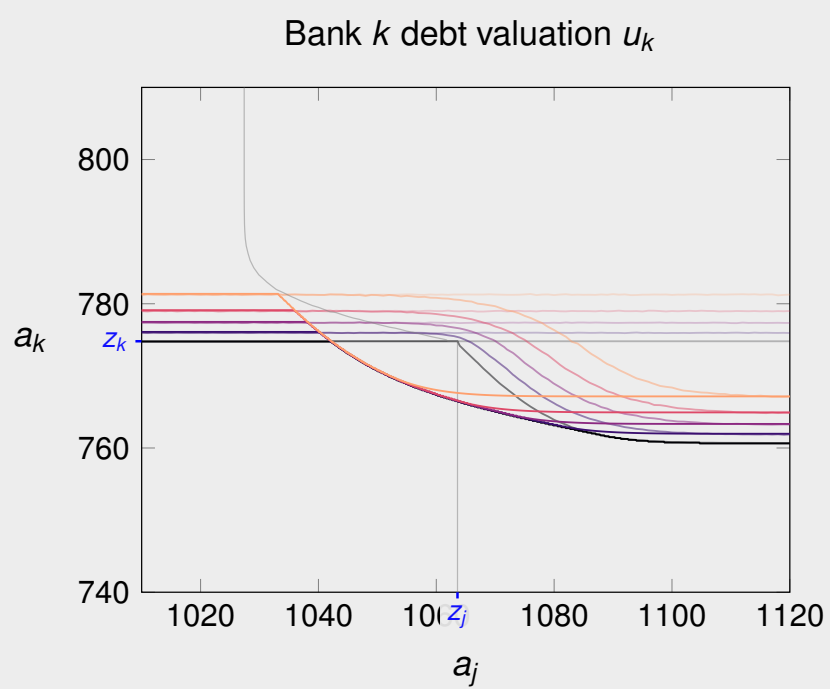
Alternative maturity profile
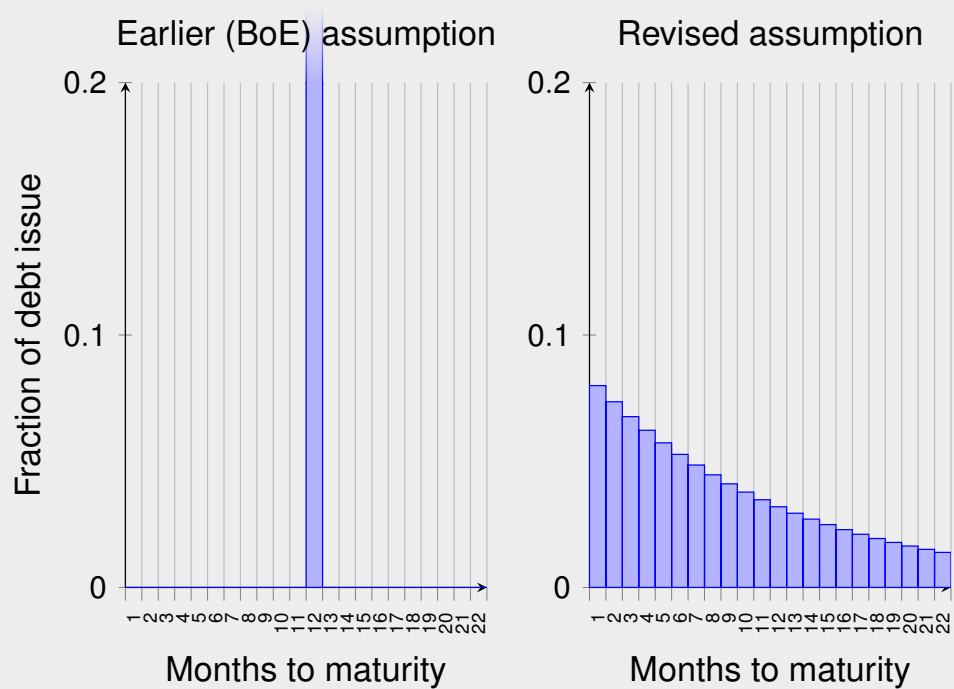
Pricing with revised maturity profile

Bank j debt valuation u_j

Pricing with revised maturity profile

Bank k debt valuation u_k
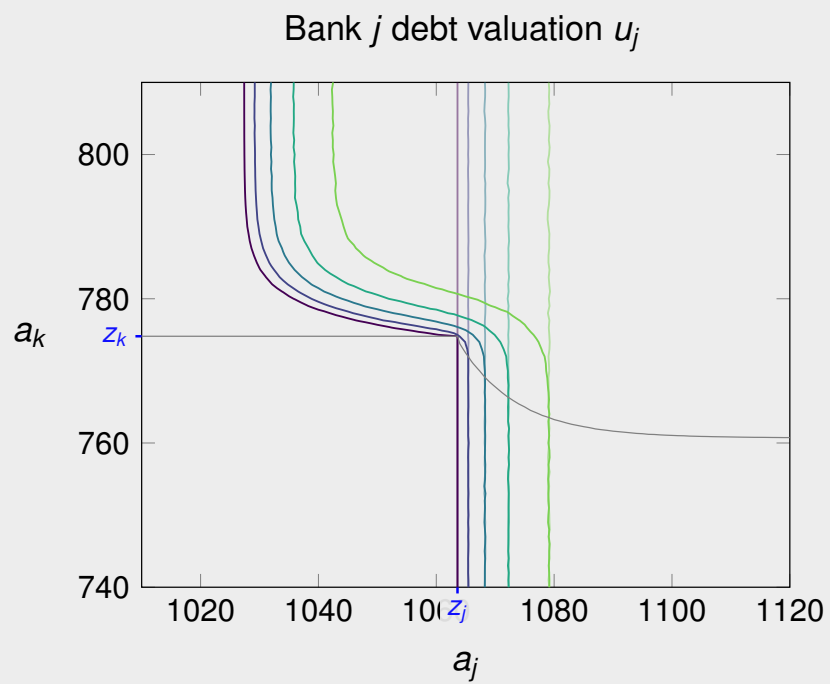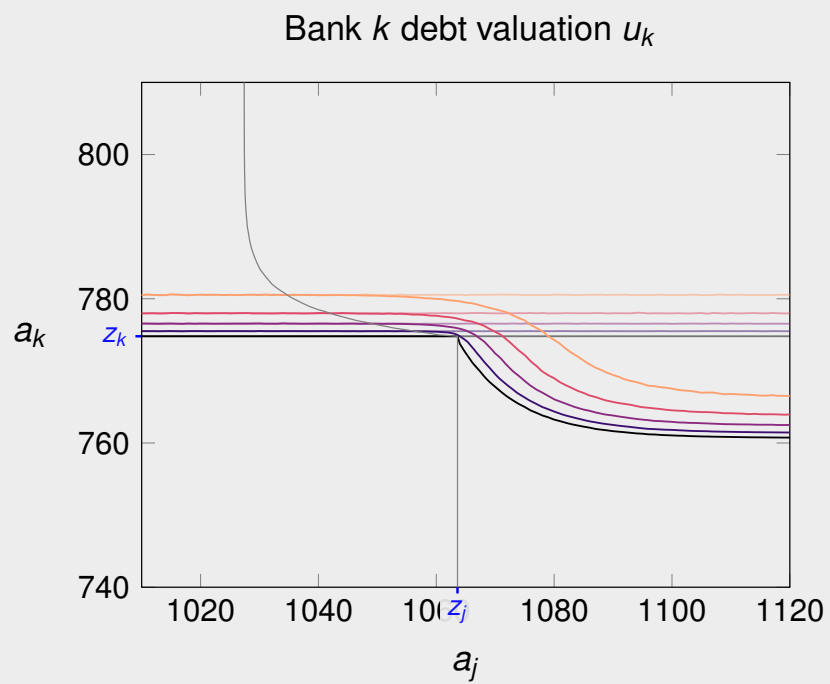
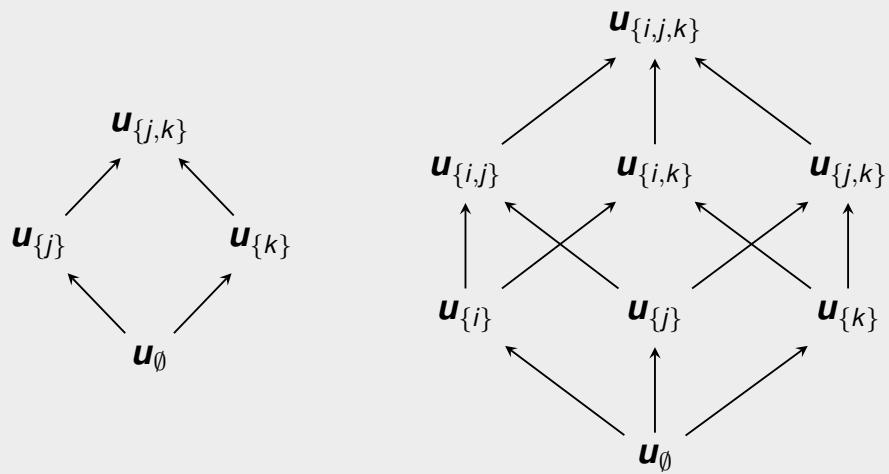# Solution dependency structure in general

Figure: Progression of the algorithm for a system of two banks and for a system of three.

A taste of the supporting theory

Theorem

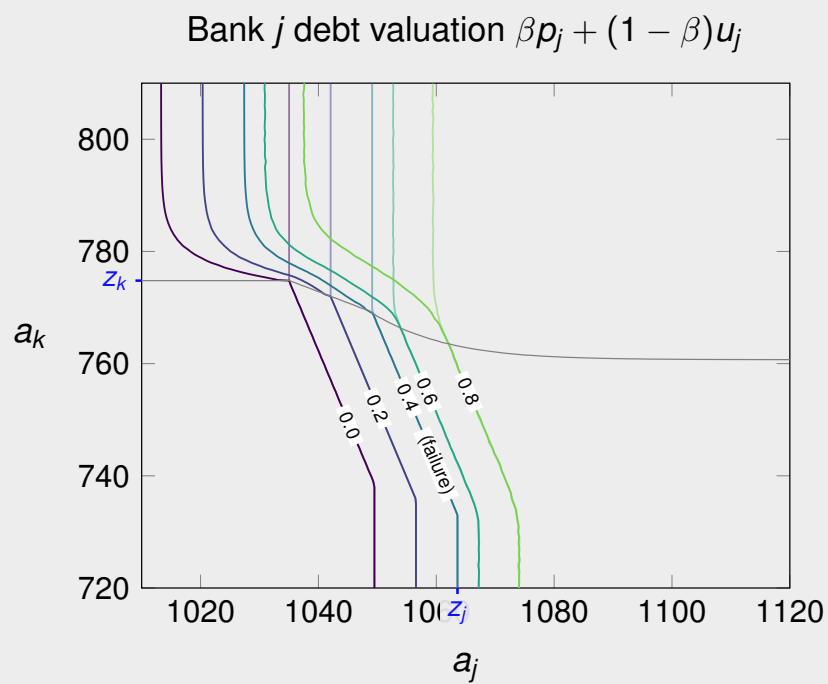*For s and s′ sets of surviving banks,*

$$s' \subseteq s \implies \boldsymbol{u}_{s'} \leq \boldsymbol{u}_s.$$

Corollary

With $\boldsymbol{E}^*$ the equity valuation function, indicating solvency,

$$\exists s' \subseteq s \quad \forall i \in s \quad 0 < E_i^*(\boldsymbol{u}_{s'}, \boldsymbol{a}, t)$$
$$\implies \quad \forall i \in s \quad 0 < E_i^*(\boldsymbol{u}_s, \boldsymbol{a}, t).$$

Pricing with recovery rate $\beta = 0.4$

Bank j debt valuation $\beta p_j + (1 - \beta)u_j$

Pricing with recovery rate $\beta = 0.4$

Bank k debt valuation $\beta p_k + (1 - \beta) u_k$