

# 'Financial Contagion'

Application of a Structural Model of Credit Risk  
to the Network of Interbank Loans

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# The importance of pricing counterparty risk

*Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one third were due to actual defaults.*

Basel Committee on Banking Supervision, 2011

# The problem of pricing risk in a network

*The Bank's **solvency contagion model** examines how deteriorating capital positions lead to revaluation of interbank debt claims, which in turn can affect banks' capital positions further ... Bank staff's judgement is that ... the overall impact on the system via this channel remains immaterial ... See Bank of England, Staff Working Paper No. 662, 'The decline of solvency contagion risk', June 2017.*

Bank of England stress testing results, 2017

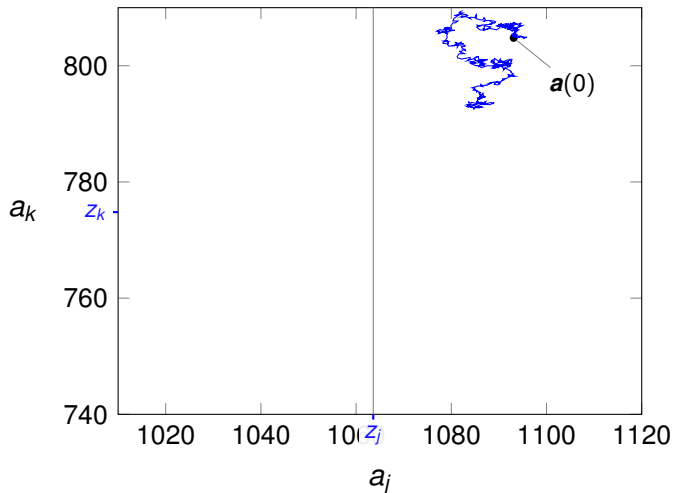
## Illustrative plots

- Two banks and zero recovery rate
- The Bank of England model

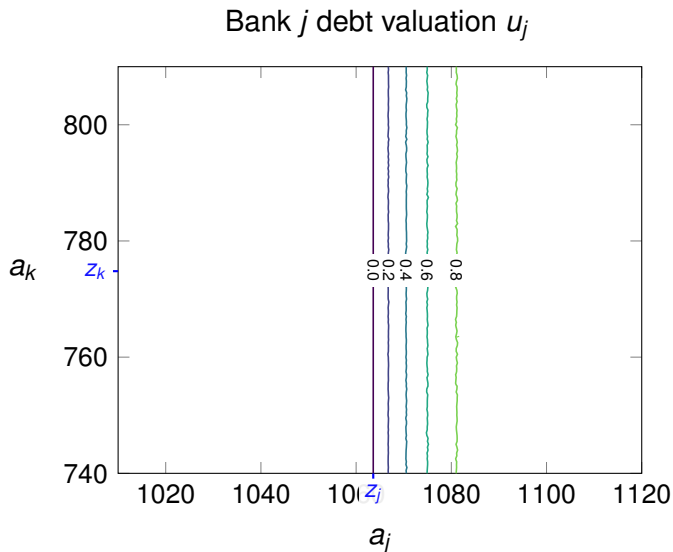
## Development

- Alternative maturity profile
- General solution structure for multiple banks
- Nonzero recovery rate

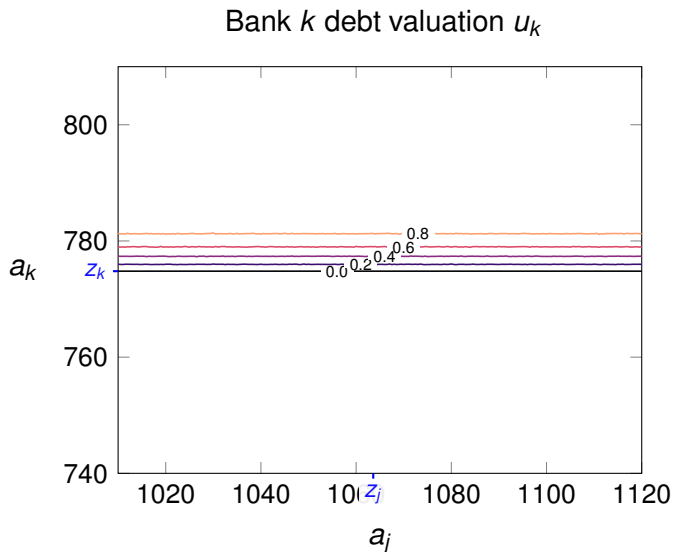
# Evolution of the real economy



# Pricing in the risk of failure



# Pricing in the risk of failure



# Balance sheets in the unconnected case

Bank  $j$

Assets	Liabilities
$a_j$	$z_j$
	$E_j$

Bank  $k$

Assets	Liabilities
$a_k$	$z_k$
	$E_k$



# Balance sheets with interbank exposures

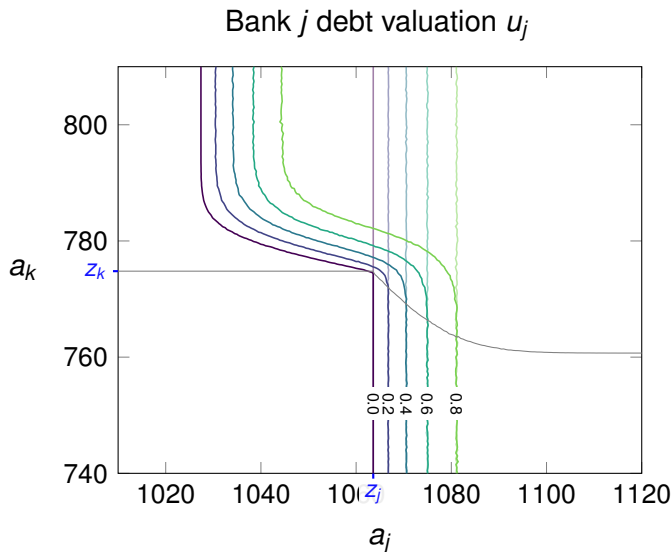
Bank  $j$

Assets	Liabilities
$a_j$	$z_j$
	$L_{jk}$
	$E_j$
$u_k L_{kj}$	

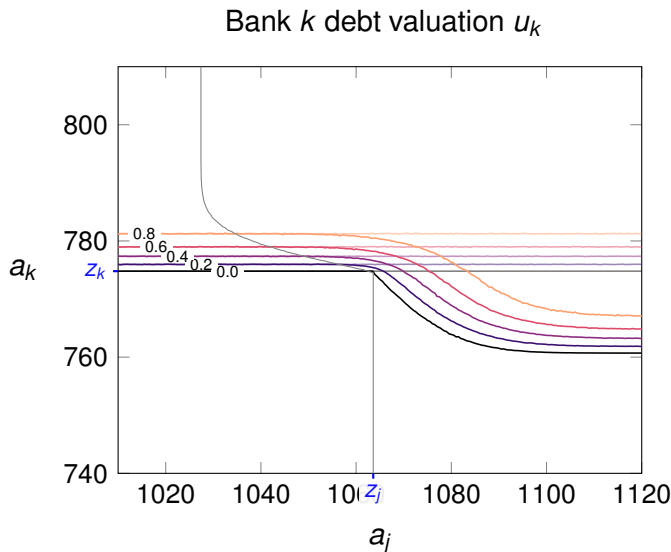
Bank  $k$

Assets	Liabilities
$a_k$	$z_k$
	$L_{kj}$
	$E_k$
$u_j L_{jk}$	

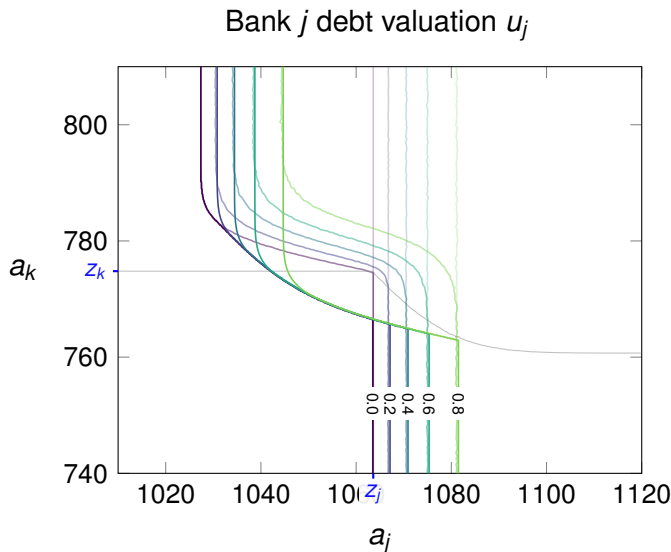
# Pricing in the effect of interbank exposures



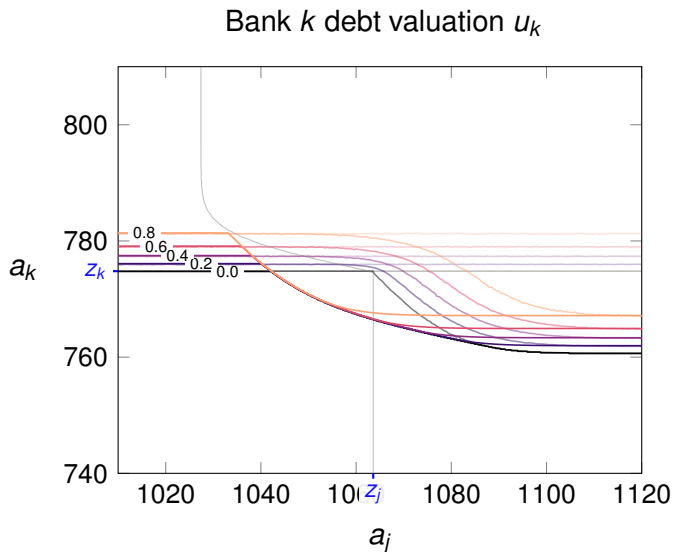
# Pricing in the effect of interbank exposures



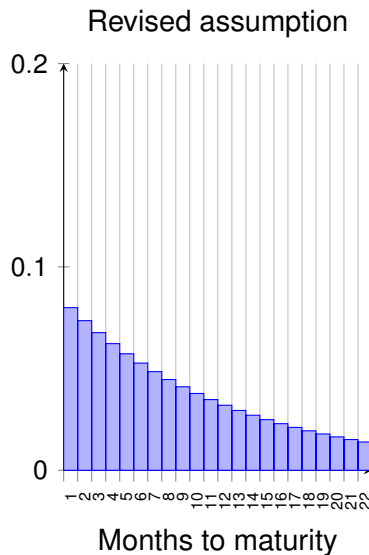
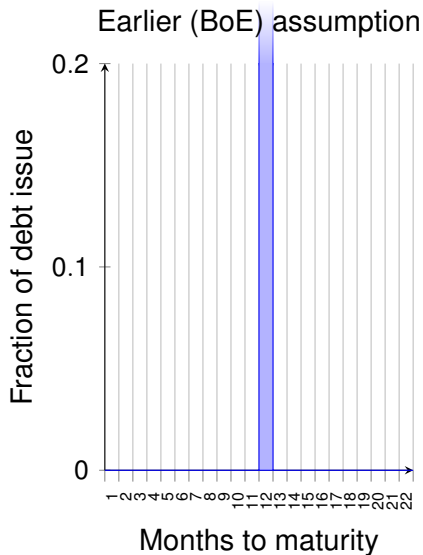
# Comparison with the Bank of England model



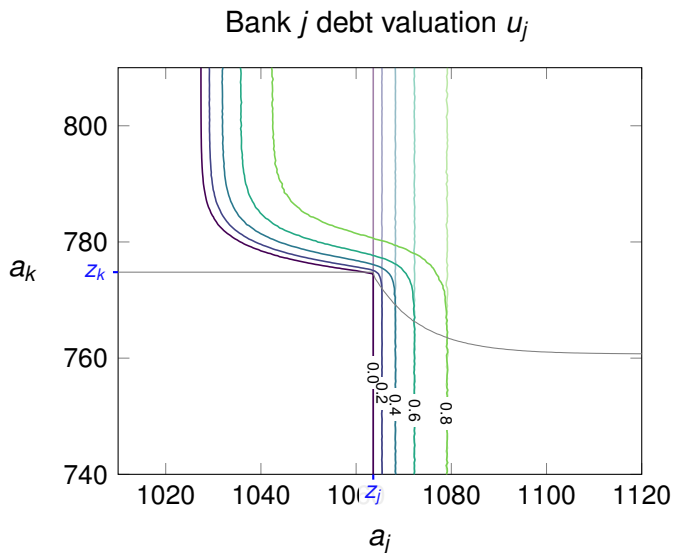
# Comparison with the Bank of England model



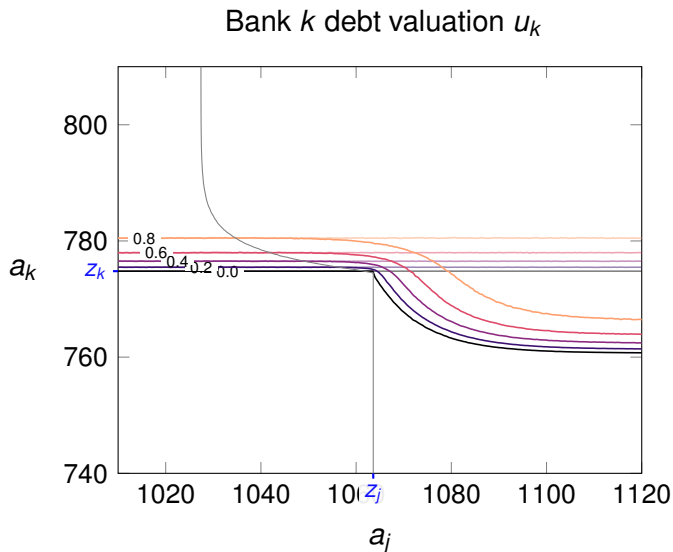
# Alternative maturity profile



# Pricing with revised maturity profile



# Pricing with revised maturity profile





# Solution dependency structure in general

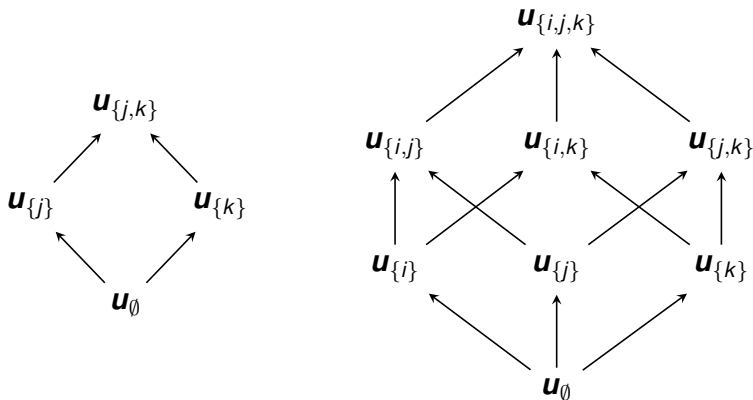


Figure: Progression of the algorithm for a system of two banks and for a system of three.

# Some supporting theory

## Theorem

For  $s$  and  $s'$  sets of (surviving) banks,

$$s' \subseteq s \implies \mathbf{u}_{s'} \leq \mathbf{u}_s.$$

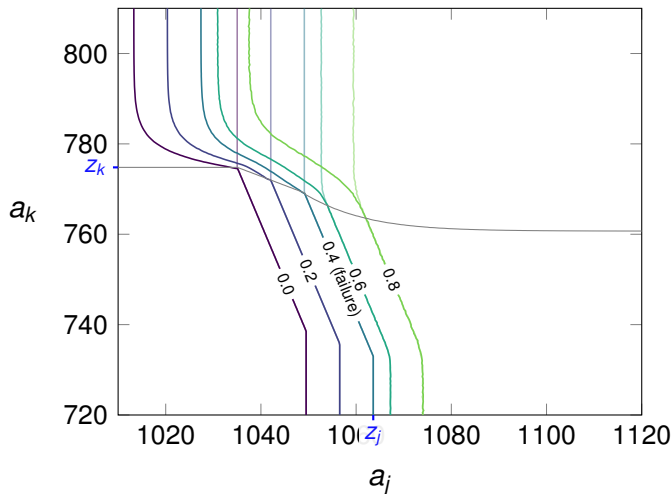
## Corollary

With  $\mathbf{E}^*$  the *equity valuation function* indicating solvency,

$$\begin{aligned} \exists s' \subseteq s \quad \forall i \in s \quad 0 < E_i^*(\mathbf{u}_{s'}, \mathbf{a}, t) \\ \implies \quad \forall i \in s \quad 0 < E_i^*(\mathbf{u}_s, \mathbf{a}, t). \end{aligned}$$

# Pricing with recovery rate $\beta = 0.4$

Bank  $j$  debt valuation  $\beta p_j / \bar{p}_j + (1 - \beta) u_j$



# Pricing with recovery rate $\beta = 0.4$

Bank  $k$  debt valuation  $\beta p_k / \bar{p}_k + (1 - \beta) u_k$

